

MATHEMATICS AT WORK

A Study of Mathematical Organisations in
Rwandan Workplaces and Educational Settings

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List of abbreviations

AT	Activity Theory
ATD	Anthropological Theory of Didactics
Frw	Rwandan francs
HSFR	Swedish council for scientific research in the humanities and social sciences
ICT	Information Communication and Technology
LP	Level of Profit
MINEDUC	Ministry of Education
MO	Mathematical Organisation
NCTM	National Council of Teachers of Mathematics
PC	Purchasing Cost
SC	Selling Cost
VAT	Value Added Tax

1 INTRODUCTION

In line with Rwanda's vision of building a knowledge based economy, there is an urgent need to get free access to basic primary education for all Rwandan citizens. To achieve the goal, an educational reform based on moving from six years to nine years of primary basic education was introduced and implemented officially in 2009. The increase of the number of students and classrooms requires a corresponding increase in the number of qualified teachers including those who can deliver mathematical knowledge. Although mathematics as a subject is given priority with the purpose to enhance the teaching and learning of science and technology (MINEDUC, 2003), few students pursue mathematics at the upper secondary school and at university. Among other reasons are based on the fact that mathematics does not seem to be seen and needed at different workplaces. Rather, the public impression is that mathematics is for those who want to become mathematicians or mathematics educators. Despite the relevance paradox which is observed in general public (Niss,1994), mathematical literacy is necessary and relevant for people

because of its concern with issues such as the goal of mathematics education, mathematics for all, the public image of mathematics, or with the role of mathematical knowledge for scientific and technological literacy (Jablonka, 2003, p. 75).

Moreover, the experience shows that even hidden or invisible to the open eyes for everyone, mathematics is used not only in the academic settings but also in several workplace settings and, therefore for the sake of challenging the discrepancy between objective social significance and subjective invisibility, these two contexts can be bridged to enhance the outcome of teaching and learning of mathematics through the context of local workplace settings. In this perspective, the current study comes in to document how mathematics teachers can be informed from their own local workplace practices to produce teaching materials for the secondary school. The study investigates specifically how mathematics that is involved in the

workplace settings is contextualised and connected to educational settings in terms of university and school mathematics classroom practices in Rwanda

1.1 Background

School mathematics is itself a form of situated learning and, thus, takes place within contexts. However, context itself is insufficient; the context must be meaningful, indeed mathematically meaningful to the learner (Massingila & Silva, 2001: p. 329).

The mentioned statement highlights the use of meaningful or significant contexts in the course of mathematics teaching and learning practices. The reason is that any society seeks to empower its subjects the quality of knowledge that is supposed to be helpful and useful for the locally specific societal development. Mathematics, as a field of knowledge, plays useful roles in society (Niss, 1994). Indeed, while investigating the adults' mathematics containing competences at work, Wedege (2004, p. 102) affirms that mathematics is integrated in workplace activities but the "transfer of mathematics between school and workplace and vice-versa is not a straightforward affair". Learning mathematics does not imply automatically that you know how mathematics is used at different workplaces (Wedege, 2004).

In this perspective researchers started to investigate how contextualised mathematics could be integrated in classroom settings to make it meaningful for learners. For instance, in Australia, Sullivan, Mousley & Zevenbergen (2003) suggest that mathematics teachers are encouraged to use contexts in order to make mathematics more meaningful and accessible for their students. In U.S, Taylor (1998) stresses that to explore mathematics through tasks which come from workplaces may support students to learn in ways that are personally meaningful. Moreover, Mohr (2008) supports that mathematics is an integral and inseparable part of daily tasks at various workplace settings. It is therefore necessary for learners to take account of both mathematical content and its contextual use in the workplace.

Although the concern of connecting mathematics to workplace contexts is focused in the Western world, in the developing world, especially in African countries, it is not very frequent. The African

educational challenge is mostly caused by reasons linked to socio-economic and political situations (Ogunnyi, 1996; El Tom, 2004; Earnest, 2004). It may imply lack of appropriate infrastructure, equipment and competent human resources in the specific field of knowledge. However, Yu et al. (2008, p. 283) realise that “the success and realisation of African Renaissance for the twenty-first century is dependent on the success of education systems in African countries”. In this perspective, the National curriculum Statement of the Republic of South Africa suggests that in school mathematical practice,

Learners must be exposed to both mathematical content and real-life contexts to develop competences. On the one hand content is needed to make sense of real life context; on the other hand, context determines the content that is needed (Republic of South Africa, 2007: p. 7).

This is to say that when it comes to assess the learners’ mathematical competencies, assessment tasks should be contextually based i.e. “based in real-life contexts and use of real-life data” (ibid., p. 7). Although Forman & Steen (2000) assume that adults rarely use much of the mathematics they learned in secondary school, Roth (2008) highlights the usefulness of teaching both content and nature of science and mathematics by providing students with opportunities to learn about how science and mathematics are practiced. In his own examples of cooking, laying tiles and hardwood floor, Roth explains how these activities allowed him to appreciate the role of the body in knowing and thus, exploited this understanding in the theories of learning and meaning with respect to mathematics in the lives of professional scientists. By then, he concluded that “what we do in everyday life generally, and how we understand ourselves specifically, mediates what we do professionally” (Roth, 2008: p. 16).

The current study focuses on the case of Rwanda as one of the poorest African countries which is nowadays facing two major challenges: (1) ensuring recovery, rehabilitation and reconciliation after the genocide of 1994 and (2) overcoming the problems associated to poverty and the massive need for sustainable development (Earnest, 2003; 2004). Those challenges are not personal rather they are collective in the sense that to deal with them requires the involvement of many people, especially a collective visible effort of Rwandan citizens.

1.2 My educational background

After the completion of my BSc (in 1994) and MSc (in 1996) in pure mathematics at the Peoples' Friendship University of Russia, I started (in 1997) to teach mathematics at the National University of Rwanda (NUR) in the department of mathematics teacher training. Three years later, I realised that there was a need to join mathematics educators to grasp more in the field of mathematics education since I was involved in a mathematics teacher training programme. From 2000 up to 2001 I went through and completed my MEd in mathematics education at the University of Western Cape in South Africa. In my MEd thesis I investigated the learning processes of university mathematics students when dealing with mathematical modelling problems. Although the problems were related to real situations, they were actually taken from a mathematical modelling book. But I dreamt to conduct research in mathematics education related to real situations. Thereafter, I came back to my old job in teacher education. The combination of having skills in both education and pure mathematics, and being involved in mathematics teacher education made me think of how to combine them in my further scientific work. Two year later, with the support of Sida/SAREC and the National University of Rwanda, I started my PhD studies in mathematics education at Linkoping University, Sweden.

1.3 Current situation of mathematics education in Rwandan schools

In Rwanda the formal education system is classified in four levels: pre-primary, primary, secondary and higher education. Schools (primary and secondary) are subdivided in three types: public (state owned), semi-independent (owned by churches) and private schools which are owned by associations. All policies related to the national education are emanated from the Ministry of Education (MINEDUC). The pre-primary education is still informal and non-compulsory because schools are created and managed under the initiatives of parents. In contrast, at the time of my data collection before 2009, primary education was compulsory and free of charge in public schools and its duration was 6 years. Secondary education comprised two levels, not free of charge. Lower secondary level was made up of three years and advanced level was also three years. In order to move from primary to

lower secondary level, from lower to advanced level, and from advanced level to higher education learners must pass national examinations.

In primary schools, learners are taught not only to read and write in general but they are also introduced to counting and elementary mathematics. At the lower secondary level, learners pursue the same timetable of different subjects and particularly in mathematics they are introduced to general elemental mathematics including geometry and algebra (such as set theory of real numbers and application in word problem solving). Finally at the advanced level, depending on their own choices and their results from national examinations, learners are grouped into different subjects such as arts and humanities (languages), professional (nursing and teaching), technical and scientific (biology-chemistry and mathematics-physics) subjects.

Public and semi-independent secondary schools are cheaper comparing to the private ones, and entrance to them is based on the learners' performance in the national examination. Due to the fact that the majority of Rwandan parents are not able to pay private secondary school fees, and the limited available places in public schools countrywide, a big number of learners in primary schools unfortunately stop their education after six years of schooling. For instance, in the following newspaper extract it is said that

.../ out of the 96,438 who passed the 2008 Primary Leaving Examination's, 20,973 pupils were selected to join different government aided schools, while at O'Level, 16,173 of the 38,527 who passed in 2008 were selected to join S.4. This was revealed by the Executive Secretary of the Rwanda National Examination Council (RNEC), John Rutayisire (The New Times, 2009).

Consequently, the rate which pupils leave school at such an early age is one of the motivating reasons for the Rwandan government to include the lower level of secondary in primary education and hence make it compulsory and free of charge. From the beginning of 2009, the duration of primary education which used to be 6 years is extended up to 9 years (Uworwabayeho, 2009). However, although the subject of mathematics is one of the prioritised subjects, in line with the Rwandan vision of building a knowledge based economy, the experience shows that there are few students pursuing mathematics at

advanced level and at universities. One reason for this is the lack of qualified mathematics teachers (Uworwabayeho, 2009). Moreover, the increase of the number of students and classrooms requires increasing the number of qualified mathematics teachers who can deliver mathematical knowledge effectively and efficiently, that is useful for the future of the beneficiaries.

1.3 Aim of the study

After the 1994-genocide, the Rwandan society was destroyed and disorganised in all sectors. In order to cater for capacity building, the Government of Rwanda has undertaken several measures in all economic sectors through its Vision 2020 for developing Rwanda into a middle-income country (Republic of Rwanda: Ministry of Finance and Economic Planning, 2000). Despite the negative consequences of the disaster, the current Government of Rwanda considers to enhance the teaching and learning of science and technology as one of several national projects for the achievement of national development. In the educational sector, the Ministry of Education (MINEDUC) has adopted the following goals: (a) eradication of illiteracy, (b) universal primary education, (c) teacher training, (d) national capacity building in science and technology and reinforcing the teaching of mathematics and sciences to provide human resources useful for socio-economic development through the education system (Ministry of Education, Rwanda, 2003; Republic of Rwanda, 2007).

In this perspective, MINEDUC suggests that teaching and learning should be not only ICT based but also context-bound to make sure that the delivered and learnt knowledge can serve the future work practice. This means that in order to serve the local society, teachers and researchers are encouraged to bring materials to the students that are taken from national local contexts. Contextualising mathematics allows students both to understand the role of mathematics in solving different workplace problems and see ways in which mathematics is used outside academic institutions. Making connections between workplace mathematical activities and classroom work supports students' mathematical thinking and learning and allows them to use mathematical concepts to interpret and understand experiences from outside the classroom (Moschkovich & Brenner, 2002). They can also

realize that such experiences can be translated into mathematical language that is taught at different academic institutional levels.

In line with MINEDUC's goals towards mathematics teaching and learning, three levels of mathematical practices are involved in the present study: the first one consists of mathematical practices that are performed by workers within their respective workplaces to generate survival means. First, examples from workplace mathematical activities with minor adaptations are brought to students who are on a mathematics teacher training programme at a university. Next mathematical tasks that are solved and re-worded by student teachers of mathematics with the ultimate purpose to adapt them for their future secondary school students are collected. Finally, the third level consists of mathematical practices that are carried out by lower secondary school students.

Workers and students of different academic levels perform their respective mathematical practices differently. Hence, mathematical knowledge is adapted from one context to another and how mathematical activities are carried out by students of different levels are core issues of the present study. It was therefore imperative to select three types of fieldwork settings: Firstly three workplace settings (taxi driving, house construction and restaurant management). The choice is based on the fact that in Rwanda there is a trend of job creation by oneself i.e independently of the people's educational background. Everyone is encouraged to make use of mind and create a small or big scale means of generating income for him/her self. Secondly a fourth year university students was chosen on the basis that in their future profession of teaching, prospective mathematics teachers should reinforce changes in mathematics curriculum to enhance the learners' deep understanding of mathematics. Finally one grade three class of lower secondary school was involved this study. This grade was chosen on the basis that before students shift from lower secondary school to upper secondary school, they should be aware in advance that mathematics is at the same time useful at work and at academic setting and this may be for some of them a significant reason to select scientific orientations for their future studies. The data collected from those fields are analysed and discussed from two perspectives: Activity Theory (AT) and Anthropological Theory of Didactics (Chevallard, 1991, 1999).

On the one hand, the workers have their own reasons and ways of using mathematics in specific workplace settings. On the other hand after adaptation, the students of two different educational settings are asked to solve contextualised mathematical tasks. The overarching aim of this study is to investigate how workplace mathematics can be contextualised and connected to university and school mathematics classroom practices so that mathematics becomes significant to the beneficiaries in both content and context. In this perspective, the current study focuses firstly how mathematics is involved in specific Rwandan workplace settings. Secondly, referring to the context in which mathematics is used at the workplace settings, the study describes and analyses mathematical contextualisation for academic mathematics purposes and how mathematical practices were organised (on practical and theoretical levels) when students of different institutional levels solved contextualised mathematical tasks.

1.4 Structure of the thesis

This first chapter introduces reason for integrating contextualised mathematics in school mathematical contexts. In the second section, the chapter outlines the challenges that the Rwandan society faces and suggested strategies to overcome those challenges. Furthermore the second section point to the need to enhance the teaching and learning of mathematics as one of the main subjects in science and technology education programme. The section ends up with the aim and overall research issues of the present study. Finally the chapter closes by structuring the content of the thesis.

In the second chapter I clarify and connect my study to the underpinning theoretical positions. In the first and second sections of this chapter, I review activity theory and anthropological theory of didactics respectively. In the third section I introduce the concept of contextualised mathematical tasks and in the final section I look at selected previous research literature related to invisibility of mathematics in the workplace settings and to in-and-out of school mathematics.

In the third chapter, I specify the research questions of the study and the methodology used to gather and analyse the data. The methodology section includes the research paradigm of the study, research design,

selection of participants, instruments used to collect data, the data collection and analysis processes as well as ethical considerations.

The fourth, fifth and sixth chapters encompasses findings and their analysis from three different fieldwork settings. In the seventh chapter I discuss the major findings from the perspectives of Activity theory (AT) and Anthropological Theory of didactics (ATD). The discussion is based on how mathematical activities are carried out at the three involved settings. Finally the chapter closes with the pedagogical implication and needs for further research.

2 THEORETICAL FRAMEWORK

In this theoretical framework part, I concentrate on different theoretical perspectives and concepts that are related to the aims and research questions of the study to analyse and discuss the findings. In the first section of this part, I introduce the concept of activity theory where a brief background of activity theory and a structure of human activity system are exposed. In the second section of this part, I embark on anthropological theory of didactics including the notion of didactical transposition and the notion of mathematical organisation. In the third section I speak briefly about contextualised mathematical tasks where the notions of mathematical task and contextualised mathematical task are defined and exemplified. Finally, related previous research on in and out-of-school mathematics is presented.

2.1 Activity theory

2.1.1 Background

Activity theory rooted from the cultural-historical theory of activity is an extension of the Russian activity theory (Julie, 2002) initiated by Russian psychologists such as Vygotsky and Leont'ev in the 1920s and 1930s. Grounded on object-oriented and artefact-mediated concepts, Russian psychologists contended that the relationship between human beings and objects of activity is mediated by cultural means, tools and signs. In other words, human actions can be described or understood with the help of the surrounding socially evolving cultural context. From that time up to nowadays, the evolution of human activity theory has been marked by three main generations (Engeström, 1996). First, instead of a simple stimulus- response model of human behaviour, Vygotsky (1978, p. 40) proposed a triangular model of “a complex mediated act”. In that model, subject, object and mediating artefacts are the major interrelated components of human activity. In his data collection he also “introduced obstacles that disrupted routine methods of problem solving” (Cole & Scribner 1978, p. 12) to challenge children's thinking.

Secondly, drawing on the work of Leont'ev, a three level hierarchical model of human activity is developed where the distinction between activity, action and operation is emphasised to delineate an individual's behaviour from the collective activity system. Depending on the level of analysis, activity theory differentiates between processes at various levels, taking into consideration the objects to which these processes are oriented.

Activity in its socio-cultural context is “an evolving, complex structure of mediated and collective human agency” (Roth & Lee, 2007, p. 198). It is a motive driven process. Activity is always oriented to *motives* of members of community who are expected to carry out the actual activity. “Motives are the objects that are impelling by themselves” (Kaptelinin, 1996, p. 108). A motive may be some material, an object or ideal that satisfies social needs for community members. A motive may be either present in perception or in the imagination or thought (Leont'ev, 1978). The shifting and developing object or motive drives activity in the sense that any activity is distinguished from another by their respective objects. This means that an activity is always connected and cannot exist without its own motive. Activity as a process evolves often over long periods and is attached to a specific socio-cultural context and time in history. However, activity is not a simple event, rather it is a complex process which “is realised by means of actions” (Daniels, 2008, p. 120).

Actions are processes functionally subordinated to activities and are directed to specific conscious *goals*. The processes of actions are channels through which the motive of activity is translated in reality. Indeed according to Leont'ev (1981, p. 59) “the basic component of various human activities are the actions that translate them into reality” purposing to achieve predefined conscious goals. Actions are carried out by individuals or groups and “are relatively short-lived and have a temporally clear-cut beginning and end” compared to activities (Daniels, 2008, p. 120). However, although objects (motives) are realised through activities as goals are realised in and through actions, goals constitute a different level of analysis which subordinate to that of activity. In this context goals differ to motives in the sense that the goals realise motives but motives give rise to goals where each presuppose the other and motives can be collective but goals are

individual (Engeström, 1999a) Moreover, objects motivate activity whereas goals are immediately directed to activity.

Actions as processes are also realised through *operations*. According to activity theory, operations are the external method used by individuals to achieve goals and are driven by the *conditions* and tools available to the action.

In a normal real-life situation, human beings can plan and predict their behaviour. However, depending on the nature, space, time and available tools for the planned behaviour, people behave differently in terms of activity, actions and operations. To understand how people behave in different settings it is necessary to take account of the status and analyse how the behaviour is driven by its motive, goal or the actual available conditions. For instance, in the provided example of hunting in Leont'ev (1981), the motive which drives the activity is that human beings are engaged in a collective hunt because they want to feed their family. Goals which drive actions is that the man performed the role of beater (the goal being to scare the prey away from himself and toward the other members of the hunting side). Conditions of hunt that drives operations will depend upon for instance the terrain, the weather, the season of year and soon.

As second generation, Leont'ev's contribution consisted of introducing the concept of division of labour to explain "the crucial difference between an individual action and a collective activity" (Engeström, 1996, p. 132) to involve the community of which subjects are members and social rules which are followed to run human activities within the community (see Fig 1). The whole procedure or process of acting to overcome the desired outcome is therefore called a 'system' of human activity. However, Leont'ev never graphically expanded Vygotsky's original model into a collective activity, (Engeström 1996).

2.1.2 Engeström's model of human activity system

Since the 1970s, the concept of activity theory took an enormous step forward in that, it turned the focus on a cultural diversity of applications where the idea of internal contradictions as driving force

of change took place in several empirical research projects (Engeström, 1996).

When activity theory went international, questions of diversity and dialogue between different traditions or perspectives became increasingly serious challenges. It is these challenges that the third generation of activity theory must deal with (Engeström, 1996, p. 133).

In the third generation the main concern of proponents of activity theory consisted of developing conceptual tools to understand dialogue, multiple perspectives and voices, and network of interacting activity systems; and the basic model of activity was expanded to include at least two interacting activity systems (Engeström, 1996), where the system stands for the whole procedure or process of acting to overcome the desired outcome of activity

Based on the ideas of the work of Marx, the model of the structure of a human activity system comprises not only subject, instrument, object, rules, community and division of labour as its main components but also production, distribution, exchange and consumption as subsystems of the activity system (Engeström, 1993).

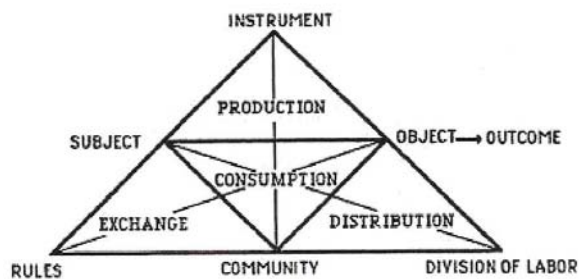


Figure 1: The structure of human activity (Engeström 1987, p. 78)

In this model, a *subject* is a person or a group engaged in an activity. An *object* is the target activity, which the subjects want to alter by using particular tools or instruments. Object is held by the subject and motivates the activity, giving it a specific direction purposing to transform the object into a desired outcome of the activity. The mediation can occur through the use of different types of artefacts or *instruments* such as physical and symbolic, external and internal mediating instruments and signs (Engeström, 1993). A *community* is

comprised of one or more people who share the object with the subject. *Rules* refer to the explicit and implicit regulations, norms and conventions to run actions and interactions within the activity system. The *division of labour* discusses how tasks are divided horizontally between community members as well as referring to any vertical division of power and status that the subjects hold in an ongoing activity system (ibid., 1993). During the course of activity, the subsystem of

production creates the objects which correspond to the given needs; distribution divides them up according to social laws; exchange further parcels out the already divided shares in accord with individual needs; and finally, in consumption, the product steps outside this social movement and becomes a direct object and servant of individual need, and satisfies it in being consumed. Thus production appears to be the point of departure, consumption as the conclusion, distribution and exchange as the middle” (Marx, in Engeström, 1987, p. 78).

Activity theory is understood in terms of closely interconnected sets of principles. Kaptelinin (1996) summarises activity theory through the following six principles: unit of consciousness and activity, object-orientedness, internalization-externalization, tool-mediation, continuous development and hierarchical structure of activity. Human activity is always goal-oriented and characterised by two major parallel actions: thinking and acting. The action is shaped by thinking and inversely through available socio-cultural tools for goal-oriented activity. Based on Marxist theory, Bannon points out that human mind and activity are always unified and inseparable. This means that

./../ human mind comes to exist, develops, and can only be understood within the context of meaningful, goal-oriented, and socially determined interaction between human beings and their material environment (Bannon, 1997, p. 1).

In activity theory, social factors and interaction between agents and their environment allow us to understand why tool mediation plays a central role. Tools shape the ways human beings interact with reality and reflect the experiences of other people who have tried similar problems at an earlier time (Bannon, 1997). Tools are chosen and transformed during the development of the activity and carry with

them a particular culture. In short, the use of tools is a means for the accumulation and transmission of social knowledge. At the same time, they influence the nature of external behaviour and the mental functioning of individuals. In other words, the behaviour or activity of human beings cannot be understood independently of their socio-cultural context.

2.1.2 Expansive learning

The concept of expansive learning that has been used and developed by Yrjö Engeström is one of the theoretical cornerstones of developmental work research. Engeström describes expansive learning as follows:

Activity systems periodically face situations in which their internal contradictions are aggravated and demand a qualitative reorganization, or re-mediation, of the entire activity. When an activity system – a workplace, for example – goes through such a reorganization and constructs a historically new mode of practice for itself, it learns something that was not there at the outset, something that no authority was able to transmit and teach. This is collective learning in which internalisation and externalisation, appropriation and creation, routinization and innovation, take place as parallel and intertwined processes. It is a type of learning that is systematically neglected in standard learning theories (1996, p. 134-135).

Expansive learning is a kind of learning in which the task is most often accomplished collectively, through the use of mediating instruments. This process manifests itself in the form of discourses, in which the participants are not necessarily supposed to be aware of some specific background of the task. The participants themselves may discover the desired outcome with help of their environmental disposition. The achievement of this goal is often a result of a long discussion. The activity is dominated by the members' discourse convictions related to the given task.

“The theory of expansive learning is based on the dialectics of ascending from the abstract to the concrete” (Engeström, 1999b, p. 382). This means that in ascending from abstract to concrete process, a basic initial idea or concept is first formed of the observed phenomenon to be learnt. This initial idea, called a ‘germ cell’,

“expresses the genetically original inner contradictions of the system under scrutiny” (Engeström, 1987, p. 245). The germ cell becomes multi-faced, enhanced and more accurate through the subjects’ engagement with the object of learning. During this engagement the initial abstract idea or concept is transformed “into a concrete system of multiple, constantly developing manifestations” (Engeström, 1999b, p. 382).

Ascending from the abstract to the concrete is not a usual method of learning. Indeed in an expansive learning process, the initial simple idea is transformed into a complex object, which is a new form of learning. Engeström (1999b) asserts that in the dialectical-theoretical thinking, based on ascending from the abstract to the concrete, an abstraction captures the smallest and simplest genetically primary unit of the whole functionally interconnected system. The expansive learning process begins with individual subjects questioning, and it gradually expands into a collective movement. The expansive learning process, in terms of dialectics of ascending from the abstract to the concrete, is accomplished through seven cyclical learning actions suggested by Engeström (1999b).

Firstly, the process starts with the action called *questioning*. The concerned participants, after being aware of the task or problem under scrutiny, have an automatic reaction of questioning, criticizing or rejecting some aspects of the accepted practice and existing wisdom.

The second stage of the process Engeström calls the action of *analysing* the situation. To analyse a situation or a phenomenon requires the involvement or intervention of the mental or discursive transformation of the situation in order to find the causes or explanatory mechanisms. Two types of analysis suggested by Engeström are historical-genetic, which explains the situation by tracing its origin and evolution, and actual-empirical, which explains the situation by constructing a picture of its inner systemic relations.

The third action in expansive learning is the *modelling* of the newly found explanatory relationship in some publicly observable and transmittable medium. This means constructing an explicit simplified model of the new idea that explains and offers a solution to the problematic of the situation (phenomenon).

Examining the model is the fourth stage of expansive learning. At this stage the participants need to run, to operate and to experiment

with the constructed model in order to fully understand its dynamics, potentials and limitations. This action is followed by the action of *implementing* the model. This supposes a fifth action of concretising the model by means of practical applications, enrichments, and conceptual extensions.

The sixth and seventh actions are those of *reflecting* and *evaluating* the process and consolidating its outcomes into a new, stable form of practice.

In sum, activity theory is relevant as a tool for analyses in the present study. The empirical parts are situated in three different settings. The first elaborates the use of mathematics by workers at three different workplace settings (taxi driving, house construction and restaurant management). The second and third empirical parts focus on mathematical task solving and task posing activities in educational settings. To understand the data collected from those three settings, comparative analyses are performed using theories of human activity where the role of mathematics employed in each setting is illuminated.

However, as the aim of the current study is to grasp not only the mathematical task solving and task posing processes related to workplace contextualised tasks but also to understand how mathematical activities are mathematically organised at both practical and theoretical levels, there is a need to use a complementary perspective, with the potential to account for such issues. For this reason, the next section of this chapter is dedicated to a brief overview of Anthropological theory of didactics, which will be used for this purpose.

2.2 Anthropological theory of didactics

2.2.1 Didactic transposition

From the beginning of the 1980s when the proponents in the field of mathematics education were mostly immersed actively in the cognitive development or genetic epistemology learning perspective (Piaget, 1968), and the socio-cultural learning perspective (Vygotsky, 1978), Chevallard (1985) developed the basic ideas on the didactical transposition. Through several research contributions (see Bosch & Gascon, 2006), didactic transposition has now reached the point where

mathematics activities are investigated at both practical and theoretical levels in terms of the anthropological theory of didactics (Chevallard, 1999).

Whenever a given society needs to develop, knowledge and society are interacting pillars (Barnet, 1994). Any society comprises a number of institutions where different pieces of knowledge are produced, used, adapted and transformed to be taught and learned. These pieces of knowledge are products of human innovations which function differently depending on targeted purposes in a focused institution. In the endeavour of spreading these products from one institutional context to another through the channel of teaching and learning processes, stakeholders for a given discipline, adapt and transform them. Since the 1980's this process of adapting and transforming objects of knowledge from one institution to another was called 'didactic transposition' by Chevallard (1991).

Un contenu de savoir ayant été désigné comme savoir à enseigner subit dès lors un ensemble de transformations adaptives qui vont le rendre apte à prendre place parmi les objets d'enseignement. Le travail qui d'un objet de savoir à enseigner fait un objet d'enseignement est appelé la transposition didactique (Chevallard, 1991, p. 39).

[A designated content of knowledge in terms of knowledge to be taught undergoes a series of adaptive transformations which then enable it to be able to take place among the objects of teaching. The work which transforms a teaching object from a knowledge object is called the didactic transposition]

The core assumption of didactic transposition bears upon associating the knowledge to be taught and learned within institutional practices. The didactic transposition perspective is therefore

aimed at producing a scientific analysis of didactic systems and is based on the assumption that the (mathematical) knowledge set up as a teaching object (savoir enseigné), in an institutionalised educational system, normally has a pre-existence, which is called scholarly knowledge (savoir savant)" (Klisinska, 2009, p. 13).

However, it is important to notice that there is a big difference between scholarly knowledge and the taught knowledge in classrooms. Indeed from the didactic transposition point of view, the *actual taught* and *available learned knowledge* in classrooms is produced and generated from outside of school environments. It is thus transposed and adapted from *scholarly knowledge* via *knowledge to be taught* (Bosch & Gascon, 2006). In this perspective, the object of didactic transposition consists of describing and explaining phenomena of transformation of knowledge from its original production to its teachable state. For instance Chevallard (1991) provides an example of teaching the concept of distance in mathematics. Within this example Chevallard (ibid., p. 40) confirms that the notion of distance between two points was spontaneously used all the time but points out that the distance as a mathematical concept or object of mathematical knowledge, was introduced in 1906 by the mathematician Maurice Fréchet. Afterwards, the scholarly mathematical knowledge of that concept passed through a number of comprehensive and adaptive transformations and since the 1970s it was introduced in the secondary school programmes in France as mathematical knowledge to be taught and learned. In the process of didactic transposition the scholarly mathematical knowledge as it is produced and used by mathematical scholars, faces a series of transformations by the noosphere members (Chevallard, 1991) with the purpose of making it teachable and understandable for the beneficiaries (mathematics students). The noosphere members could be for instance: a set of experts, politicians, curriculum developers, educators, textbooks, didactical materials and recommendations to teachers.

To analyse the mathematical knowledge as set up by the teacher or a researcher in mathematics education (a member of the noosphere), one must consider the institutional conditions and constraints under which the knowledge to be taught is constituted (Bosch & Gascon, 2006). Those conditions and constraints usually guide the mathematics educator while reconstructing or transforming the mathematical scholarly knowledge to come up with the teachable mathematical knowledge. Some of those constraints and conditions are for instance the kind of questions that are asked, such as Why teaching this? What will the beneficiaries gain from this kind of knowledge?

In the present study, mathematical practices involved in different workplace settings represent a contextualized mathematical work to be

experienced by students while working with mathematics in school, in order to enhance the cultural relevance of school mathematics. Among different ways to try to achieve this, an approach focusing on tasks has been chosen in this thesis. As a consequence, the kind of problems at the workplaces which are solved by the workers with mathematical tools and techniques will need to be transposed to tasks for school mathematics. As the aim is to "teach" a mathematical knowledge pre-existing outside the teaching institution, in this case a contextually based or situated knowledge, such task construction can be described as a didactic transposition process. However, in contrast to the national curriculum work for school mathematics, the noosphere and its role in this 'micro' process are clearly defined in terms of the researcher and the students involved.

In the current study, the types of didactic transpositions first concern the description of the contextualisation of the collected workplace mathematics materials to be given to the university mathematics student teachers in terms of tasks. Thereafter, a description of their formulated tasks to be given to secondary students follows. However, the study does not stop there; it goes further to investigate how the contextualised workplace mathematics tasks are solved at both technico-practical (know-how) and technologic-theoretical (know-why) levels. These two concepts will be detailed in the following subsection in terms of mathematical organisation or mathematical praxeology

2.2.2 Mathematical organisations

The scope of the theory of didactic transposition was in the mid 1990s widened into the anthropological theory of didactics by studies of the ecology of mathematical knowledge within institutions. The unit of analysis used for such studies was set up by the notion of a mathematical organisation. The theoretical model from the anthropological theory of didactics (ATD), views teaching and learning as a human activity situated in an institutional setting (Chevallard, 1999b; Bosch & Gascon, 2006). By engaging in this activity, the participants elaborate a target piece of knowledge for which the activity was designed. This perspective sets focus on the knowledge itself as an organisation system (a *praxeology*), including a

practical block (know –how) of types of tasks and techniques to work on these tasks, and a *theoretical block (knowledge or know-why)* explaining, justifying, structuring and giving validity to work in the practical block (Barbé, Bosch, Espinoza & Gascon, 2005). This is to say that in order to solve any type of task or problem within an institutional context, the available appropriate technique is more or less explained and justified by the theoretical discourse related to why it is reasonable to apply the chosen technique. Chevallard explains this in the following quote (Chevallard, 1997, p. 14):

En toute institution, l'activité des personnes occupant une position donnée se décline en différent type de tâches T, accomplis au moins d'une certaine manière de faire, ou technique, τ . Le couple $[T / \tau]$ constitue par définition, un savoir - faire. Un tel savoir-faire ne saurait vivre à l'état isolé :il appelle un environnement technologico-théorique $[\theta / \Theta]$, ou un savoir (au sense restreint), formé d'une technologie, θ , « discours » rationnel (logos) sensé justifier et rendre intelligible la technique (technê), et à son tour justifié et éclairé par une théorie, Θ .

[In any institution the activity of persons occupying a given position takes the shape of different types of tasks T, accomplished by means of at least a specific way of acting, or technique, τ . The couple $[T / \tau]$ constitute by definition a know-how. Such know-how cannot live in an isolated state: it requires a technological- theoretical environment $[\theta / \Theta]$, or know-why (in a restricted sense), consisting of technology, θ , a rational « discourse » (logos) supposed to justify and make the technique (technê) understandable, and in turn to be justified and clarified by a theory Θ].

In this quote, Chevallard illuminates the structure of an institutional body of knowledge. This structure includes two inseparable parts while engaged in any kind of activity: a practical-technical part $[T / \tau]$ and a technological-theoretical part $[\theta / \Theta]$.

Le système de ces quatre composantes, noté $[T / \tau / \theta / \Theta]$, constitue alors une *organisation praxeologique* ou *praxeologie*, dénomination qui a le mérite de rappeler la structure bifide d'une telle organisation, avec sa partie pratico-technique $[T / \tau]$ (savoir-faire), de l'ordre de la praxis, et sa partie technologico-théorique $[\theta / \Theta]$ (savoir), de l'ordre du logos (Chevallard, 1997, p. 14)

The system of these four components, written $[T / \tau / \theta / \Theta]$, constitutes a *praxeological organization* or *praxeology*, a naming which illustrates the two-part structure of such organization, with its practical-technical part $[T / \tau]$ (know-how), at the level of praxis, and its technological-theoretical part $[\theta / \Theta]$ (know-why), at the level of logos.

According to Chevallard (1999b) at the basis of a praxeology there is the notion of task that normally belongs to a set or type of tasks T , i.e. the artefacts constructed within an institution. In the praxeology the level of task or type of tasks T is mostly recognised through by the use of verbs such as compute, find, solve, construct and so forth. To solve or find out the answer to the constructed type of tasks requires at least one way to solve it, i.e. an applied technique τ (know-how). However, while solving T , it is important to justify that the used technique τ works and explain why it works. Of these two roles of the technology θ in mathematics, the function of justification dominates over the function of explanation (Chevallard, 1999b, pp. 226-227). The third role of technology is the production of new techniques (ibid. 227). The role of theory, Θ , while solving the type of the tasks T , is similar to the one of technology, i.e. justification, explanation and production.

To analyse the institutional didactical processes through the ATD framework, Chevallard (1999b, p. 228-229) classifies organisations (organisations) as point (ponctuelle), local, regional and global. Most often within an institution, a given particular type of tasks, T , defines a triplet of technique, technology and theory. A point (ponctuelle) or specific organisation is generated by a unique type of tasks. But these kinds of organisations are very rare. A local organisation is generated by the integration and connection of several specific organisations i.e. when a type of tasks can be solved through the use of different techniques. A regional organisation is obtained as the result of the coordination of several local organisations with a common theory. A global organisation emerges when several regional organisations are added together. In a structural common understanding, at the regional and global level, the technological-theoretical component dominates over the technico-practical component. This is due to the fact that the type of tasks T genetically precedes the technological-theoretical block $[\theta / \Theta]$, which is constructed with the purpose to produce and justify the use of appropriate technique τ to T . In other words, $[\theta / \Theta]$ allows generating τ for the given T . For that reason, $[T / \tau]$ may be viewed as

an application of the discursive component $[\theta / \Theta]$. To clarify this, the following example is provided in Chevallard (1999, p.229):

Dans l'enseignement des mathématiques, un thème d'étude (« Pythagore », « Thalès, etc. ») est souvent identifié à une technologie θ déterminée (théorème de Pythagore, théorème de Thalès), ou plutôt, implicitement, au bloc de savoir $[\theta / \Theta]$ correspondant, cette technologie permettant de produire et de justifier, à titre d'applications, des techniques relatives à divers types de tâches. On notera cependant que d'autres thèmes d'étude (« factorisation », « développement », « résolution d'équations », etc.) s'expriment, très classiquement, en termes de type de tâches.

[In the teaching of mathematics, a theme of study (« Pythagoras », « Thales, etc. ») is often identified as a specific technology θ (the theorem of Pythagoras, theorem of Thales)], or rather, implicitly as the know-why $[\theta / \Theta]$ corresponding to this technology, allowing to produce and justify, as applications, the techniques in relation to different types of tasks. In this context one can also note that other themes of study (« factorisation », « expansion », « solving equations », etc.) are classically expressed in terms of types of tasks.]

This organisation of knowledge can be used to describe very systematic and structured fields of knowledge (such as mathematics or any experimental or human science) and its related activities, with explicit theories, a fine delimitation of the kind of problems that can be approached and the techniques to do so. Considering the mathematics teaching and learning process, we can find two different, intimately related, kinds of organisations: mathematical ones, corresponding to the subject knowledge taught, and didactical ones, corresponding to the pedagogical knowledge used by teachers to perform their practice. For the purpose of the present study I look into the mathematical organisations observed in the different settings.

2.3 Mathematical tasks

2.3.1 *The notion of mathematical task*

A mathematical task can be viewed, in general terms, as any piece of mathematical work to be done by an individual or a group. In mathematics education, especially in teaching-learning context, a mathematical task normally refers to mathematical work or problems that are assigned to students, teachers or other concerned people (such as parents and mathematics curriculum makers) to be performed for the purpose of societal knowledge development in the subject of mathematics. For instance, on the one hand, at home students learn to solve their homework tasks. Furthermore, parents can formulate appropriate mathematical tasks or problems to be coped with when helping their children to perform better. On the other hand, at school, mathematics teachers most of the time test mathematical performance of their students through various assigned tasks.

With reference to Doyle's academic task definition (1983), Stein et al. (1996) define mathematical task as a classroom activity that "includes attention to what students are expected to produce, how they are expected to produce it, and with what resources" (Stein et al., 1996, p. 459-460). A mathematical task is a "classroom activity, the purpose of which is to focus students' attention on a particular mathematical concept, idea, or skill" (Henningsen et al. 1997, p. 528). In this perspective Stein et al. (1996) concur with Henningsen et al. (1997) that a mathematical task can be viewed as passing through three phases: 1) as represented in curricular or in instructional materials, 2) as it is set up by a teacher in a classroom, and 3) as it is implemented by students in the classroom. In this perspective, the product or outcome of the process which combines the above mentioned three phases constitutes the students' learning outcomes (ibid.).

Within the second phase, while setting up a mathematical task, teachers must take into account two dimensions: task features and cognitive demands. Here the task features refer to identified aspects of tasks by mathematics educators as important considerations such as multiple solution strategies, multiple representations and mathematical communication for enhancement of mathematical understanding, reasoning and meaning making during the implementation phase.

Cognitive demands refer to two types of thinking processes: one that is suggested by the teacher to solve the task while setting it up and one in which students engage during the implementation phase (Henningesen et al., 1997). For example, if the teacher wishes to investigate the performance or skills of the students in solving the quadratic equation $ax^2 + bx + c = 0$ with $a \neq 0$ (a polynomial equation of second degree) as it may be required in the mathematics curriculum; the teacher may state the mathematical task as follows: Solve the following quadratic equation $ax^2 + bx + c = 0$. In this case, depending on the level or grade of the students, the teacher will expect students to check if the constants a , b and c are real or complex and therefore to proceed to find out the roots (solutions) of the quadratic equation. In addition the teacher will expect students to use correctly the quadratic formula $x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ where x_1 and x_2 are the first and second roots (solutions) of the quadratic equation, respectively.

Mathematical tasks play a core role to mathematics learners in the sense that they convey messages about “what doing mathematics entails” (NCTM, 1991, p. 24). Findings from studies in mathematics education reveal that tasks in which students engage provide the contexts in which they learn to think about mathematics, and different tasks may place differing cognitive demands on students (Doyle, 1983; Hiebert et al., 1993; Marx & Walsh, 1988 & Henningesen et al., 1997). Indeed “the nature of tasks can potentially influence and structure the way students think and can serve to limit or to broaden their views of the subject matter with which they are engaged” (Henningesen et al. 1997, p. 525). To some extent this is to say that by engaging in doing various mathematical tasks, students develop their sense and way of doing mathematics. For instance a variety of mathematical tasks in calculus, algebra and geometry facilitates students to broaden their mathematical views and at the same time the way in which the task is stated or the type of mathematical task implies how the engaged students invest their cognitive demands in mathematics. A study on academic work (Doyle, 1983, pp. 162-163) points out four general categories of academic tasks:

(1) *Memory tasks* in which students are expected to recognize or to reproduce information previously encountered. For example in mathematics a question such as ‘enunciate the Pythagorean theorem’ requires students to reproduce word by word the formulation of that theorem as it is stated in mathematics books or as it was stated by the teacher in the classroom during the lesson.

(2) *Procedural or routine task* in which students are expected to apply a standardised and predictable formula or algorithm to generate answers. For example: ‘Solve a set of linear equations’, ‘compute the limit or derivative of the following functions’, are mathematical tasks that require students to use the known appropriate algorithms and formulas.

(3) *Comprehension or understanding tasks* in which students are expected to (a) recognise transformed or paraphrased version of information previously encountered, (b) apply procedures to new problems or decide from among several procedures those which are applicable to a particular problem (for example solving mathematical word problems), and (c) draw inferences from previously encountered information or procedures such as for instance devise an alternative formula for squaring a number.

(4) *Opinion tasks* in which students are expected to state a preference for something. For instance, it could be a task that requires students to select a favourite story which can help to illustrate a mathematical situation.

In this study, I will not focus on all these four types of tasks; rather my focus is oriented to the comprehension or understanding tasks. In fact, as it is stated in the aim of the present study, the starting point of the study consists of investigating phenomena related to the use of mathematics at three different workplace settings. Those phenomena were subsequently transposed in written form with appropriate adaptations in terms of mathematical tasks and were assigned to university mathematics student teachers. To deal with those kinds of tasks students must direct their “attention to the conceptual structure of the text and to the meaning that the words and sentences convey” (Doyle, 1983:163). During the solving process, ideas represented in the surface structure of a text are abstracted from their immediate context and organised into high psychological functions. In this case students “must build a high-level semantic structure or schema that can be

instantiated in several ways as particular circumstances demand” (ibid., p. 164). Although comprehension tasks are of various types, such as open ended, exemplifying, closed and context based (word problems), in the current study I will focus especially on contextualised word problems in the next subsection.

2.3.2 Contextualised mathematical tasks

Studies in mathematics education have shown that it is imperative to encourage mathematics teachers to include authentic or realistic word problems (Palm, 2009) that are provided to their students (Kramarski et al., 2002; Palm, 2008). Niss (1992, p. 353) describes an authentic-extra mathematics situation as “one which is embedded in a true existing practice or subject area outside mathematics, and which deals with objects, phenomena, issues, or problems that are genuine to that area and are reorganised as such by people working in it”. The reason to include contextualised mathematical materials in school is that, “textual descriptions of situations are assumed to be comprehensible to the reader, within which mathematical questions can be contextualised” (Verschaffel et al., 2000, p. ix). Moreover, they “are equally important to ensuring that learners perceive that mathematics does contribute to working at and resolving issues of living” (Burton, 1993, p. 12). Therefore contextualised mathematical tasks establish a relationship between school tasks and real life situations in the sense that they are understood as context-based tasks, and at the same time they facilitate students to engage in critical thinking and reasoning and to use tools which may be at their environmental disposition. The common feature of contextualised mathematical tasks is based on the fact that there are no ready-made algorithms to solve them as is the case for standard tasks (Forman et al., 2000; Kramarski et al., 2002). Depending on the nature of the task and the subject to whom the task is assigned, each authentic task can be approached in different ways and requires the solver to be skilled with a wide range of mathematical knowledge.

However, to study the relationship or concordance between a school task and a real life task or an out-of-school situation, Palm (2009) suggests a framework which focuses on the central idea that “if a performance measure is to be interpreted as relevant to real life performance, it must be taken under conditions representative of

stimuli and responses that occur in real life” (Fitzpatrick et al., 1971, cited in Palm, 2009, p. 8). In this context, it is assumed that the venture of developing tasks with the above mentioned relationship may be viewed as a matter of *simulation* where *comprehensiveness*, *fidelity* and *representativeness* are seen as fundamental concepts of the framework (Palm, *ibid.*).

Comprehensiveness refers to the range of different aspects of the situation that are simulated. *Fidelity* refers to the degree to which each aspect approximates a fair representation of that aspect in the criterion situation. *Representativeness* refers to the combination of comprehensiveness and fidelity” and is “used as technical term for the resemblance between a school task and a real-world task situation” (Palm, 2009, p. 8).

In this framework, Palm (2009) emphasises that the restriction of comprehensiveness is always necessary in simulation or modelling processes because while simulating, it is impossible to simulate all aspects involved in a situation in the real-world and this implies that we cannot expect from the simulated out-of-school situation that the conditions for the solving of the task will be exactly the same in the school situation.

The more the task is closer to the situation source, the degree of fidelity to it is greater and consequently the more the concordance or resemblance between a school task and a real-world task is represented.

2.4 Previous research on in-and-out of school mathematics

In striving to grasp how people conceptualise the role and practice of mathematics in their work, studies related to how people behave in workplace settings revealed diverse findings. On the one hand some of them provide two types of discrepancy: 1) between objective relevance and subjective irrelevance, 2) between workplace mathematics practice and school mathematics practice. On the other hand, in other sets of studies the use of formal mathematics strategies is clarified.

2.4.1 Objective social significance and subjective invisibility of mathematics

From the everyday people's practice point of view, mathematics plays various important roles. It is used at workplace settings, learned and practiced at academic and school settings. However, studies reveal that despite its social significance, mathematics seems to be invisible and unrecognised by the general public including those who work in extra-mathematical fields (Niss, 1994). For instance, Noss (1998, p. 3) argues that

by carefully studying what kind of problems people actually solve in a variety of workplaces,/.../ there exists a rich source of mathematical activities which people exhibit in their working lives, even though these are flatly denied by those involved”.

The same idea is argued by Wedege (2002) where she observes two possible conflicting approaches within the *subject* area of adults and mathematics. Wedege suggests that the objective society's requirements and the subjective adults' needs for math-containing competences are two possible lines of research because people are *very* convinced that mathematics is important in society but they confirm at the same time that they do not use it and do not need it in their life. The main argument behind this is that mathematics is not seen on surface in working practices as it is displayed in classrooms. Indeed, in many cases the users have social goals instead of mathematical *educational* ones. In that context the pragmatic social perspective is more focused because if the mathematical knowledge taught “*becomes less relevant to working practices*” comparing to how it is observed in school contexts, then “*working practices show less evidence of making use*” of it (Noss (1998, p. 3). In the current modern society, where ICT is a driving tool in many workplaces, the use of mathematics is hidden in technology and “*mathematics as visible tool disappears in many workplace routines*” (Wedege, 2002, p. 27).

2.4.2 Workplace mathematics and school mathematics

Over the last thirty years, researchers have done studies with the aim to investigate how mathematics in everyday practices differs from what is taught at school and in academic institutions. In this endeavour Lave

(1988) found that mathematics practices in everyday settings were structured in relation to ongoing activities. Based for example, on the use of shoppers' "best-buy" strategies, she points out that mathematical practices in workplaces do not require any imposed regulation. Rather, adults use any available resources and strategies which could potentially help to solve a problem. Also, in a collection of studies related to informal and formal mathematics, Nunes, Schliemann and Carraher (1993) found that there was a discrepancy between street mathematics practices and school mathematics practices. This is demonstrated through a mathematical test which was given to the same children who performed better out of school than in a school setting. This discrepancy is due to the fact that at school children tried to use formal algorithms whereas in real situations they did arithmetic based on quantities. It should be noted though that the requested arithmetic procedures were quite simple. In the results from a study related to college mathematics and workplace practice, Williams et al. (2001) found that the conventions of school and workplace graphs might be different. Indeed, in a chemical industry, school graph knowledge was not enough to allow a college student to interpret a graph of chemical experiments. However, the college student was able to interpret it with the help of an experienced employee. In a recent study, Naresh and Presmeg (2008) followed a bus conductor in India in his daily practice, where they observed that though he performed significant mental mathematical calculations the bus driver's attention was fully concentrated on the demands of his job, making his mathematical work more or less invisible to him.

2.4.3 Use of formal mathematics strategies

Researchers have also done studies with the aim to find out what mathematical concepts and processes that are used in different workplace settings. In a study on mathematical use in a group of carpenters, Millroy (1992) found that not only are many conventional mathematical concepts embedded in the everyday practices of the carpenters, but their problem solving is enhanced by stepwise logical reasoning similar to the reasoning used in mathematical proofs. Also, a study by Massingila (1994) revealed that mathematical concepts and processes are crucial in carpet laying practices such as estimation and installation activities. Furthermore, she found that measuring and

problem solving are two major processes in the carpet laying practice. Abreu (1999b) also found that Brazilian sugar cane farmers used indigenous mathematics to control their income. However, over time, technological innovations in measuring quality requested change to more school-like problem-solving strategies which made farmers prone to abandon traditional units of analysis and value their children's success at school mathematics.

In their exploratory study related to how mathematics is used and described in workplaces in the context of employees in an investment bank, paediatric nurses, and commercial pilots, Noss, Hoyles, and Pozzi, (2000) found that practitioners use mathematics in unpredictable ways. Hence, their "strategies depend on whether or not the activity is routine and on the material resources at hand" (p. 17). However, the same others (Noss et al., 2001) found that experienced nurses use a range of correct proportional-reasoning strategies in their study about proportional reasoning in a nursing context.

A common point to all these studies is that mathematical strategies that are used at workplaces differ from those taught at academic institutions. A mathematical strategy for solving a problem refers to a 'roadmap' that consists of identifying the problem to be solved and the appropriate technique(s) that allow solving that kind of task. In some of the above mentioned studies mathematical strategies are described as applied by the participants without details about how they are used or may be underpinned by mathematical justifications. In others, mathematics is seen as a tool to mediate human activity through the lens of the participants' goal achievement with clear mathematical justifications (Noss et al., 2000). However, in these studies the same participants act both as workers and students. That is, their mathematical strategies are investigated in one authentic workplace setting where they are competent and in one educational setting which is abstract and less familiar. In contrast, the aim of the current study emphasises to investigate mathematical organisations embedded in mathematical activities found in Rwandan workplaces and educational settings where the participants are part of their authentic settings and are solving similar tasks in ways relevant for their specific settings.

3 RESEARCH QUESTIONS AND METHODOLOGY

3.1 Aims and research questions

The research problem of the present study concerns ways to contextualise school mathematics within cultural mathematical practice in Rwanda. Hence, in line with the above background, the overarching aim is to investigate how workplace mathematics can be contextualised and connected to university and school mathematics classroom practices so that mathematics becomes significant to the beneficiaries in both content and context. Also, there is an interest to meet a theoretical challenge that attempts to combine the second and the third generations of activity theory with Chevallard's anthropological theory of didactics to find out what types of knowledge could be generated. The following research questions are posed:

- (1) How is mathematics involved in workplace and educational activity systems?
- (2) What mathematical organisations are used when solving mathematical tasks in workplaces and educational settings?
- (3) What transpositions are made when contextualising mathematical tasks from workplaces to educational settings?

3.2 Research methodology

3.2.1 The study in relation to an interpretivist research paradigm

A research activity primarily involves the discovery or creation of knowledge that was not previously known or understood. With respect to the aims and research questions, the research findings allow getting knowledge about the involved population sample, phenomena and events. For the sake of this purpose, the engaged researchers should be

clear on the ontological, epistemological and methodological assumptions (Guba & Lincoln, 1994; 2000) because the three axiomatic components are interrelated in terms of the research process. The ontological assumption is needed for the purpose of seeking the form and the nature of the reality embedded in the researched phenomenon or event, whereas the epistemological assumption emphasises the relationship between the knower and what can be known. When it is assumed that the relationship is established, the methodological account comes in to clarify how the knower goes about finding out whatever s/he believes can be known. In other words, while ontology involves the philosophy of reality (existence), epistemology addresses how individuals come to know about that reality and at the same time methodology identifies the particular practices used to achieve knowledge about it (Krauss, 2005).

The interpretivist paradigm is one among several research paradigms that contrast with the positivist paradigm. Interpretivist researchers make a clear distinction between the subjects of social sciences (people and their institutions or organisations) and the ones of the natural sciences (positivist). An interpretivist research view claims that

social reality has a meaning for human beings and therefore human action is meaningful – that is, it has a meaning for them and they act on the basis of the meanings that they attribute to their acts and to the acts of others that is the job of social scientist to gain access to people’s ‘common-sense thinking’ and hence to interpret their actions and their social world from their point of view” (Bryman, 2004, p. 14).

From this quotation it is clear that an interpretivist point of view requires a researcher to know not only the people’s (participants in the research) intentions and motives but also how they understood or interpreted the situation or phenomenon. That is why researchers call the outcomes “subjective meanings of those whom they are researching that is, the different understandings and interpretations which the participants bring with them to the situation” (Pring, 2000, p. 98). People perceive and thus construe the world in ways which are often similar but not necessarily the same. Therefore there can be different understandings of what is real. Interpretivist researchers reject the positivists’ view that the social world can be understood in terms of general statements about actions. According to them, the descriptions

of human actions are based on social meanings, and people living together interpret the meanings of each other, and these meanings change through social interaction.

Interpretivism rests upon idealism. Indeed, first the social world is interpreted through the people's mind and reality is a social construct of the human mind (Bassey, 1991). Second, the social world cannot be described without investigating how people use language and symbols to construct what social practices and experiences mean for them. This means that it is only when a researcher comes to understand the individuals' experience and their subjective interpretation that s/he begins to understand why social actors behave in particular ways. This implies that any social explanation is complete when it describes the role of meanings in human actions. In the interpretivist view, the major issue is to understand, not to explain and predict as it is in a scientific (positivist) view.

The interpretivist research paradigm involves an ontology in which social reality is the product of processes by which social actors negotiate meanings of and for actions and situations. In its epistemology, knowledge is derived from everyday concepts and meanings. The interpretivist researcher enters the everyday social world in order to grasp the socially construed meanings, and then construct these meanings in a social scientific language. In other words, understanding should begin from the presupposition that there is at least common ground between the researcher and the researched. It begins from a commonality that assumes that from the shared experience, the researcher is required to empathise.

The current qualitative study may therefore be underpinned by an interpretivist research paradigm. Indeed the study is embedded in mathematics education, a research field which traditionally utilises methodologies of the scientific research paradigms (Ernest, 1994). This is to say that researchers in that domain make use of certain philosophical assumptions about what there is (ontology), how and what we can know (epistemology) and the appropriate methods for gaining knowledge (methodology). Interpretative research paradigm seeks to explore real human and social situations and uncover the meanings, understandings and interpretations of the actors involved.

In the current study, firstly the research seeks to uncover what kind of mathematical problems workers meet and how they solve them.

Secondly, it seeks to understand how mathematics student teachers solve tasks related to workplace and design tasks for secondary school students and finally to understand how secondary school students solve tasks designed by using culturally embedded workplace contexts. From the data observed, interviews, participants' group discussions and produced written works, the reality 'the use of mathematics at workplace and solving and designing mathematical tasks', the knowledge about 'participants' mathematical techniques used and justified will be uncovered through the interpretation of 'the participants' meanings and actions. Furthermore as far as the researcher interacted with the participants while gathering data, this implies that knower (researcher) and the known are interactively linked.

3.2.2 Research design

A research design is a logic plan for getting from here to there, where here may be defined as the initial set of questions to be answered, and there is some set of answers about these questions. A research design is a plan that guides the investigator in the process of collecting and analysing research evidences. It is a plan that guides a researcher to deal with what question to study, what data are relevant and how to analyse the results (findings). In qualitative research, the choice of methods of data collection and analysis should be coherent and consistent with the research questions (Yin, 2003; Bryman, 2008). Indeed research questions are so crucial in the sense that they guide researchers, to take decisions about the kind research methods or instruments to employ, what data and from whom they should be collected, and the underpinning literature for data analysis.

Following the interpretivist research paradigm guidelines the current qualitative study explores the things in their natural settings, purposing to make sense of or to interpret events or phenomena in terms of meanings the research participants bring to them. In qualitative research, social phenomena and their meanings are continually being negotiated by social actors (Bryman, 2004). In qualitative study the researchers "seek answers to questions that stress how social experience is created and given meaning" (Denzin & Lincoln, 1994, p. 4) Qualitative research emphasises on processes that are not rigorously measured in terms of quantities.

In this study I approached, observed, and listened to workers and students while they were at their respective workplace settings. I then interpreted their meanings which they were attributing to the processes of using and experiencing to solve and contextualise workplace mathematics. In this perspective, no measurement was required, and research questions are answered according to the contextual understanding of the point of view and written works of the participants. Ontologically, this research study is located in a constructionist position instead of an objectivist position as the realities are local and specifically constructed by the participants. Epistemologically it refers to an interpretivist position where the findings are subjectively created instead of a positivist.

3.2.3 Selection of participants

After the final decision on the aims and research questions of the study, I went first to a taxi driving station based in Kigali (capital of Rwanda) and contacted five taxi drivers to participate in the study. Three of them refused for their own reasons. Fortunately one of the two who accepted was driving his own car and the other one was employed. I could contact more than two but with the interviews I conducted with them, I felt that I had enough information for designing one task for the mathematics student teachers. Secondly, I contacted a builder who works privately in his own small construction company. The reason was based on the idea about how mathematics facilitates a private person to manage his own house construction activities. The restaurant manager was selected for the same reason I contacted three but only one accepted. From the raw data I got from both the house builder and the restaurant owner, I was able to design two tasks related to their businesses. The second category of the case study is a sample of 16 mathematics student teachers at a HEL. Two of them are females and 14 are males. The selection was made from a class of fourth year mathematics student teachers. There was no special criterion of selection because after a short introduction about the nature, aims and research questions of the study, 16 of them responded positively to participation.

The third category of the case study is a third year class of 37 secondary school students. 24 of them were female and 13 of them are male. The participants' age varied between 14 and 20 years with the

exception of one of them who was 32 years old. The choice was made on the basis of the national mathematics curriculum for secondary school with respect of the designed tasks. Table 1 summarises the number of participants at different field settings.

Table 1: Number of participants at different field settings

Setting	Number of participants		
	Females	Males	Total
Taxi driver	0	2	2
House construction	0	1	1
Restaurant management	1	0	1
University students	2	14	16
Secondary school pupils	24	13	37
Total	27	30	57

3.2.3 Instruments

The instruments used to collect the data varied depending on the settings with respect to the related research questions. First, from all my chosen field work settings, I made observations and took field notes. Individual semi-structured interviews were performed at the three workplaces (taxi driver, house construction and restaurant management). In addition, the house builder and the restaurant owner made two pieces of written work for me (one page each) where they provided authentic examples of their previous activities related to mathematics. Second, at the university, I tape recorded group discussions, semi-structured group interviews and collected the participants' written work (produced written work) of their respective given tasks. Finally, at the secondary school setting, I also tape recorded a semi-structured class interviews and collected the students' written solutions to given tasks (see interview guides in Appendix 1) The instruments used in data collection process are summarised in Table 2 below.

Table 2: Instruments used in the data collection process.

Method \ Setting	Observations and fields notes	Individual semi-structured interviews	Group discussions	Participants' written work	Semi-structured group/class interviews
Taxi driver	X	X	-	-	-
House construction	X	X	-	X	-
Restaurant management	X	X	-	X	-
University	X	-	X	X	X
Secondary school	X	-	-	X	X

3.2.5 Data collection

First of all, four workers from the three workplace settings volunteered to participate in this study. Three visits were made at each workplace. The purpose of the first visit was to inform the participants why and how I wanted them to be involved in my research. On this occasion, they agreed that I was permitted to observe and interview them about the use of mathematics in their daily activities. On the second occasion, after three weeks, the purpose was to observe and make the first semi-structured interview in order to understand how mathematics helps the workers to achieve their goals in their respective work sites. Three months later, a third visit was conducted to strengthen the understanding of the mathematical organisations within the workplace settings. On that occasion, supplementary semi-structured interviews and observations were conducted and two pieces of written work from the house builder and restaurant owner respectively were collected. The interviews were performed in Kinyarwanda, a common language to all involved parties. Field notes were taken and interviews were tape recorded, transcribed and translated into English at all visits.

Second, at the first step of the second phase of the data collection, I visited to the fourth year (final) university mathematics students and requested them to participate in my research. After a short introduction about the nature, aims, research questions, and the kind of data I wished to collect; 16 of them (two females and 14 males) out of 36 responded positively to participation and we agreed to meet three weeks later for solving contextualised mathematical tasks. I split the

participants in four groups with four participants in each group. I then agreed with them that the first and second group could come in the morning and the third and fourth groups in the afternoon.

Before the participants of the first and second groups started the process of solving tasks related to taxi driving and restaurant management, I introduced them to the nature of the tasks and what is going to be done: read the task, discuss how to solve it, and a write down report. Moreover, they were told that after the mathematical problem solving activities, I would like to have group interviews with them so that they can explain verbally about the mathematical techniques (Chevallard, 1999b) they used while solving the tasks and what they think about inclusion of that type of materials in their teaching programme. I also asked for permission to tape record the sessions. The group discussion lasted about an hour and a half and the semi-structured group interviews lasted about half an hour.

In the afternoon of the same day, I did the same with the participants of the third and fourth groups who were dealing with the tasks related to taxi driving and house construction. The group discussions and semi-structured group interviews were performed in a mixture of French and predominant Kinyarwanda whereas the written work was in French. Field notes were taken; group discussions and interviews were transcribed and translated into English in both groups.

Finally, at the secondary school, I first made a visit to the principal of the school with the purpose of addressing my research concern. After being permitted, I met the students of the third year and informed them about why and how I wanted them to be involved in my research. On the second visit, I split the students in 18 groups of two students each with the exception of one group which was constituted by three students. Thus, in total I had three tasks and each task was solved by six groups. The students were seated in a manner so that the neighbouring groups were solving different tasks. Each group received one task and they submitted one copy of their produced written work at the end of the problem solving process by the time assumed by curriculum standards. Afterwards I conducted a class interview. Tasks and produced written materials by students were written in French whereas the interview was done in Kinyarwanda because it was a common language to all involved parties. Field notes were taken and interviews were tape recorded, transcribed and translated in English at

all visits. The design of the data collection process is summarised in Figure 2 below:

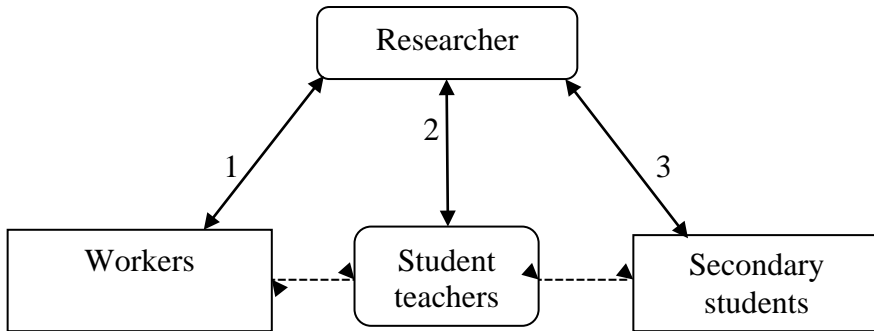


Figure 2: Research design for data collection

In the current study, I am focusing the filled arrows because in Rwanda it is still problematic while in the western countries this issue is already sorted out. In other words, Rwandan researchers in mathematics education are at the starting point for encouraging the understanding of connecting real world practice and mathematical word activities.

3.2.6 Data analysis

In the analysis of the collected data I will use the mentioned different conceptual framework such as activity theory, anthropological theory of didactics and contextualised mathematical word tasks. In the analysis of data from workplace settings, I have used ideas from activity theory in which I draw on the object of activity to elucidate mathematics as one among the involved mediating tools in the activity. This part of the analysis illuminates the mathematical problems that are embedded in the workers' activity. Regarding how mathematics is used by workers on workplaces, the analysis draws on ideas of the anthropological theory of didactics (ATD), especially on its notion of mathematical organisation (MO). To perform this analysis I will build on a reference MO (Bosch & Gascon, 2006, p. 57), based on my own knowledge of academic and applied mathematics, in order to be able to analyse the observed MO in the workplace settings and on the interview data. In the analysis of data from university and secondary school settings I am using ATD and activity theory perspectives.

Also in the analysis of data from university and secondary school settings I will draw on activity theory and ATD. Here the concept of didactic transposition as well as the notion of MO will be used. The didactic transposition to be analysed comprises a three-step process:

- by the researcher from collected information about workplace mathematics to contextualised mathematical tasks for student teachers;
- by the student teachers from given contextualised mathematical tasks, and the experience of solving those tasks, to contextualised mathematical tasks for secondary school students;
- by the researcher from the combination of collected information about workplace mathematics, own constructed contextualised mathematical tasks for student teachers, and given contextualised mathematical tasks constructed by student teachers for secondary school students, to contextualised mathematical tasks for secondary school students.

To compare the outcomes of the three didactic transpositions a reference will be made to a qualitative measure by way of the notion of representativeness (see section 2.3.2) in relation to the source of the transpositions (as presented in chapter 4), and to a quantitative measure in terms of number of words (in the English translation, measured by the word count function of the text editor software; the French originals do not differ much) used in task formulations. In addition, as one of the aims of engaging in contextualised activities in school mathematics is to educate students about the role of mathematics in society, including the development of a critical attitude towards its use (cf. Jablonka, 2003), elements of the constructed contextualised tasks that enable such critical discussion will also be compared.

3.2.3 Ethical considerations

For the sake of research ethics in this study, I referred to ethical guidelines set up by the Swedish council for scientific research in the humanities and social sciences (Swedish HSFR, 1990). During the three phases of my data collection, the participants who took part of the study can be split in three categories: workers from three workplace settings (taxi driving, construction and restaurant management settings), university students and secondary school students. First of all, at each workplace setting, I made a first visit with the purpose of negotiating with workers to allow me to get information

about their ways of using mathematics while their work activities are ongoing. During that visit before I requested them to participate in my study. I explained to them the research interest of my study and why I wanted them to be involved in it. At the same time, the workers were informed that they were allowed to agree or to disagree to participation and to withdraw at anytime. After their agreement, I guaranteed them the confidentiality that no one shall know their real personal identity. Therefore in the next part of the study they were anonymously represented by drivers A and B, the builder and the restaurant owner respectively. Furthermore, they were guaranteed that the product of the research project is of scientific purpose.

Secondly, the information that I got from the workplace settings was supposed to be given to the university students. But before that, at the first step of data collection of the second phase, I had a meeting with the fourth year university mathematics students. During that meeting, they were introduced to the purpose and aim of my study. Thereafter, I explained why and what kind of data collection I wished to get from them for my research project. In addition they were made aware that the participation is voluntary and the confidentiality is guaranteed in written and oral forms. In fact the names that are mentioned in the thesis are not their real names rather they are fictional. Also participants were informed that they can withdraw from participation at any time and that their contribution would be for scientific purposes.

Thirdly before I met the secondary school students, visit to the principal of the school was made with the purpose of addressing my research project and request to be permitted to collect data from her school. Like in the two previous data collection phases, information, agreement, confidentiality and usage were guaranteed in any form of transmission. Once again for the sake of confidentiality, the secondary school students were given the fictional names in the interviews.

4 MATHEMATICS USE AT THE WORKPLACE SETTINGS

To grasp the motives that are the basis for workers to involve mathematical knowledge, the current empirical part of the study reports findings and analyses of data from the observed activities at three workplace settings. It provides knowledge about the mathematical basis the workers use to perform their daily activities in their respective workplaces. First, ATD is used for analysing mathematical organisations at the respective workplaces. Secondly, the participants' information that concern mathematics and other aspects of their work is analysed using activity theory in line with Leont'ev and Engeström's models.

4.1 At the drivers' workplace

The taxi driving profession in Rwanda is mostly exercised by citizens with limited school background. The majority of taxi drivers consider the driving license as their core means of generating income. Some of them drive their own cars whereas others are employed. Taxi driving is mostly done in towns where you find financially potential people able to use taxi as a means of transport. Rwanda has not yet an explicit policy or norms and regulations that taxi drivers should follow to charge their customers. Because of lack of taximeters in the cars, the cost is negotiated between the taxi driver and the costumer.

From the transcripts of the interviews conducted with two taxi drivers, an employed (A) and a car owner (B), their main concern seems to be a non fixed level of profit and to avoid the risk of loss. Due to the difficulty of determining the number of customers every day, the estimation of costs depends mainly of considering control of factors such as road condition (good/bad), trip distance (in kilometres), quantity of petrol that the car consumes for a given trip (measured by money spent), waiting time (if necessary), and the time of the day (different day and night tariffs). Following an agreement between driver A and the employer, A was not responsible for expenses such as taxes, insurance, spare parts and so on. Also, A and his employer had agreed that A must deposit 5000 Frw every day to B and A's monthly

salary was 30000 Frw. When the drivers were asked about their mathematical reasoning process while estimating costs, they always referred to authentic examples like pre-fixed estimations and rounded numbers without detailed calculations. In the interview, A gives an example of how he calculated the costs for a trip Kigali – Butare on a high quality tarmac road.

Interviewer: OK. Let's take an example. Has it happened to you that you have taken a client from here [Kigali] to Butare?

Driver A: Yes, many times.

Interviewer: Could you explain to me how you have estimated the price?

Driver A: A one way of that trip is about 120 kilometres. The estimated cost for that trip was 30000 Frw. It means that I considered the cost of the petrol about 12000Frw and I remained with 18000 Frw /.../ But sometimes it happens that while I am on my way of returning back, I meet customers and depending on how we negotiate the cost I charge him 3000 or 5000, it depends /.../ But when estimating the price with the customer before the departure, I ignore this case because there is no guarantee to have this chance.

This extract shows that the estimation of cost was made with respect to the cost of petrol and the driver's profit only. Road conditions were probably not mentioned as both interviewer and interviewee were assumed to be familiar with it. Transports between Kigali and Butare are frequent as contacts between the National University in Butare and official administrators or foreign aid agencies and others in Kigali take place on a daily basis. The next example is taken from a less frequented distance.

Interviewer: OK. OK let's take the case of a Kigali- Bugesera trip. Although the road is now becoming macadamized it was always used as a non macadamized road. How much do you estimate for instance, when you bring somebody there?

Driver A: The distance is almost 50 kilometres...then the return trip is 100 kilometres. But because of the poor road conditions, the cost is estimated at 15000 Frw. In that case I assume that the car is going to consume petrol for 5000 and I remain with 10000.

In the above extract, the estimation of the trip cost was made according to road condition, cost of petrol and the driver's profit. A seems to assume that more petrol is needed if the road is of bad standard but looking at Example 1 the same unit (10 km for 500 Frw) is used. However, in Example 2 the driver does not seem to expect to be able to pick up a new passenger for the return trip.

In the second interview with B, the owner of the taxi, he explains how he estimates costs in relation to distance, price of petrol and time.

Interviewer: Let me ask you one explanation... for example, when you charge a customer a cost of 1500 Frw ... what is your basis for that price?

Driver B: Do you remember I told you that with the petrol of 1000 Frw, I usually go 20 kilometres? Now when the customer tells me the destination I start to think of the number of kilometres to reach there. Then you say this time one litre of petrol costs for example, 550 Frw... Approximately my car consumes 50 Frw to go one kilometre. This means that to go a distance which is not more than 10 kilometres for a return trip my car uses 500 Frw. So if I transport the customer to that destination without any waiting time I should have 1000 Frw for a work time less than 20 minutes... Do you get my point?

Like driver A, B calculates with rounded thirds, one third for petrol, one third for time spent and one third as a profit. As he is the car owner he could also have calculated with taxes and other costs involved with keeping a car.

To characterise the MO observed in this taxi driving workplace setting, the type of problems involved could be described as varying versions of calculating the value of a function symbolically written as $W = F(x, y, z, t) + P$, where W is the estimated cost that the driver suggests to the customer. This cost consists of a non-fixed profit P and a cost F for the driver, estimated from all or a few of the four variables road condition (x), covered distance (y), petrol consumption (z) and time (t). Referring to the examples shown above, in the case of waiting for the customer the problem simplifies to $W = F(t) + P$, while the

case with a short distance on a bad road will increase both the time and petrol needed: $W = F(z(t(x))) + P$. When the road is good but the distance longer it is the distance which is the deciding variable, $W = F(z(t(y))) + P$, which in the case of also a bad road changes to $W = F(z(t(x, y))) + P$. The techniques used by the drivers to solve these different types of problems are based on rounded estimations of basic costs, without providing a rationale of the amounts mentioned, and when needed elementary arithmetic operations are performed on these rounded numbers. For example, for the Kigali-Butare trip the model $W = F(z(t(y))) + P$ was used, with $y = 2 \times 120$ km and $W = 30000$ Frw with $z = 12000$ Frw and $P = 18000$ Frw. In the case of the Kigali-Bugesera trip the road was not macadamized and thus in a bad condition and the model $W = F(z(t(x))) + P$ was applied, where $W = 15000$ Frw and $P = 10000$ Frw with $y = 2 \times 50$ km. Technologies included number facts of addition and subtraction of natural numbers, and simple multiplication facts such as doubling. All numbers used were contextualised with units of distance and currency and no justification of the mathematical techniques used was referred to. Rather, it could be described as silent knowledge, adopted by experience and exchange with colleagues.

4.2 At the builder's workplace

In contrast to the taxi driver profession, becoming a builder by profession requires a minimum of upper secondary level. Although in Rwanda there are big construction companies, you also find people working on individual basis, and sign contracts with a person/institution who wish to build a small house/building. However before the agreement takes place between the two counterparts, it should be understood that it is a question of competition between different builders for earning the job. The one who suggests the lowest cost is the one who gets the job. The concern of the current study is to understand how the builder uses mathematics to determine the estimated cost to become an earner of the job.

From the transcripts of interviews conducted with a house builder, his main concern is to get a certain level of profit and avoids risk of loss. For that reason, the builder estimates cost and profit margin after

considering local conditions of availability of materials, labour and prices. Secondary, the use of books, calculator and mathematics (geometry, algebra, trigonometry etc.) which is learnt at school, facilitate the computations of the cost of each part of the house such as for instance foundation, elevation of walls, wall painting, doors, and windows. However, to make sure that those calculations are performed in a right manner, it is better to reassure that the real quantities are found according to the plan of the house that is intended to be built. According to the builder, the construction of the house requires considering many factors but the consideration of (1) the quantities of materials per unit (2) the costs unit for each material delivered to the building site (3) the payment of local labour (4) builder's profit margin (a certain percentage of the purchase and transport cost of the delivered materials to the building site) and (5) taxes are very crucial for calculating the estimated cost. Furthermore, the builder revealed that the estimated cost includes two main components: the installation cost and the cost of the building site itself.

Interviewer: /.../ you remember last time you explained how you proceed to estimate the cost of the building site when you wish to earn the job. However, I found that we did not speak about while estimating that cost how the taxes and installation of the building site are included...

Builder: Well as I told you last time, to calculate the costs per unit I calculate the quantity of materials per m^3 and surface area per m^2 but in our job we consider also other costs such as the installation cost of the building site /.../

Interviewer: /.../ Mmm, what does it include?

Builder: /.../ For that additional post, we include the costs of the number of visits to the building site which should be made by the chief or other involved persons, the costs of temporal toilets for local labour and the costs of the temporal office which will be removed by the time the construction will be finished. In short in our job we predict the installation price and the price of building site itself. But all those costs are included in the estimated cost which is submitted to the job provider.

Regarding the relationship between the payment of taxes and the estimated cost, the builder explained it in the following extract:

Interviewer: How do you include taxes in your calculations?
 Builder: / / In my job it is only the Value Added Tax (VAT) which is included in our calculation.
 Interviewer: Oh really could you tell me...
 Builder: Yes /.../ now regarding /.../ there is exempted and non-exempted markets. For the non-exempted markets we calculate on our basis the unit costs and we add the total cost for VAT. Briefly this VAT is paid by the job provider”.

When asked to illustrate how the calculations are performed, the builder explained by providing, in a written way, an authentic example related to the calculation of the estimated cost for brickwork of cement blocks realised in November 2006 as follows:

An example of estimated costs of brickwork of one m³ of cement blocks

Regarding the quantities of materials per unit, I considered one m³ of cement blocks as unit of calculation of materials. From my experience, in one m³ of cement blocks, you find approximately 50 cement blocks.
 At that time, the cost of one block was estimated at 400 Frw. Therefore the cost of one m³ was estimated at $50 \times 400 = 20000$ Fr.

Regarding the cost per unit of material for the brickwork of 50 blocks we needed cement, sand and some water. The needed quantity of cement for that brickwork was estimated at 90 kg and the cost of one kg cement was fixed at 150 Frw. Therefore, the needed quantity of cement was estimated at $90 \times 150 = 13500$ Frw.

The needed quantity of sand was estimated at 0,65 m³ and the cost of one m³ of sand was estimated at 8000 Frw. Therefore the needed quantity of sand was estimated at $0,65 \times 8000 = 5200$ Frw. The needed quantity of water was estimated at 0,3 m³. The cost of one m³ of water was estimated at 2500 Frw. Therefore the needed quantity of water was estimated at $0,3 \times 2500 = 750$ Frw. The total cost is $20000 + 13500 + 5200 + 750 = 39450$ Frw for the brickwork of one m³ without the payment of the labour

We assumed that one bricklayer and two assistants-bricklayers should finish the brickwork of 50 blocks per day. A bricklayer was paid 1500 Frw and his assistants were paid 600 Frw each per day. In total the labour cost was estimated at $1500 + 600 \times 2 = 2700$ Frw per day.

The choice of profit margin was based on the idea that it might not favour higher cost of brickwork of blocks per m³ because I had to earn the job. For that reason, referring to the information from the site ground, when I took a 25 % profit margin of 39450 Frw, I got 9863 Frw. As a result, the cost of the brickwork of one m³ was estimated at 39450+2700+9863 =52013 Frw. When I tried to take a 30 % profit margin of 39450, I got 11835 Frw. Finally I preferred to estimate the cost of brickwork of one m³ at 39450+11835= 51285 Frw without payment of labour. This type of reasoning was applied to all parts of the house such as the foundation, elevation of walls, wall painting, roof, roughcasting and so on. When I submitted the estimated total cost of the house I fortunately earned the job.

The builder explains how the calculations are made for each part of the house and the cost of the whole house is determined by the total sum of sub-costs of each part. The given example illustrates how the builder proceeds to calculate the cost of the building site. The cost of installation and 18% VAT are not mentioned in the example but in the earlier extracts, it was explained that those expenses are normally included in the final cost rather than in the cost of each part of the house.

In the case of the house construction workplace setting, the builder is responsible for all activities related to the building site. Within those activities the builder's motivation consists of generating a certain level of income. However, before he embarks upon the activities, he has to compete with other builders to earn the job. To earn the job the builder is requested to estimate the cost of the house and submit it to the job provider. The question is how the builder estimates (calculate) the cost of the house so that there is a certain level of profit. While calculating the total cost of the house, two major sub-costs are considered with their respective VAT. The calculations that are illuminated in the builder's example above involved mathematics as a tool to achieve the goal.

The mathematical problem that the builder must solve is the calculations of the cost of the whole house to be submitted to the job provider i.e. $C = C_1 + C_2 + 18 \% (C_1 + C_2) = (1+18\%) (C_1 + C_2)$ where C is the total cost of the house to be submitted to the job provider. This cost consists of an installation cost of the building site (C₁) and a cost of the building site itself (C₂). However, in the case of the provided example, C₁ and VAT are missing because the builder did not explain

the entire calculations he has done before the construction of that house. Rather the only information I get is the cost for one day of brickwork using one m^3 of cement blocks. In other words, the example illustrates only the calculations of $C_2 = C_{21} + C_{22} + C_{23} + C_{24}$ (without its VAT) where C_{21} is a cost of 50 cement blocks contained in one m^3 , C_{22} a cost of the needed delivered materials (cement, sand and water) to the site for the brickwork of 50 cement blocks, C_{23} a cost of the payment of local labour for the brickwork of 50 cement blocks and C_{24} a value of the profit margin for the brickwork of 50 cement blocks. Here two alternative values of C_2 were calculated: $C_2 = 39450 + 2700 + 9863 = 52013$ Frw and $C_2 = 39450 + 11835 = 51285$ Frw.

The techniques used by the builder to solve that type of problems are based on real quantities estimated in units of each material with reference to detailed house plan and local conditions. The performance of those techniques is strengthened by the use of calculator and pre-knowledge in mathematics and civil engineering fields. In our example, the use of elementary arithmetic was the core means to find out the strategic thinking to get a job without losing the profit. Technologies included natural numbers facts of addition, subtraction and multiplication. Such natural numbers facts are for instance the use of the logic of the rule of three. All numbers used were contextualised with units of volume, weight and currency with reference to and justification of the mathematical techniques used. In the example of the estimated cost of the brickwork of cement blocks of one m^3 , two values of C_2 were suggested but the builder explained why it was necessary to choose the second one instead of the first one. This could be described as the use of pre-knowledge in mathematics as it is suggested in the 'reference' MO.

4.3 At the restaurant owner's workplace

Managing a restaurant is another income generating profession which is mostly exercised in towns. The restaurants are classified in three categories with respect to the customers' financial means: high, middle and low categories. In the present study we focused on the middle class restaurant where the owner decides about the weight and cost while purchasing and selling different products that are consumed in the restaurant. On the open market, located in the suburbs, purchasing is done through negotiation between the restaurant owner or employees

and the farmer whereas in the super markets and shops, the products are purchased at marked prices.

From the transcripts of interviews that were conducted with the restaurant owner, the main focus of the concern is a certain level of profit. The interviewee revealed that normally, the level of overall profit in percentage set by experience is a guarantee of not losing. In this regard, the restaurant owner claimed that the most important thing is that if the calculations show that, after selling everything, the overall profit is between 30% and 40%; there is a guarantee that in normal conditions, all changing cost factors should be covered and a certain level of profit remains. Furthermore she revealed that to achieve the goal, her experienced knowledge plays an important role in keeping control of changing cost factors. Such factors are for instance: money spent on each product while purchasing, prices at competing restaurants, number of employees, taxes that are paid regularly and expected number of customers. When asked how purchasing is done, the restaurant owner explained that there are two alternatives. Some products which are available on the open market are purchased through negotiation and others which are found in super markets or shops are purchased at their marked prices. However, due to those changing cost factors, the restaurant owner claimed to prefer the first alternative in the following extract:

Interviewer: How do you purchase different food products?

Rest. owner: You know for me the best way of purchasing is negotiating with the estimated weight and price because in that case I gain much more than to purchase the products with the well known price and weight measured on a scale /.../ Let's say for example, if in town one kg of plantains costs 100 Frw, you could find it at 60 or 70 Frw on the suburb markets and then you can compare. However, that does not mean that the one who purchases that kg here in town necessarily would lose, rather in this case he gains, but comparing to me, I will gain much more...

Interviewer: OK can you just tell me how you negotiate while buying the plantains?

Rest. owner: The thing is that the moment you are on that kind of market, first you take a look and choose the plantain stock which you like, you pick it up in your hands and you estimate its weight...

Interviewer: /.../ How?

Rest. owner: For example, the one which is approximately 20 kg. Here in town that plantain price is $20 \text{ kg} \times 100 \text{ Frw} = 2000 \text{ Frw}$. You suggest the plantain owner /.../ let's say 1000 Frw /.../ Slowly you increase the price up to 1500 Frw depending upon how you find its quality and weight.

In this extract, there is no relationship between the estimation of weight (quantity) and price rather the estimation of the price is based upon the comparison with the price in town. The negotiation process is done in a way that the restaurant owner always refers to the prices in town.

Regarding the estimation of the price of a meal, the restaurant owner claimed that despite the changing price factors, the estimation is also based on considering the amount of money spent on purchased products and the approximation of the number of meals in the purchased products:

Interviewer: Now how do you estimate the price of a meal in the restaurant?

Rest. owner: Here also it is a question of estimating by saying that all purchased products are valued at a certain amount of money. Then I approximate the number of meals from those products and then I decide the cost of one meal.

Interviewer: Can you give me an example?

Rest. owner: Mmm it happens for example, that I purchase products of 20000 Frw and when I arrive here in the restaurant, I make my estimation and I find definitely that from those products it is possible to make 40 meals /.../ then I decide the price of each meal.

When asked to explain how the estimation of quantities of products is done, the restaurant owner said that it is difficult to explain because it requires a certain experience in the profession of managing a restaurant. She mentions that sometimes she estimates quantities according to previous purchase, and considers the days because she does not receive the same number of customers all days.

However, to be more concrete she explained through a written concrete case of an order of 70 people who had celebrated a party in

her restaurant. The customers' order included various food products and drinks, beer and soft drinks. Regarding the food products, the restaurant owner mentioned 10 kg of rice at 700 Frw per kg, five kg of cow meat at 1100 Frw per kg, various vegetables at 1300 Frw, five cabbages at 100 Frw each, five kg of onion at 200 Frw per kg, eight kg of macaroni at 500 Frw per kg, two big stocks of plantains at 3000 each and five kg of beans at 350 Frw per kg. The subtotal cost of purchase was 27050 Frw. Regarding the drinks, beers and soft-drinks, purchasing and selling prices are always pre-established by the brewery and the subtotal purchase cost was 41250 Frw. The overall purchase cost was $27050 \text{ Frw} + 41250 \text{ Frw} = 68300 \text{ Frw}$. After cooking, each meal was sold at 800 Frw i.e. $70 \times 800 \text{ Frw} = 56000 \text{ Frw}$ and from the drinks she got 59000 Frw. The total selling cost was $56000 + 59000 = 115000 \text{ Frw}$. Finally the overall profit was $115000 - 68300 = 46700 \text{ Frw}$. In this example, the overall Purchasing Cost (PC) and Selling Cost (SC) are obtained by adding subtotals of the purchased and sold products respectively. The Level of Profit (LP) is then obtained by subtracting the overall selling cost and purchasing cost.

In the restaurant management workplace, serving the customers requires purchasing, preparing, cooking and selling various products. The coordination of those activities is mostly oriented to a certain level of profit. This challenge requires the restaurant owner to keep controlling the changing cost factors. However as far as it is difficult to express all of them in terms of numbers at any purchasing and selling case, the estimation of quantities and costs for various products and meals is based on the purchasing and selling costs and the overall level of profit. In other words, to achieve the optimum goal, the owner tries to purchase products at the lowest possible prices for the purpose of maximising the level of profit. The level of profit is calculated in terms of difference between SC and PC i.e. $LP = SC - PC$. That is a type of mathematical tasks which the restaurant owner keeps on tackling. In the given example of the party of 70 persons, $PC = 68300 \text{ Frw}$, $SC = 115000 \text{ Frw}$, $LP = 115000 - 68300 = 46700 \text{ Frw}$. PC was performed through the addition of all costs of the several purchased products.

The techniques used by the restaurant owner to solve that type of problems are based on real quantities estimated in units and currency. The performance of those techniques is strengthened by the use of

good calculations and thinking (purchasing on the lowest price to maximise the profit). In this example elementary arithmetic was applied. Technologies included natural numbers facts of addition, subtraction and simple multiplication. Such natural numbers facts are for instance the use of the logic of the rule of three. All numbers used were contextualised with units of weight and currency. Although in this example the restaurant owner tries to justify the mathematical techniques referred to, but on the other hand, due to changing factors, she does not explain how she estimates quantities to satisfy her clients. Rather she referred to the use of experience in her profession. To some extent this can be described as a silent knowledge adopted by experience and exchange with colleagues, like in the case of the taxi drivers.

4.4 Activities in running small scale enterprises

In this section data from the three workplace settings are analysed with the support of activity theory. Two analyses, guided by Leont'ev and Engeström's concepts, are made to find out what the different models can contribute to the analysis when it comes to the use of mathematics in everyday settings. Moreover, they also give an overview of and a context to the coming mathematical tasks. Only examples expressed by the participants are included.

According to Leont'ev (1981, 61) "human activity exists only in form of an action or a chain of actions", and the goal-directed actions are selected as components of concrete activities. This study does not look at the activity in its entire historical state, rather the study looks at activity at the level of actions at different workplace settings such as leading cooking work, purchase food, negotiate prices, compete with other builders to earn a job. In Table 3 components of concrete activities at the three workplace settings are analysed according to Leont'ev.

Table 3: Analysis of activity components at the workplace settings guided by Leont'ev (1981).

Settings	Taxi drivers	Builder	Restaurant owner
Components			
Activity	Running a small scale enterprise within the respective areas		
Motives	To generate income for earning a good living		
Action	Negotiate prices with different customers, buy petrol	Estimate price for job competition, negotiate costs, lead construction work	Estimate prices and weight of food products for purchase, negotiate prices, sell the cooked food
Goal	A good income	A good income	A good income
Operation	Estimate distance, estimate time, check prices at petrol station	Plan, visit concerned stakeholders	Plan, discuss with customers, visit different market places
Conditions	Road conditions, weather forecast, number of customers, type of customer and time	Time available for construction, Number of competitors, current prices of building materials, the cost of labour	Number of customers, prices in the neighbouring restaurants, day of the week

In his model, Leont'ev starts with the activity and its motive which is common to all the workers, that is, to run a business and earn money to be able to cater for the survival of themselves and those who depend on them. Then the mathematical actions are analysed to reach the goal of a good income which in turn depend on how workplace specific operations are performed under the current conditions. The description is broad and there is no room for elaboration of how to reach the goals.

In workplaces, the participants are estimating costs, distance, and weight and other factors that influence the pricing of what is offered on

the market. However, in their context most of the mathematics is performed “at a glance” or what the restaurant owner called “by experience”. Hence, Leont’ev’s model shows that the mathematics is highly situated and is embedded in actions so that it becomes invisible. At the same time it shows how individual actions, including mathematics, are important to avoid the risk of losing money. In Engeström’s terminology (1998, p. 78),

activity is seen as a collective, systemic formation that has a complex meditational structure. Activities are not short-lived events or actions that have clear-cut beginning and end. They are systems that produce events and actions and evolve over lengthy periods of sociohistorical time.

Although Engeström’s model includes an overarching context he starts by pointing out the individual. Also, mediational means, i.e. instruments or tools, become central (Table 4).

Table 4: Analysis of activity components at the workplace settings guided by Engeström (1987).

Settings	Taxi drivers	Builder	Restaurant owner
Components			
Subject	Two taxi drivers	One builder	One restaurant owner
Object	Estimating prices when negotiating with different customers	To make mathematical calculations related to the estimation of costs for job competition	Estimate prices and weight of food products. Calculate costs for meals
Tools	Language, mathematical calculations	Language, mathematical calculations, pen, papers, calculator, books	Language, calculator, pen, papers, food products, and mathematical calculations
Rules	Standard norms and regulations which are applied within the community of taxi drivers	Standard norms and regulations which are applied within the community of house builders	Standard norms and regulations which are applied within the community of restaurant management
Community	Other drivers, passengers, sellers of petrol	Other competitors, job providers, labour	Visitors of the restaurant and other restaurant owners
Division of labour	Roles of drivers, roles of passengers and sellers of petrol	Roles of builder and roles of labour	Role of restaurant owner, role of restaurant visitors
Outcome	Gaining profit in terms of money	Gaining profit in terms of money	Gaining profit in terms of money

Seen from Engeström's perspective, the three workplaces resemble each other to a high degree. However, they shift in complexity and the involvement of risk. The taxi driver who does not own his car depends on getting a big enough number of clients and charge prices that leaves

him with a profit. In contrast, those who employ others also have to consider risk of damages on the car, mistakes or accidents when building and failure to get enough guests to the restaurant respectively. As they have made greater investments their risk of loss is greater than the employed. Common to all the participants is that they have strong driving forces to be effective and efficient in their respective production to gain as much as possible from the consumption of their services.

The activity of running small scale enterprises requires the concerned subjects to carefully understand the focused community in which they are based when operating in it. Through their daily habit of working, while sharing the same activity with co-workers or other stakeholders, their experienced understanding is always goal-oriented to achieve the pre-determined outcome of the activity. Depending on the nature of the enterprise and the achievement of the daily end-product of the activity, subjects are imposed to follow the respective common norms that are standardised by the members of the community. In the process of achieving outcome, subjects are always helped by the available instruments. Those instruments can be physical, symbolic, external or internal mediating tools.

The choice of enterprise depends on the need and available means of starting to run it. For instance, although the majority of taxi drivers in Rwanda have a limited schooling level (primary education), the first requirement is to be a morally and physically normal holder of a driving licence. Then the holder can apply for a job or drive her/his own car. In this case it is assumed that the required documents of the car are provided from the authority in charge of it. This means that becoming a driver does not necessarily require to invest an amount of money initially. In contrast to start a business of running a house construction company or managing an own restaurant requires the concerned subject to invest an amount of money from the beginning. The choice of tools and how they are used depend on the circumstances and constraints in which the subjects are working in.

For instance, taxi drivers, house builder and restaurant owner are working in different social communities where the rules and division of labour are applied differently. However, they are motivated by the same object of generating income for earning a good living. This implies that in their respective chain of actions to achieve outcome,

that is, gaining a certain level of profit in terms of money, they have in common an action of estimating prices which subsequently oblige them to use mathematical calculations as tools to work effectively and efficiently.

The question of grasping how the workers used and justified mathematical techniques is the issue of anthropological theory of didactics and it was illuminated in each sub section of this chapter. For instance, drivers estimate prices with respect to the road conditions, cost of petrol, time and distance to be covered. The builder and restaurant manager consider other parameters which are specifically pertinent to their respective professions. Those parameters are for instance a builder's costs for the house including the installation cost on building site and the cost of the building site itself, whereas a restaurant owner considers the difference between selling cost and purchasing cost as a level of profit.

5 MATHEMATICAL ACTIVITIES AT A UNIVERSITY SETTING

The current chapter aims to describe the nature of mathematical activities that are carried out by prospective mathematics teachers when dealing with mathematical tasks that are contextualised in workplace settings. I start to analyse the transposition of workplace mathematics into university tasks. Then follow mathematics student teachers' processes of solving contextualised mathematics tasks. Thereafter, an analysis of a second transposition is performed which takes place when students construct tasks for secondary school students. Also, students' reflections are reported. Finally, the experience of mathematical activities at the university setting is analysed.

5. 1 Transposing workplace mathematics for university students

Mathematics student teachers are expected to become qualified mathematics teachers soon. This requires from them at the same time to be able to understand the subject and demonstrate it before they can teach it. Due to the fact that they have not been in the field themselves, the mathematical knowledge used at the workplace settings, as analyzed in chapter 4, was transposed for them by the researcher. The principles for changes in the construction of tasks were twofold: First, I wanted to use the data from the workplaces in a respectful way which was done in the builder and restaurant management tasks. However, in the construction of the tasks related to taxi driving I had to move from oral descriptions in dialogue with the drivers to written tasks for the students. Here I decided to formulate two tasks, one based on the norms of the context of the taxi drivers except for the use of fractions and percentages. In the other task I purposely disrupted the information as I refrained from mentioning the cost of the petrol for a certain distance in order to challenge the students' thinking.

By 'transforming' a situated mathematical practice at a workplace into tasks to be solved in a didactical institution in the format of word problems, it is impossible to keep more than a surface level of

authenticity. Some essential differences relate to the methods or techniques used and the criteria for what counts as a successful solution.. In the situated practice methods may be tried out and adapted to the conditions and the result may have immediate consequences for the individual such as economic profit or loss, while in the educational institution only the use of a specific set of techniques and technologies are defined as legitimate in order for a solution to be assessed as a successful. To describe relationship between a contextualized mathematics task for the educational setting and the out-of-school situation referred to in the task, the concept of representativeness, comprised by comprehensiveness and fidelity (Palm, 2009), was discussed in section 2.3.2. In addition to the 'respectfulness' mentioned above, a principle for the didactic transposition work of the researcher in constructing the mathematical tasks for the student teachers at the university, was to aim at representativeness by maintaining as much as possible of the components of the situation considered by the workers (comprehensiveness) as well as the authentic measures in terms of quantities and costs (fidelity).

Hence, according to the findings related to estimating prices while driving a taxi, a task was constructed on the basis of: (1) taking account of some changing factors (the quantity of petrol that the car would consume to cover a given distance, i.e. the distance that the customer intend to cover) and the cost of the petrol to cover any trip. That cost was compared, in terms of fraction ($\frac{1}{3}$) and percentage (40%) respectively, to the total amount of money paid by each customer for both trips. (2) The notion of the estimated cost of the consumed petrol for one kilometre for both trips was introduced to show how many times the first trip is cheaper compared to the second one. With this information mathematics student teachers were asked (a) to find the amount of money that the driver remained with after paying the petrol used for both trips and (b) to formulate a related task for third year secondary school students. This can be seen in details in the following mathematical task related to estimating prices while driving a taxi car:

In order to comprehend how mathematics could be useful in estimating prices while driving, an interested researcher approached a taxi driver in Kigali city and requested him to explain how he manages to estimate the prices. The driver said that in general, the prices are estimated in relation to

- (1) quantity of petrol that the car would consume
- (2) distance that the customer intends to cover
- (3) road condition
- (4) sometimes waiting time .

On the basis of the above factors he gave the following concrete example: One day, the driver had two clients (passengers). The first wished to go to Nyamata center (Eastern Province of Rwanda) and return back to Kigali whereas the second wished to go to Butare city (South Province of Rwanda) and return back to Kigali. According to what driver explained to the researcher:

For Kigali-Nyamata-Kigali trip, the car has consumed around 1/3 of the total amount which the costumer was charged. The distance Kigali-Nyamata is about 50 km.

For Kigali-Butare-Kigali trip, the car has consumed around 40% of the total amount which the client was charged. The distance Kigali-Butare is about 120 km.

Curiously according to the data given to the researcher, the results have shown that (1)

the estimated cost of the consumed petrol for one kilometer is the same for both trips (version for groups A and B)

the estimated cost of the consumed petrol for a distance of 20 km is 1000 Frw for both trips (version for groups C and D).

(2) The cost of petrol for the 2nd trip (Kigali-Butare-Kigali) is 2,4 times the cost of petrol for the 1st trip (Kigali-Nyamata-Kigali).

Now, as students at a university in Rwanda who soon intend to become mathematics teachers in secondary school,

(a) Your task is to find the amount of money that the driver remained with after the payment of petrol for both trips.

(b) By assuming that you are a mathematics teacher in the 3rd year (lower secondary school); using the given data in your task, you are asked to formulate a task which could be solved by your future students.

Referring to the findings of the example of the estimated cost of brickwork of cement blocks of one m^3 , a task related to house construction was designed on basis of considering four main factors (quantity of materials per units, costs of unit for each material delivered to the building site, the payment of local labour and builder's profit margin) with respect to the figures provided by the builder for elevation of a wall of 50 cement blocks contained in one m^3 . Thus, mathematics student teachers were asked (a) to find the builder's profit

margin that was submitted to a job-provider and (b) to formulate a related task for third year secondary school students. Full details can be seen in the following mathematical task related to house construction

In order to comprehend how mathematics could be useful in house construction, an interested researcher visited one builder and requested him to explain how he manages to estimate the prices for participating in the competition on the job-market for the construction of a house. The builder told him that the estimation of prices is mostly based on the following four factors:

- quantities of materials per unit
- price per unit of material and transport costs (when the materials arrived at construction site)
- payment of local labour (on site)
- profit margin (where the profit margin is in percentage of the purchase and transport cost of materials when the materials arrived at the construction site)

In principle, the builder said that he must reassure the real quantities by conforming to the plan of the house that is to be built. In order to be clearer about the estimation of prices, the builder provided the following concrete example related to the bricklaying of cement blocks.

- (1) Regarding the consideration of quantities of materials per unit, the builder considered one cubic meter (m^3) of cement blocks as unit of materials. From his experience, in one cubic meter of cement blocks, there are approximately 50 cement blocks (50 pieces per m^3). At that time, the cost of one block was estimated at 400Frw.
- (2) Regarding the consideration of the price unit per material and transport, for the bricklaying of 50 cement blocks, the builder needed cement, sand and water.
 - The needed quantity of cement for that bricklaying was estimated at 90kg. At that time, the cost of one cement kg was estimated at 150 Frw.
 - The needed quantity of sand for that bricklaying was estimated at $0,65m^3$. At that time, the cost of one cubic meter of sand was estimated at 8000 Frw.
 - The needed quantity of water for that bricklaying was estimated at $0,3m^3$. At that time, the cost of one cubic meter of water was estimated at 2500 Frw.
- (3) At the moment when he was elevating that house, one bricklayer and two assistants were able to elevate the wall of 50 cement blocks per day. A bricklayer was paid 1500 francs per day whereas each assistant was paid 600 Frw per day.

- (4) When one wishes to earn the job-market, s/he must be somehow strategic in estimating the unit price, depending on the cost of local labour on site, sometimes the estimated unit price should not include the value of labour payment but in that case you must increase the profit margin. In the present concrete case, it was advantageous to increase profit margin and omit the local labour payment. Finally the estimated unit price of one cubic meter of bricklaying of cement blocks which allowed him to earn the job-market was 51285 Frw.

Now, as students a university in Rwanda who soon intend to become mathematics teacher in secondary school,

- (a) Your task is to find the benefit margin that the builder has fixed to earn the job.
- (b) By assuming that you are mathematics teachers in the 3rd year (lower secondary school); using the given data in your task, you are asked to formulate the task which could be solved by students of your students.

No changes of the data from the builder were done as he provided a written example of his calculations. However, I provided the information in a more narrative form and stated the tasks.

Finally with reference to the findings related to restaurant management a task was designed according to the quantities of food products and drinks with their respective purchasing prices. However, instead of revealing the concrete selling costs of drinks, as it was done for the cost of one meal (800 Frw each meal), there were computed percentages of profit for each bottle of each drink. Therefore, mathematics student teachers were asked (a) to find the total amount of money the restaurant owner was paid and the total profit she gained from the party and (b) to formulate a related task for third year secondary school students. Full details can be seen in the following task related to restaurant management.

In order to comprehend how mathematics could be useful in restaurant management, an interested researcher visited a manager (a lady) of one restaurant in Kigali city and requested her to explain how she manages to estimate the quantities of products so that she can make sure to satisfy her customers. The manager explained these issues through the concrete following example of reception party that held in her restaurant.

In the reception party, 70 people wished to have meal and drinks. Each meal included rice, macaroni, beans, green bananas, cow meat and salad that included tomatoes onion, cabbage, pepper, salt and mayonnaise. Therefore she said that she purchased the following quantities of products:

- (1) 10 kg of rice at a rate of 700 Frw per kg
- (2) 8 kg of macaroni at the rate of 500 Frw per kg
- (3) 5 kg of beans at the rate of 350 Frw per kg
- (4) 5 kg of cow meat at the rate of 1100 Frw
- (5) 7 kg of onion at the rate of 200 Frw each kg
- (6) 2 kg of pepper at the rate of 300 Frw each kg
- (7) 2 big green banana stocks at the rate of 3000 Frw per banana stock
- (8) 5 cabbages at the rate of 100 Frw per cabbage
- (9) 1 basket of tomatoes at the rate of 800 Frw

After cooking, the price of each meal was fixed at 800 Frw.

In addition, the drinks included:

- (1) 3 cases of small Mützig at the rate of 7200 Frw per case
- (2) 2 cases of lemonades (Fanta, Coca, Sprite, Tonic) at the rate of 3650 Frw per case
- (3) 1 case of Primus at the rate of 4650 Frw per case
- (4) 1 case of Amstel at the rate of 7700 Frw each case

According to the records that she revealed the researcher, the calculations shown that she got a profit of 25% on each bottle of small Mützig, 38,8% on each bottle of lemonade, 29,5% on each bottle of Primus and 35,8 % on each bottle of Amstel.

Now, as students at a university in Rwanda who soon intend to become mathematics teachers in secondary school,

(a) Your task is to find out an approximated total amount that the restaurant manager was paid and the total profit she gained from that party.

(b) By assuming that you are mathematics teachers in the 3rd year (lower secondary school); using the given data in your tasks, you are asked to formulate the task could be solved by students of your students.

Like the builder, the restaurant owner provided a written example of her calculations. No changes of the costs stated by her were done. However, from the information given I calculated the costs for drinks per bottle and stated the percentages of profit per bottle instead of providing the sales prices. Also, I provided the information in a more narrative form and stated the tasks.

In sum, the researcher's designed tasks to be solved by student teachers provide a rich description of the context, components and mathematical work as presented in the source of the didactic transposition, with high degree of representativeness. There is information provided which could serve as a starting point for critical

discussions related to several societal issues such as cost levels of different products, workers' wages and levels of profit for small-scale enterprises. The quantitative word count measure of the tasks related to the three different workplace tasks have an average of 400 words, distributed as follows:

- Taxi driving: 311 words
- House construction: 500 words
- Restaurant management: 389 words

Mathematical knowledge used at the workplace was thus first modelled as tasks to be solved by student teachers within the university institution by way of a didactic transposition employing a principle of representativeness. Drawing on the information data collected at the workplaces, this didactic transposition of knowledge has a twofold purpose. The first is to be able to analyse mathematical organisations used when mathematics student teachers deal with real-life word problems. The second purpose is to encourage mathematics student teachers, based on their experience in solving these problems, to employ information from workplaces that involve mathematics in constructing contextualised tasks that may be devolved to students in secondary school mathematics classrooms.

5.2 Solving contextualised mathematical tasks

As mentioned in chapter 3, each group of students solved two tasks. On their task sheets, the participants were advised to read the task, reflect on it and discuss within members of their respective group. In addition, they were allowed to use any tools or ask for clarification if necessary. The morning groups solved the taxi driving and restaurant management tasks and the afternoon groups solved the taxi driving and house construction tasks. From the fact that all four groups had the same mathematical background, the reason behind that variation was based upon investigating if there was any difference in the mathematical organisations between morning and afternoon groups. All tasks were designed so that they contained two sub-questions (a) and (b). The focus of the current subsection of the study pinpoints not only the (a) sub-question in each of the three contextualised tasks where participants were acting as problem solvers but also describes mathematical organisations that are embedded in the university mathematics students' practices.

5.2.1 Task related to a taxi driving workplace

In the group interviews, all four groups claimed that they solved the task through formulation and resolution of a system of two equations with two unknowns x and y and this is depicted in the following extract of the group interview which I had with them.

- Interviewer: Ok I see here your written works about problem solving process of both taxi driving and restaurant management tasks, please tell me how did you solve them.
- Roza (A): In our group [group A] for the task related to taxi drivers, we tried to use the system of two equations with two unknowns, you will see on our report sheet but it did not work as we wished because the information concerning the price of petrol to cover a distance of one kilometre was not enough. Fortunately when you [interviewer] told us to decide as mathematics teachers, we assumed that, that cost should be known in terms of a value given because we thought the task is linked to the real situation /.../ and then it worked.
- Jonas (B): That is exactly what happened in our group [group B] too /.../.
- Kalisa (D): In our group [group D] while solving taxi driving task, we introduced x and y as unknowns /.../ so it was a kind of system of equations which allowed us to find the solution/.../.
- Aaron (C): For us [group C] /.../ we too used the system of equations with two unknowns and solved it /.../.

In the beginning of the task solving process, the morning groups (A and B) assumed that the use of a system of two linear equations would definitely allow them to solve the task. But it was difficult for both groups when instead of finding a heterogeneous system, in all attempts they tried, they found a homogeneous system and two similar linear equations.

In the following, examples from the dialogues in group A are analysed in line with Engeström's concepts of expansive learning. The students started by reading the task and raise questions about how to solve it:

- Roza: We find a homogeneous system /.../ it is homogeneous
- James: Yaaa they are similar.

The contradiction was due to the insufficiency of information related to the concrete value of the cost of petrol to cover a distance of one kilometre. Thus, their problem solving process led to intensive discussions between the members of the groups. The following dialogue occurred when the students analysed the task and tried to fit it into their existing mathematical models:

- Jackson: Okeee as the cost of the consumed petrol for one kilometre /.../ is the same for both trips... therefore we have $\frac{x}{300} = \frac{y}{600}$ which means $2x = y$.
- Roza: yees
- Jackson: /.../ Then we shall analyse the second case where it is said that /.../ the cost of petrol for the 2nd trip is 2,4 times the cost of petrol for the 1st trip/.../.
- Roza: /.../ This means that $\frac{x}{3} \cdot 2,4 = \frac{40y}{100}$.
- Jackson: No we have already made calculations for that /.../
- Felix: But what she is saying is true.
- James: Yes.
- Jackson: So then we finally end up with the same equation /.../?
- Roza: /.../ that is what I told you few minutes ago /.../look here!
- Jackson: This means that $\frac{24x}{30} = \frac{40y}{100}$ isn't?
- James: That is true
- Jackson: $24x = \frac{120y}{10} \Leftrightarrow 24x = 12y \Leftrightarrow 2x = y$.
- Roza: Yees we find the same equation.
- Felix: So the two equations are equivalent!
- Roza: Yes they are /.../
- Felix: But /.../
- Jackson: Unless we introduce the third unknown z and express it as a dependent variable of x and y/.../ but how can we do it?

In the following dialogue, the students start analysing their model and the implications of using it.

- Roza: Listen to me /.../ let us leave them like they are
because if we take $x + y - \frac{x}{3} - \frac{40y}{100}$ we find z
which is the balance of the driver /.../ it is okeee /.../
- Jackson: Ooh no we could not allow the unknowns in the final result
- Roza: So what can we do?
- Jackson: /.../ actually, I think this task requires mathematical logic /.../
- Roza: /.../ But you can be convinced that finally what I am saying should be our final result as long as we do not have the concrete cost of petrol for one kilometre as Jackson is saying.
- Felix: Do you have an idea of how the result can look like?
- Roza: Of course I do. Look here on this sheet: from the first assertion to cover a distance of one kilometre the
driver must spend $\frac{x}{100} = \frac{x}{300}$ OK? /.../ then from the
second one to cover a distance of one kilometre the
driver must spend $\frac{40y}{240} = \frac{y}{600}$ OK? Now the total
amount of money he remained with after paying the
petrol of both trips is $x + y - 340\frac{x}{300}$ or
 $- 340\frac{y}{600}$ because the third term of the expression
represents the total amount of money he spent to pay
the petrol for both trips /.../ how do you think?

After examining Roza's justification of her reasoning, the group agrees and continues to reflect on how to implement the model.

- Felix: Well according to the time we spent on those calculations /.../ I find it is well explained.
- Jackson: Apparently it is a good idea according to our given data in the task.

James: For me it is OK

Roza: /.../ So then the expression $x + y - 340\frac{x}{300}$ should be symbolised for instance by z . In that case whenever the cost of one kilometre is fixed at certain concrete value, you can easily deduce the value of x and y . That is why I am trying to convince you how our result should look like this.

James: /.../ Yes I think you are right because the fact that the cost of one kilometre is missing implies somehow a lack of an independent term in our different equations.

Jackson: Ooh James thanks a lot! Really the independent term is missing... yes it is true I agree with her.

After reflecting on the solution, the other group members contributed with more explanations and finally they evaluated the solution positively.

In this extract, the students had a problem of solving a system of two equivalent equations and a homogeneous system with two unknowns. However, by assuming that whenever the petrol cost to cover one kilometre is known, there will not be any problem to find a solution. Therefore, they ended up to be convinced that their mathematical expression should contain the variables x and y , that is:

$$x + y - 340\frac{x}{300} = z$$

This fact is also confirmed in the content of their task solving written work below.

Résolution:

1. 1^{er} Premier cas: Trajet: Kigali - Nyamata - Kigali.

soit x le montant total payé par le 1^{er} client
le prix du carburant est $\frac{x}{3}$. (K-NY-K).

Distance aller-retour = $50 \times 2 = 100 \text{ km}$

Pour dire que: $100 \text{ km} \longrightarrow \frac{x}{3} \text{ Frs.}$

$1 \text{ km} \longrightarrow \frac{x}{3 \times 100} \text{ Frs/km.}$

$$= \frac{x}{300} \text{ Frs/km.}$$

2^{ème} Deuxième cas: Trajet Kigali - Butare - Kigali.

soit y le montant total payé par le 2^{ème} client.

$\frac{407}{100}$ est le prix du carburant. (K-B-K).

Distance (K-B-K) = $120 \text{ km} \times 2 = 240 \text{ km.}$

Cela signifie: $240 \text{ km} \longrightarrow \frac{407}{100}$

$1 \text{ km} \longrightarrow \frac{407}{100 \times 240} \text{ Frs/km} = \frac{4}{600} \text{ Frs/km}$

Comme la valeur du prix du carburant consommé par km est la même pour les 2 trajets; on a

$$\text{alors } \frac{x}{300} = \frac{4}{600} \Leftrightarrow 2x = 4. \quad (1)$$

On a aussi que la valeur du prix du carburant consommé pour le deuxième trajet est

24 fois la valeur du prix du carburant d'un 1^{er} trajet. de cette proposition on a:

$$24 \frac{x}{3} = \frac{407}{100} \Leftrightarrow 2x = 7. \quad (2)$$

Les équations (1) et (2) étant équivalentes, il nous faut de déterminer d'autre équation pour avoir un système de deux équations à deux inconnues.

Établissons l'équation entre le montant total et le prix total du carburant :

$$\text{On a } (x+y) - \frac{340x}{300} = \frac{1x}{300} \text{ F.}$$

Comme la valeur de x et y ne sont pas définies ; le montant d'argent resté dans la poche du chauffeur reste toujours l'exprime, en fonction de x et y .

$$\text{ie } z = \frac{40x + 300y}{300}$$

In the above written work, the members of group A assumed x and y to be the total amounts paid by the first and second customers respectively. By following the information given in the task, they proceeded to solve the tasks as follows:

In the first place $\frac{x}{3}$ is the cost of the petrol to cover the distance of the first trip (Kigali-Nyamata-Kigali) which is about 100 km. From that information, according to their reasoning, the cost to cover a distance of one km is

$$\frac{\frac{x}{3}}{100} = \frac{x}{300} \text{ Frw/km}$$

In the second place $\frac{40y}{100}$ is the cost of the petrol to cover the distance of the second trip (Kigali-Butare-Kigali) which is about 240 km. Using

the same reasoning, the cost to cover a distance of one km is

$$\frac{40y}{240} = \frac{y}{600} Frw/km$$

According to the given task information, on the one hand, the cost of petrol to cover one km is the same for both trips. Therefore, this means that

$$(1) \quad \frac{x}{300} Frw/km = \frac{y}{600} Frw/km \Leftrightarrow 2x = y$$

(this is the first equation of the targeted system of two equations).

On the other hand, the cost of the petrol for the second trip is 2,4 times the cost of the petrol for the first trip. From this proposition (mathematical statement) the participants concluded that

$$(2) \quad 2,4 \frac{x}{3} = \frac{40y}{100} \Leftrightarrow 2x = y$$

(a mathematical expression that represents the second equation of the same system of equation).

Then, according to the information from their written work, the participants concluded that the equations (1) and (2) are equivalent and for them to find out the values of x and y they need another equation which should include x and y. With the assumption that unknowns x and y are not defined, participants concluded that the total amount of money remained within the driver's pocket will be expressed in the function of x and y under form of z , that is

$$x + y - 340 \frac{x}{300} = z \Leftrightarrow$$

$$z = \frac{-40x + 300y}{300}$$

The mathematical dialogues in both morning groups became rich due to the fact that the researcher disrupted the information originally given by the taxi drivers.

In contrast, the dialogues in the afternoon groups (C and D) were less elaborated as the problem of missing information was no longer a constraint. This is seen for instance in the following extract of discussions in group D.

- Justice: /.../ Then let us think about the issue of covering a distance of 20 kilometres. Here it is explained that the cost of petrol for that distance is 1000 Frw/.../ this means that covering a distance of one kilometre, the car consumes $1000/20$ /.../ i.e. 50 Frw for both trips.
- Martin: Oh yes... wait I am writing it...
- Kalisa: /.../ Yes good idea /.../
- Bugingo: Yes yes
- Bugingo: Let's assume that the unknown x be the total amount paid by the first client.
- Kalisa: What is the value of x ?
- Justice: /.../ Well for the moment it is not easy to find it because we do not know the cost of petrol paid for the first trip /.../ you see?
- Martin: /.../ It is true /.../ rather /.../ one third times x ($\frac{1}{3}x$) is the amount paid for the petrol /.../ this means that $\frac{1}{3}x = 50 \text{ Frw} \times 100 \text{ km}$ OK?
- Kalisa: /.../ It is 5000 Frw?
- Bugingo: /.../ No it is 15000 Frw.
- Martin: OK $x = 15000$ Frw.
- Bugingo: Please Martin write it on the sheet /.../.
- Martin: Yes I am doing it /.../.
- Justice: Good let's then look at unknown y
- Kalisa: /.../ Truly /.../ it is true if you take somebody there, he or she should pay 15000. Let's assume y to be the total amount paid for the second trip /.../ by the second client.
- Martin: OK
- Bugingo: OK
- Justice: OK

- Martin: $\frac{40}{100}y$ or $40\% \times y$ is the cost of petrol /.../ that is, it corresponds to $240 \text{ km} \times 50 \text{ Frw}$ /.../
- Bugingo: It equals /.../ make calculations.
- Kalisa: It equals 12000 Frw.
- Justice: 12000 times 100 equals 1 200 000 Frw.
- Martin: Then y equals /.../ $40y = 1\,200\,000$ and $y = 1\,200\,000/40 = 30\,000$ Frw.
- Bugingo: The money remained in the driver's pocket for the first trip is /.../ if the cost of petrol is one third then two third of x remained in his pocket /.../
- Kalisa: Mm
- Justice: Mm
- Martin: Mm /.../ therefore $\frac{2}{3}x = \frac{2}{3} \times 15000 = 10000$ Frw
- Kalisa: Yes
- Bugingo: Yes
- Justice: Yes
- Martin: For the second trip /.../
- Kalisa: $\frac{60}{100}y = \frac{60}{100} \times 30000 = 18000$ Frw /.../ OK.
- Justice: The total amount after two trips is /.../
- Kalisa: /.../ it is 10000 Frw plus 18 000 Frw which means 28000 Frw

After reading the task one of the students took on the responsibility to construct the task while participating in the discussion. They found the cost of the petrol to cover a distance of one kilometre straight away and proceeded to find out the money that remained within the driver's pocket through the introduction of unknowns x and y. Thereafter they found the values of unknowns x and y. Finally the addition of the values of x and y was seen as the final solution to the task as it was requested in the problem, that is 28 000 Frw.

This fact is also confirmed in the content of their task solving written work below.

a) Réalisation du problème
 Soit x le montant total payé par le 1er client:
 $\frac{1}{3}x$ le montant payé pour le Carburant
 $\frac{1}{3}x = 50 \frac{\text{Fr}}{\text{km}} \times 100 \text{ km}$
 $\frac{1}{3}x = 5000 \text{ Frs}$

$$x = 3 \times 5000 \text{ Frs}$$

$$x = 15000 \text{ Frs}$$

Soit y le montant payé par le deuxième client
 $\frac{40y}{100}$ le prix payé pour le Carburant
 $\frac{40y}{100} = 240 \text{ km} \times 50 \frac{\text{Fr}}{\text{km}}$
 $\frac{40y}{100} = 12000 \text{ Frs}$

$$y = \frac{100}{40} \cdot 12000 \text{ Frs} = \underline{30.000 \text{ Frs}}$$

Le montant resté dans la poche du chauffeur
 - pour le 1er trajet: soit $\frac{2}{3}x$

$$\frac{2}{3}x = \frac{2}{3} \cdot 15000 \text{ Frs} = 10.000 \text{ Frs}$$

- pour le 2nd trajet: soit $\frac{60}{100}y = \frac{60}{100} \times 30.000 \text{ Frs}$
 $= 18.000 \text{ Frs}$

Le montant total resté dans la poche du chauffeur après
 avoir payé le Carburant pour les deux trajets:

$$10.000 \text{ Frs} + 18.000 \text{ Frs} = \underline{28.000 \text{ Frs}}$$

Like their classmates of group A, participants of group D assumed x and y to be the total amounts paid by the first and second customers respectively. They then found the values of x and y . Finally their logical reasoning to find answer was that if $\frac{1}{3}x$ and $\frac{40}{100}y$ are the

costs of petrol which the driver paid to cover two trips respectively, therefore $\frac{2}{3}x$ and $\frac{60}{100}y$ are the total amounts remained within the driver's pocket after paying petrol for both trips.

The mathematical organisations involved in the problem solving of the contextualised taxi driving task are characterised in advance by a known type of problem '*find the amount of money that the driver remained with after paying the petrol for both trips*'. As the prospective mathematics teachers were not playing the role of workers, the problem solving process required to recall from their mathematical background, a mathematical technique (practical level) which allowed them to come up with negotiated solutions.

In the taxi driving task, all four groups opted to formulate a system of two linear equations with two unknowns and recalled one of the different possible methods. Indeed, all four groups assumed the unknown x to be a total amount of money paid by the first customer and y by the second customer.

However, due to the information given related to the concrete value of petrol cost to cover a distance of one kilometre, the morning and afternoon groups worked differently in terms of mathematical justifications (theoretical level). From their own common understanding, the morning groups claimed that it is impossible to find the values of two unknowns simultaneously when its equations are equivalent. From the mathematical point of view, in that case the system solver should write the answer in a linear expression such as for instance $y = ax + b$ or $x = ay + b$ where one variable is seen as an independent (free) variable, the second is seen as a dependent one and a and b are seen as real constant values. The participants of the morning groups did not work in that way. As they assumed that the task was designed from a real situation at the time the research was being carried out; they assumed that there was a concrete value of the cost of the petrol to be consumed by the car for covering the distance of one kilometre. To find the answer they needed absolutely to find the values of x and y . They therefore thought that the solution of the task should look like the following mathematical expression

$$z = x + y - \frac{340}{300}x \quad \text{or} \quad z = \frac{-40x + 300y}{300}$$

where z is the total amount of money remained within the driver's pocket after paying the petrol for both trips.

For instance, if the cost of petrol for one kilometre is 50 Frw as it was the case for the evening groups, according to their reasoning

$$\frac{x}{300} = 50 \Leftrightarrow$$

$$x = 300 \times 50$$

$$x = 15000$$

since $y = 2x$ therefore $y = 2 \times 15000 = 30000$.

as $z = x + y - \frac{340}{300}x$.

therefore $z = 15000 + 2 \times 15000 - \frac{340}{300} \times 15000$

$$z = 28000$$

This result corresponds exactly to the one found by the afternoon groups. However, although the afternoon groups claimed to use a linear system of two equations, the problem solving process was actually based on the resolution of one linear equation of two cases (two different trips). Indeed, according to their written reports and group discussions, there was not an explicit algebraic method (for instance elimination of variables, row reduction or Cramer's rule) for solving a system of two linear equations, rather to find the value of x and y it was a matter of rethinking about the number of kilometres of each trip and multiply it by the cost of petrol for one kilometre. For instance, in the extract of group D, as the members of the group assumed x be the total amount paid by the 1st client and y to be the total amount paid by the 2nd customer, then the system was supposed to look like

$$(3) \quad \left\{ \begin{array}{l} \frac{1}{3}x = 50 \times 100 \\ \frac{40}{100}y = 240 \times 50 \end{array} \right.$$

and finally they found that

$$\begin{cases} x = 15000 \\ y = 30000 \end{cases}$$

Then having found the values of x and y they used the following fractions reasoning such that if x is the total amount paid by the 1st client then

$$x - \frac{1}{3}x = \frac{2}{3}x = \frac{2}{3} \times 15000 = 10000$$

is the money remained within the driver's pocket after the first trip. The same reasoning was applied to the second trip and then they found

$$y - \frac{40}{100}y = \frac{60}{100}y = \frac{60}{100} \times 30000 = 18000$$

as an amount of money remained within the driver's pocket after the second trip. Finally they added 10000 and 18000 and found 28000.

However, if one can have a look at the system (3) above, one can notice that it is actually two cases of a linear equation with one unknown. Here one can think of the discrepancy between what the participants expressed in the interviews and the meaning of their real content of their discussions and written works. In other words, what is on the surface does not reflect exactly what they meant by their explanations. The fact that the participants introduced these two unknowns, it was due to the information given in the task which mostly likely misled them to think of the use of a system of two equations instead of the double use of one linear equation with one variable.

5.2.2. Task related to the house construction workplace

This task was assigned only to the afternoon groups (C and D). From their group interviews, both groups claimed to use the rule of three.

- Interviewer: I see here your written works about problem solving process of both taxi driving and restaurant management tasks, please tell me how did you solve them? If you don't mind we can start with question (a) and end up with (b) question for both tasks.
- Kalisa (D): /.../ The task about house construction, it was really too long but nothing special. The main problem consisted of understanding the concept of profit margin, when you [Interviewer] explained us, it was just matter of adding, multiplying and applying the rule of three. That is all.
- Justice (D): That is true /.../
- Martin (D) Oh yaa really personally I was surprised how the people make so many calculations in their activities.
- Interviewer: Thank you. Let us allow the members of other group to express their views
- Aaron (C): Regarding the house construction task, we had the same problem of profit margin but after your intervention it was easy. However we acknowledge that we did not write many details on our written report because we spent much time in writing on the draft sheets. We just wrote the final calculations results.
- Joys (C): Mmmm (yes)

But in their problem solving processes they do not show explicitly how the rule of three helped them as such. In fact according to their group discussion transcripts, after reading and making sense of the task the participants consensus was that they shall put together all given data and try to use them as they are given in the task.

- Martin: Let us try to put together all given data /.../ the builder has to look at the cost of materials plus transport and payment of labour/.../ OK.
- Justice: Yes
- Bugingo: Yes
- Kalisa: Yes
- Martin: /.../ a) Cement blocks /.../ remember one m^3 is our guiding unit: $400 \times 50 = 20000$ Frw
 b) materials:
- | | |
|----------|--|
| - cement | $90 \text{ kg} \times 150 = 13500$ Frw |
| - sand | $0,65 \times 8000 = 5200$ FRW |
| - water | $2500 \times 0,3 = 750$ Frw |
- [using calculator]
- Justice The sum is $13500 + 5200 + 750 = 19450$ Frw

Martin: /.../ Yes 19450 Frw /.../ now
 c) one bricklayer /.../ 1500 Frw per day and two
 bricklayer assistants /.../ $600 \times 2 = 1200$ the sub-total is
 $1200 + 1500 = 2700$ Frw.
 The total for one m^3 is $20000 + 19500 + 2700 = 42150$
 Frw /.../ therefore, the profit margin is $51285 - 42150 =$
 9135 Frw per day /.../
 The question is finished.

Martin Eeeh he gains much money

Kalisa: So you think that this is the profit margin for the
 builder?

Bugingo: Oh yes

Justice: Oh yes

Bugingo: /.../ Let's then formulate the question.

Kalisa: I am not sure /.../ what do you really mean by profit
 margin?

Justice: /.../ It is the difference /.../

Bugingo: Justi... how do you differentiate profit margin and
 gross profit?

Here there was some confusion about the concept of profit margin and I was obliged to intervene for explanation. My intervention allowed them to remember about the hint the builder used in increasing profit margin and omitting the labour payment, then they reviewed their task

Bugingo: /.../ In this case we remove this subtotal of labour
 payment /.../

Kalisa: Yes

Justice: Yes

Martin: Yes

Bugingo: This means that instead of 42150 Frw we have 42150
 Frw - 2700 Frw = 39450 Frw

Martin: OK, then $51285 - 39450 = 11835$ Frw. Now the profit
 margin is $\frac{11835}{39450} \times 100 = 30\%$

Bugingo: OK that is all?

Martin: Yes

Bugingo: OK good

In this extract the principle of the task solving was that the participants took the cost unit of each construction material and multiplied it by the needed quantity of the same material to find out the amount of money that was spent on each kind of material. This strategy was applied to all materials and the addition of all amounts was seen as the money

that the builder was supposed to spend. Therefore, to find out the value of the profit margin the participants subtracted the total cost of all materials from the cost which was submitted to the job provider and then they computed the percentage of it in relation to the total cost of all materials.

The mathematical organisations involved in the solving of the contextualised house construction task are characterised in advance by a known type of problem ‘find out the profit margin that the builder has fixed to earn job’. As the mathematics student teachers were not playing the role of workers, the task solving process required to recall a mathematical technique (practical level) which allowed them to come up with the negotiated solutions. In that particular house construction task, both groups opted to recall the mathematical rule of three in elementary arithmetic. Indeed, from a mathematical point of view, the rule of three derives from the cross-multiplication process. In elementary arithmetic, given an equation between two fractions or rational expressions, one can cross-multiply to simplify the equation or to determine the value of variables in the equation (proportion) like the following

$$\frac{a}{b} = \frac{c}{d}$$

(note that ‘b’ and ‘d’ must be non-zero for these to be proper fractions)

Then, by cross-multiplying you get

$$ad = bc \quad \text{or} \quad a = \frac{bc}{d}$$

This means that we multiply the numerator of each (or one) side by the denominator of other side. The mathematical justification for the method is from the following mathematical procedure. If we start with the basic equation:

$$\frac{a}{b} = \frac{c}{d}$$

We can multiply the terms on each side by the same number and the terms will remain equal. Therefore, if we multiply the fraction on each side by the product of the denominators of both sides ‘bd’ we get:

$$\frac{a}{b} \times bd = \frac{c}{d} \times bd$$

Then we can reduce the fractions to lowest terms by noting that the b's on the left hand side and the d's on the right hand side cancel, leaving:

$$ad = bc$$

This is expressed as the product of the means 'ad' equals the product of extremes 'bc'.

Also, we can divide both sides of the equation by any of the elements, in this case we will use 'd' and we get:

$$a = \frac{bc}{d}$$

This is a common procedure in mathematics, used to reduce fractions or compute a value for a given variable in a fraction. Participants' computations were also based on the same principle. For instance, when they computed the money spent for water they proceed as follows:

The cost of one m³ of water was 2500 Frw. They need the cost of 0,3m³ of water. By applying the rule of three where the three known numbers are 1m³, 0,3m³ and 2500 Frw.

$$\frac{1m^3}{0,3m^3} = \frac{2500Frw}{xFrw} \Leftrightarrow xFrw = 2500Frw \times 0,3m^3 = 750Frw$$

$$x = 750Frw$$

5.2.3 Task related to the restaurant management workplace

This task was assigned only to the morning groups A and B. In the group interviews and group discussion both groups claimed that compared to the previous problem, the restaurant management problem was clear and understandable. When they were asked about the technique used to solve it, they claimed that it was an elementary number calculation application such as the rule of three, addition, subtraction, multiplication, division and calculation of percentages. For instance, in the interview they expressed it as follows:

- Interviewer: OK. Then, how about the restaurant management task?
- Roza (A): Well, with this task /.../ there was nothing really special, maybe the hard work was about to coordinate the pile of numerical data, otherwise it was a matter of adding, subtracting, multiplying, dividing, calculating percentages and so forth. Just simply elementary number calculation application /.../
- Serge (B): Yes it was just a kind of recalling somehow to apply the rule of three /.../
- Bana (B): Yes you are right.

The problem solving process was characterised by several steps which is summarised by one participant of group A in the following extract:

- Roza: Listen Felix [...] take a sheet [...] this task is clear and understandable. It is not like the previous one.
- James: Are you sure?
- Roza: /.../ Yes. Look first of all we calculate the amount of money. She spent to purchase all drinks, then we calculate the profit which she has got from all drinks and the total amount of money she has got after the party is the sum of the two. Secondary, we do the same for meals. Finally we add the total amount of money from the drinks and the meals [...] briefly that is all.

In the first place, by following the rate of the cost per unit of each product given in the problem, the participants found the total amount of money for the needed quantity of each product. This strategy was applied to both food and drink products as can be seen in the following extract:

- Roza: Eeeh pardon, wait please. We need first of all to find out the cost of each bottle of drinks, right? Then for example, in the case of small Mützig we calculate 20% of that cost for each bottle. Finally the result should be multiplied by 72 i.e. 24×3 [the number of bottles of small Mützig required in the party]
- Felix: Yeee
- Jackson: Yeee
- James: Yeee
- Felix: One case of small Mützig was purchased at the rate of 7200 Frw each. Therefore one bottle was purchased

at $\frac{7200}{24} = 300$ Frw. Then 20% of 300 is

$$\frac{300 \times 20}{100} = 60 \text{ Frw.}$$

Now we have in total 72 bottles of small Mützig, that is the total profit from small Mützig is $60 \times 72 = 4320$ Frw.

Roza: Very good

Jackson: This is nice

Felix: This technique should be applied for all drinks I think.

Roza: Yes

Jackson: Yes

James: Yes

Felix: Now /.../ one case of lemonades was purchased at a rate of 3650 Frw each. Therefore one bottle was

purchased at $\frac{3650}{24} = 152,08$ Frw. Then 38.8% of

$$152.08 \text{ is } \frac{152,08 \times 38,8}{100} = 59,00704 \text{ Frw.}$$

Now we have in total 48 bottles of lemonades i.e. the total profit from lemonades is $59.00704 \times 48 = 2832.4$ Frw /.../. One case of Primus was purchased at the rate of 4650 Frw. Therefore one bottle was purchased at

$$\frac{4650}{12} = 387,5 \text{ Frw. Then } 29.5\% \text{ of } 387.5 \text{ is}$$

$$\frac{387,5 \times 29,9}{100} = 114,3125 \text{ Frw.}$$

Now we have in total 12 bottles of Primus i.e. the total profit from Primus is $114.3125 \times 12 = 1371,75$ Frw /.../. One case of Amstel was purchased at the rate of 7700 Frw. Therefore one bottle was purchased at

$$\frac{7700}{24} = 320,83 \text{ Frw. Then } 35,8\% \text{ of } 320,83 \text{ is}$$

$$\frac{320,83 \times 35,8}{100} = 114,85714 \text{ Frw. Now we have in}$$

total 24 bottles of Primus i.e. the total profit from Primus is $114,85714 \times 24 = 2756,6$ Frw /.../. The total

profit is $4320 + 2832,4 + 1371,75 + 2756,6 = 11280,75$ Frw.

Jackson: Fine now we have got the profit from the drinks. But we need also the total amount of money she spent to purchase those drinks because we are asked to calculate the total amount that she received after the party.

James: That one I think it is not difficult to calculate/.../ let see $3 \times 7200 + 2 \times 3650 + 1 \times 4650 + 1 \times 7700 = 21600 + 7300 + 4650 + 7700 = 41250$ Frw. Then at the end of the party she received $41250 + 11281 = 52531$ Frw from the drinks

Roza: OK! Now let look at the food

Rice:	$10 \times 700 = 7000$
Macaroni	$8 \times 500 = 4000$
Beans:	$5 \times 350 = 1750$
Cow meat:	$5 \times 1100 = 5500$
Onion:	$7 \times 200 = 1400$
Pepper:	$2 \times 300 = 600$
Big green bananas:	$2 \times 3000 = 6000$
Cabbages:	$5 \times 100 = 500$
A basket of tomatoes:	$1 \times 800 = 800$

The total is 27550 Frw and the money she received from the meals is $70 \times 800 = 56000$ Frw.

James: So $56000 - 27550 = 28450$ Frw is the profit she gained from the meals.

Jackson: You know, now I see those people apply lot of mathematics really!!!

James: Yes this is a good example

Felix: Now we are asked to calculate the total amount that the restaurant owner has received after the party

Roza: That is simple she has received $56000 + 52531 = 108531$ Frw and the total profit from both meals and drinks is $= 28450 + 11280,75 = 39731$ Frw /.../
Now you can write the final draft to be submitted.

To calculate the total profit (from drinks and food), participants proceeded through two steps. On the one hand while calculating the profit from drinks according to the information at hand first they considered the rate of purchasing cost of one case of each drink. Second they divided it by the total number of bottles contained in one case to find out the purchasing cost of one bottle and third they calculated the corresponding total profit from each drink with respect

to the percentages given in the task. This way of calculating was applied to all kind of drinks. Thereafter a sum of all total profits represented the required profit from the drinks.

On the other hand, according to the figures of several costs of food products; first participants calculated the purchasing cost of all food products with the use of multiplication of the needed quantity of each product by its cost unit. By adding all those purchasing costs, they found 27550 Frw as a total purchasing cost of all food products. After the calculation of the total amount the restaurant owner received from the food ($70 \times 800 = 56000$ Frw), the profit from food was calculated through the subtraction of 27550 Frw from 56000 Frw which is 39731 Frw. Finally the total amount of money restaurant owner received from the party was found through the addition of the total amount received from both drinks and food ($56000 + 52531 = 108531$ Frw).

The mathematical organisations involved in the problem solving of the contextualised restaurant management task are characterised in advance by a known type of problem *'find out an approximated total amount that restaurant manager was paid and the total profit she gained from that party'*. As their classmates of afternoon groups when dealing with the task related to house construction workplace, groups A and B recalled also the Rule of Three as their mathematical technique. For instance, if the cost of one kilogram of rice is 700 Frw then to find the cost of 10 kilograms, according to the rule of three, participants proceeded implicitly as follows:

$$\frac{1kg}{10kg} = \frac{700Frw}{xFrw} \Leftrightarrow xFrw = 700Frw \times 10 = 7000Frw$$

$$x = 7000Frw$$

Or if one case of Primus beer is 4650 Frw and that it contains 12 bottles, therefore the cost of one bottle should be found as follows:

$$\frac{12}{1} = \frac{4650Frw}{xFrw} \Leftrightarrow 12x = 4650 \times 1 \Leftrightarrow$$

$$x = 387,5Fr$$

However, in addition to the use of the rule of three they also calculated percentages of profit for each kind of drink which allowed them to find the total profit from the drinks.

5.3 Transposition of mathematical tasks for secondary school students

In the current section, the transpositions that took place while prospective mathematics teachers acted as task posers are analysed. The subsections emphasise on task (b) where it is assumed that the participants are mathematics teachers of 3rd year lower secondary school. Referring to the data I presented in their tasks, the student teachers were asked to formulate tasks for their students. In total, eight tasks were constructed. Tasks related to the taxi driving workplace were formulated by both morning and afternoon groups whereas house construction and restaurant management tasks were formulated by afternoon and morning groups respectively.

5.3.1 Task related to the taxi driving workplace

The group interviews were performed separately with the morning (A,B) and afternoon (C,D) groups. As can be seen below, all four groups asserted that they designed tasks related to the taxi driving profession keeping in mind that the secondary school students should use a system of two equations with two unknowns when solving them.

- Interviewer: Good. Now how do you think about the mathematical techniques used for sub-question (b) of both tasks?
- Roza (A): In our group, there was nothing special, we thought that we shall formulate the task keeping in our minds that students of third year secondary school should think of the system of two equations with two unknowns/.../ but a clever student can solve it without thinking of the system if the cost of petrol to cover a distance of one kilometre is given. In our case we suggested 300 Frw.
- Jonas (B): Yaaa there is a common point of thinking about a system of two equations with two unknowns but in our case we did not precise a concrete cost of petrol to cover a distance of one kilometre rather what we did is just to say that if that cost is known, then the task is solvable in any case.

- Bugingo (D): In our group, as we assumed to expect students to apply the knowledge about the system of equations we thought that it is better to formulate the task in that way /.../
- Justice (D): Yes/.../ but
- Interviewer: Thank you. Group C would you like to tell us something about (b) sub-question of both tasks?
- Joys (C): For us, there is nothing special. For example, in the taxi driving task we only mentioned the concept of proportionality and suggested 20.000 Frw instead of writing the concrete number that we found from the results of our calculations.

Rather than discussing on how to bring in the workplace context this group emphasises on what school mathematics concepts and methods the secondary school students should bring in while solving the task, in this case the concept of proportion and the method of setting up a system of linear equations. The idea of formulating a task so that secondary students would think of the use of x and y in terms of a system of two linear equations while dealing with that problem was raised in both morning and afternoon group discussions and in their respective written works. For instance, in the morning group A discussion, the following debate took place:

- Felix: OK colleagues I think that this should be our final conclusion .Then let us think of how we can formulate the task for secondary school students.
- Roza: Yes yes
- James: Yes
- Jackson: Yes
- James: OK. /.../ let assume that you are a teacher in secondary school [3rd year]. Yes because really it is there where the systems of equations of that kind are taught. Having found that amount, you are asked to formulate the task from the data above that should be solved by students of your class.
- Felix: The data are those ones so what we need is to use them with modification and reduce them in our formulation.
- Roza: Actually we shall start with that unpredicted missing cost of one kilometre.
- Felix: Yeee I think so
- James: Should we keep the data?

Roza: Of course
 Felix: Yes
 Jackson: Yes
 James: We can change so that the students will solve the system nicely.
 Felix: Yeee
 James: I suggest that we shall keep our unknowns so that in our formulation the students will be required to find the values of them.
 Felix: In our formulation we shall even precise the total amount of money he gained after the two trips.
 Jackson: OK yes we shall think of for example, that the first client paid x amount of money and the second one paid y amount of money and he gained 20.000 Frw after the covering of the two trips.
 Roza /.../ So that $z = 20.000$ Frw

For this group it is important to use the data given but with modification in terms of a reduction to adapt to the level of the students so that they "will solve the system nicely". The debate resulted in the following transposed written task

b) Un chauffeur a fait deux trajets: Kigali-Nyamata-Kigali et Kigali-Butare-Kigali. Sachant que le trajet K-N-K est de 100 km et le trajet K-B-K est de 240 km. Trouver le montant que chaque client lui a payé, si après avoir payé l'essence qu'il a utilisée, il lui reste au total 20000 Frs et le prix d'essence par 1 km est de 300 Frs.

[A driver made two trips: Kigali-Nyamata-Kigali and Kigali-Butare-Kigali. The distances of the two trips are 100 km and 240 km respectively. Given that the cost of the petrol per kilometre is 300 Frw and after the payment of the petrol for both trips, the driver remained with 20.000 Frw. Find the amount of money that each client paid the driver.]

The number of calculations in the task formulated by the students in group A are reduced as they stated the return distance for both trips. They also provided the students with the cost of petrol per kilometre and the total amount of money the taxi driver remained with. Finally they changed the formulation of the task. This task is formulated as a traditional word problem.

The task formulated by the students in group B is very close to the original task.

Group B

b) posons la question comme suit :

Un chauffeur d'une taxi-voiture transporte les passager vers Nyamata. La route Kigali-Nyamata vaut 50 km. Le prix du Carburant pour le trajet Kigali-Nyamata-Kigali vaut $\frac{1}{3}$ de la somme totale payée par ce passager.

Le chauffeur transporte un autre passager vers Butare. La distance Kigali-Butare est de 120 km. Le prix du Carburant consommé pour le trajet Kigali-Butare-Kigali vaut $\frac{2}{5}$ de la somme payée par ce passager.

Le prix de Carburant pour 1 km est le même pour les deux trajets. Le prix total du Carburant de la route Kigali-Butare-Kigali vaut 2,4 fois le prix du Carburant consommé dans la route Kigali-Nyamata-Kigali.

Calculer le prix montant resté dans la poche du chauffeur après ces deux trajets.

Discussion: Cette question est faisable dans le seul cas où le prix du Carburant par km est donné comme nous l'avons déjà fait.

[A taxi driver transported a passenger towards Nyamata and the distance Kigali-Nyamata is about 50 km. The cost of the petrol to cover that distance is 1/3 of the total amount of money that the passenger paid. The same driver transported another passenger towards

Butare and the distance of Kigali-Butare is about 120 km. The cost of the petrol to cover the second distance is $\frac{2}{5}$ of the total amount of money that the second passenger paid. The price of the petrol is the same for both journeys. The total cost of the petrol of the trip Kigali-Butare-Kigali is 2,4 times the cost of the petrol to cover the trip Kigali-Nyamanta-Kigali. Calculate the amount of money that the driver remained with after the two trips.

Warning: This task is solvable only if the price of the petrol per kilometre is given as we assumed while solving our original task.]

The only transposition the students do is to change the percentages into fractions. In addition, a specific price of petrol is provided in line with how they solved the original task where they assumed that the task is solvable if the price of petrol per km is given and is the same for both trips. Also this task is constructed as a regular word problem.

In the afternoon groups the task construction is different as the students got information about the cost of the petrol in the original task. However, the idea that the secondary students should solve the task by way of their knowledge of linear systems of equations was raised also by the afternoon groups, as can be seen in the following group discussion:

- Bugingo: The finished one was a question, now write b OK!
 Martin: /.../ Wait, wait! Kali..., what are you suggesting...
 Kalisa: /.../ I am saying that we should formulate the task related to the system of two equations with two unknowns.
 Martin: /.../ Are you sure that it is part of the program of the third T.C.(troc commun)?
 Kalisa: Oh yes
 Bugingo: Oh yes
 Justice: Oh yes
 Bugingo: /.../ Using the given data in our original problem we are going to formulate the requested task...
 Kalisa: /.../ Yes so that the students in secondary school will be able to solve it with the help of solving a system of two equations with two unknowns.
 Justice: OK
 Martin: OK , we too we used x and y.
 Bugingo: Yes

- Justice: /.../ If we formulate it in that way how can we make use /.../ that the students will be able to know immediately the unknowns?
- Martin: /.../ We must formulate it so that the exercise will be understandable on the level of the students /.../ so, we must adapt it to their level.

Group D

4/
 Un chauffeur fait un transport de 2 passagers:
 le 1^{er} de Kigali - Nyamata - Kigali
 le 2^{ème} de Kigali - Butare - Kigali.
 Les distances Kigali - Nyamata - Kigali et Kigali - Butare -
 Kigali sont respectivement 100 km et 240 km. Le prix de
 par 1 km pour les deux trajets est le même et vaut 50 Frw.
 Sachant que le 1^{er} client a payé 1
 Sachant que le prix du carburant pour le 1^{er} trajet
 vaut $\frac{1}{3}$ du montant total payé par le 1^{er} client et
 le prix du carburant pour le 2^e trajet vaut $\frac{40}{100}$ du
 montant total payé par le 2^e client.
 Déterminer le montant total resté dans la poche
 du chauffeur après avoir payé le carburant
 de deux trajets.

[A driver transports two passengers: the first to Kigali - Nyamata - Kigali, the second to Kigali - Butare - Kigali. The distances Kigali - Nyamata - Kigali and Kigali - Butare - Kigali are 100 km and 240 km respectively. The price of petrol per one km is the same and is equal to 50 Frw. Knowing that the price of petrol for the first trip, equals $\frac{1}{3}$ of the total amount that the first client paid and the the price of petrol for the second trip is $\frac{40}{100}$ of the total amount paid by the second client. Find out the total amount that remained in the driver pocket after he has paid the petrol for both trips.]

Students in the afternoon group D tried to stay close to the data given in the original task. In fact, they kept two trips with their respective number of kilometres as well as the question that the students were

expected to answer. However, instead of mentioning information related to the price of petrol to cover a certain distance, they provided the exact cost for travelling one kilometre. Also, instead of using both fractions and percentages, as in the task given to them, they used only fractions. The task is formulated as a traditional word problem exercise.

The task constructed by group C resembles the one of group A.

Group C

b) Un chauffeur a fait deux trajets Kigali-Nyamata-Kigali et Kigali-Butare-Kigali. Sachant que le trajet Kigali-Nyamata-Kigali est de 100 km et le trajet Kigali-Butare-Kigali est de 240 km. Trouver le montant que chaque client lui a payé si après avoir payé le carburant qu'il a utilisé il lui reste au total 20000 Frw et le prix du carburant par km est de 50 Frw sachant que le montant payé par trajet est proportionnel au nombre de km parcourus.

[A driver made two trips: Kigali-Nyamata-Kigali and Kigali-Butare-Kigali. The distances of the two trips are 100km and 240km respectively. Find the amount of money that each client paid the driver given that the cost of the petrol per kilometre is 50 Frw, after the payment of the petrol for both trips, the driver remained with 20.000 Frw and the amount of money paid for a trip is proportional to the number of kilometres covered.]

Although, this task resemble the one of group A it is made slightly more complex by group C as they provided a new mathematical concept when they stated that the amount of money paid for a trip is proportional to the number of kilometres covered.

In sum, what is common in both interviews, group discussions and students task constructions is that the students expected the secondary school students to solve a system of two equations like they did in their task solving. Also, they made several transformations which they

motivated with arguments that the tasks should correspond with the curriculum and be understood by the secondary school students.

5.3.2 Task related to the house construction workplace

In the group interviews concerning the formulation of tasks related to the house construction workplace, the afternoon groups asserted that they designed tasks related to house construction profession.

- Interviewer: How do you think the calculations of profit margin should require any kind of mathematical techniques?
- Justice (D): Well/.../ rule of three, addition, multiplication and subtraction. I think.
- Interviewer: Thank you. Group C, would you like to tell us something about (b) question?
- Molice (C): For us, you know some arithmetic operations such as addition, subtraction and multiplication and /.../ Yaa there was also the use of the rule of three.

However, in their group discussion both groups insisted that the formulation of the task should be based on the idea that the secondary school students should find out the profit margin. This is confirmed in the following discussion from group D:

- Justice: Now our next step /.../ we can formulate a task for secondary school students.
- Kalisa: I think the question will be based on the profit margin/.../
- Justice: /.../profit margin?
- Kalisa: Yes we assume that there is a builder who wishes to construct a house and he needs cement, transport etc/.../ but we must make the question shorter than that one.
- Bugingo: Listen could we take this money which was spent for everything and /.../
- Martin: Please let us formulate the question and go home
- Bugingo: Yes let us do it...
- Martin: While you are busy in formulating it I am going to write the final draft of a) task of this second exercise.
- Kalisa: OK
- Bugingo: OK
- Justice: OK
- Bugingo: /.../ A builder wishes to construct a house of cement blocks. The expenses and transport of materials are/.../

- Kalisa: /.../ OK if one m^3 of cement blocks is estimated at 20000 Frw and the transport of materials is estimated/.../. No this can be solved mentally let us try to formulate the task which requires some serious thinking while solving it.
- Martin: Listen what I wrote /.../ a builder wishes to construct a house of cement blocks /.../ OK
- Bugingo: Mm
- Justice: Mm
- Justice: Mm
- Martin: He has chosen one m^3 of cement blocks as unit. One m^3 contains 50 blocks and each block is estimated at 400 Frw....Do you follow me?
- Bugingo: Yes
- Kalisa: Yes
- Justice: Yes
- Martin: /.../For that reason he needs 0,65 m^3 of sand, 0,3 m^3 of water. If one m^3 of sand costs 8000 Frw, one m^3 of water costs 2500 Frw and the labour payment is estimated at 2700 Frw /.../, you are requested to find out the profit margin.
- Justice: But the cement is not included...
- Martin: Ya we did not
- Bugingo: Tell me how old are the students of third year secondary school T.C?
- Kalisa: Mostly between 15 and 18 but you can find some particular cases.

In this discussion, the participants did not consider all items while designing the task as they wanted to reduce it compared to their own original task. At the same time, they wanted to make the task more complex so that it would require secondary students to think seriously while they are solving it. Also, the students' division of labour is seen in how they distribute the tasks between themselves. Moreover, although they noticed that they did not include the component of cement in their formulation they did not seem to bother, probably because the task is written by hand and they wanted to go home. The result of the discussion of group ended up with the following formulated task:

Group D

Un constructeur veut construire une maison de bloc ciment à son client.
Il a estimé l'unité 1m^3 de blocs ciments, or 1m^3 a 50 blocs ciment et le prix d'un bloc ciment est de 400 Frw.
Il a besoin aussi $0,65\text{m}^3$ de sable et $0,3\text{m}^3$ d'eau et 1m^3 du sable coûte 8000 Frw et 1m^3 d'eau coûte 2500 Frw et que la main d'oeuvre lui faut 2700 Frw. Alors il a fixé le prix à 40 000 Frw pour la construction de cette maison. Déterminer la marge bénéficiaire.

[A builder was asked to construct a house of cement blocks for someone. The builder has chosen one m^3 as a unit of the quantity of materials and assumed that 1m^3 includes 50 cement blocks and the cost of 1m^3 is estimated at 400 Frw. The builder needs $0,65\text{m}^3$ of sand and $0,3\text{m}^3$ of water. If the cost of 1m^3 of sand is 8000 Frw and 1m^3 of water is 2500 Frw, and the cost of labour is 2700 Frw; then the price to construct that house is 40.000 Frw. Find out the profit margin.]

The students in group D started with a short narrative statement to introduce the task. They mentioned all the units at the same prices as stated in the original task, but left out the cement. They transformed the information given and stated the total cost of the salaries. Also, they changed the total cost of raising the wall, and assumed that it was the cost of building the whole house. The task, to find the profit margin

is similar to the original. Seemingly, they assumed that their students know what a profit margin is.

This task is transposed to a high degree by the students and the text would be very difficult to understand by secondary school students as it lacks coherence by not respecting comprehensiveness and fidelity towards the original contextualised task. The cost stated for the house is quite unrealistic and it would be impossible to erect the wall without cement.

Group C formulated a minimal task where all figures are included except salaries for the workers.

Group C

b) Un constructeur veut construire une ~~maison~~ ^{un mur} ~~et~~
Pour ce faire il a besoin de:
- 50 blocs ciment (400 Frw par bloc)
- 90 kg de ciment (150 Frw/kg)
- 0,65 m³ de sable et (8000 Frw/m³)
- 0,3 m³ d'eau (2500 Frw/m³)
Quel est l'intérêt si on lui a payé au total 51285 Frw.

[A builder wants to build a wall. For that reason he will need :

- 50 cements blocks (400 Frw per block)
- 90 kg of cement (150 Frw per kg)
- 0,65 m³ of sand (8000 Frw per m³)
- 0,3 m³ of water (2500 Frw per m³)

What is the constructor's profit if he was paid 51285 Frw?]

This task is formulated as a regular word problem easy to solve and lacks the local cultural context. Fidelity is kept in the quantities provided but there is a lack of comprehensiveness in relation to the original task and the workplace setting as the labour costs are not included and the context being strongly reduced.

5.3.3 Task related to the restaurant management workplace

Concerning the formulation of a task related to the restaurant management workplace, in the group interview with the morning groups, they affirm that they formulated shorter tasks comparing to the original one and assumed that in order to solve that task secondary school students are expected to apply elementary mathematics such as addition, subtraction and multiplication.

- Interviewer: Fine how do you think about the restaurant management task on that issue?
- James (A): On the basis of our calculated results we tried to make a shorter task for the students and instead of 70 people we have chosen 100 people.
- Charles (B): We in our group we referred to our calculated results and thought that students of third year should be able to apply elementary mathematics like addition, subtraction and multiplication.

In the discussion in group A, one member of the group proposed a task and the others agreed that it could be their task for the secondary school students.

- Felix: Wait we did not yet formulate a task for secondary school students I mean b) question.
- James: This is also for secondary school students?
- Jackson: Yes it is
- Felix: Before we start to formulate let us look at what was our question. Then I think it can be our target to formulate it /.../
- James: Look down here ... the question is about the total amount she received and profit/.../
- Jackson: Formulate a task from the data above /.../ OK listen what I suggest to formulate /.../
A restaurant owner has paid: 7000 Frw for 10 kg of rice, 4000 Frw for 8 kg of macaroni, 10.000 Frw for vegetables, 5500 Frw for 5 kg of cow meat. After cooking the food for her 100 customers, the cost of one meal was fixed at 800 Frw. Calculate the total amount of money she received at the end of the party.
- James: We did not include drinks
- Roza: There is no problem
- Felix: OK!

In the dialogue, the participants wanted to produce a quick formulation of the task. When they realized that they did not include costs of the drinks they did not bother about it. Instead they accepted the task as it is formulated below.

Un restaurateur a acheté: 10 kg de riz à 7000 Frs
, 8 kg de macaroni à 4000 Frs, les légumes
à 2000 Frs, 5 kg de viande à 1100 Frs/kg.
Après avoir préparé la nourriture à des
100 clients, le prix du repas a été
fixé à 800 Frs.
Trouver le montant total qu'il a reçu
après la réception et le bénéfice.

[A restaurant owner purchased 10 kg of rice at 7000 Frw, 8 kg of macaroni at 4000 Frw, 5 kg of cow meat at 1100 Frw each kg. After cooking the food for the 100 customers, each meal was sold at 800 Frw. Find the total amount and profit that the restaurant owner gained after the reception party.]

As can be seen in the above written work of group A, several transpositions were made. The participants based their task on calculated results of some food products and specified the cost of one meal and the number of clients which they changed to 100 instead of 70. They reduced the task by not mentioning any kind of drinks. They kept the idea of the task as they asked the secondary school students to find the total amount of money and the profit as was requested in their original task.

However, group B proceeded differently as can be seen in the task below.

Exercice n° 6.

b) Un restaurateur achète les nouritures à 2750 F et les boissons à 41250 F. Il vend toutes les boissons à une somme totale de 52480,7 Frw. Il s'écrit 70 personnes et le prix d'un repas est estimé à 800 F. Calculer la somme totale d'argent qu'il a reçue après la réception et l'intérêt total du restaurateur.

[A restaurant owner purchased a variety of food product at 2750 Frw and various drinks at 41250 Frw. Then the drinks were sold in total at 52480,7 Frw. 70 people bought a meal at 800 Frw in that restaurant. Calculate the total amount of money and profit that the restaurant owner gained after the reception.]

It should be pointed out here that the quantity of 2750 Frw provided is a mistake as the given quantity was 27550 Frw.

In the transpositions the students mentioned the total purchased costs of all food products and drinks respectively, the number of clients and the cost of one meal. Then they requested the secondary students to calculate the total amount of money and profit. The situation and context in the original task given to these student teachers is strongly reduced with a low degree of comprehensiveness. The suggested task is more similar to a traditional school mathematics word problem than to a contextualised task with relevance towards mathematics involved in local workplaces.

In sum, the student teachers' designed tasks to be solved by secondary school students can be characterised as condensed word problems with a focus on contextualised quantitative data with a minimum of contextual descriptions with low degree of representativeness. There is a low fidelity on activity (examples: 'driver' instead of 'taxi driver', 'constructor' instead of 'house constructor', 'wall' instead of 'house'), and in some cases low comprehensiveness (examples: weak reference to local placement, food costs not separated on different products; drink costs not included in one account). Components included in all three workplace contexts that allow a critical discussion are the levels of profit and of costs for different products. In the case of the taxi driver the latter refers to the

use of cars, for the house construction workers' wages, and for the restaurant owner the drinks in relation to the food. The number of words in the different tasks designed by the students was for:

- Taxi driving: A: 58 words, B: 148 words, C: 74 words, D: 110 words (average: 98)

- House construction: C: 57 words, D: 100 words (average: 79)

- Restaurant management: A: 55 words, B: 64 words (average: 60)

With an average word count of 83, the word problems that the student teachers constructed for the secondary school students were thus much shorter than the original ones given by the researcher to the student teachers. In particular, the restaurant management task was made short by excluding many details of the specific products.

To characterise the mathematical organisations that are embedded in problem posing processes, the participants' tasks consisted of formulating mathematical tasks that could be solved by the third year secondary school students. In the processes of formulation of the tasks, they thought about mathematical techniques which could help secondary school students to solve the tasks. The mathematical techniques that they suggested are similar to those ones which they used themselves while they were solving the a) sub-question i.e. the use of a system of two linear equations, the rule of three and the use of some arithmetic operations of the real numbers (addition, subtraction, multiplication and division). In their problem posing processes, prospective mathematics teachers do not explicitly justify why secondary school students are supposed to use their suggested techniques. This could be explained by the fact that in their original contextualised tasks, they are no longer playing the role of problem solvers; rather they are playing the role of the members of the noosphere (mathematics teachers). This means that prospective mathematics teachers assumed that the mathematical technology and theory (included in mathematics curriculum) are supposed to be taught to the secondary school students, i.e. the content of mathematical knowledge to be taught is assumed to be delivered to the secondary school students before they will solve the tasks.

5.4 Student teachers' reflections on tasks related to workplaces

When the students were asked about their views about including tasks from workplace contexts in their mathematics teaching programme, they responded positively in the sense that a real situation can help to explain how mathematics works outside the school contexts by providing examples of how workers use mathematics while they are estimating prices, purchasing and so on.

Interviewer: Very good. Just a last question: How do you think if you are asked to include that kind of materials in your teaching programme?

Jackson (A): Personally I found it interesting and for us who are expected to teach mathematics, it can help us to explain to the students with real examples how mathematics works in different workplace activities.

Jonas (B): Yes I agree with Jackson, but the question is that it is time consuming. Look for example, [pin-pointing interviewer] before you brought and formulated the tasks, it was necessary that you went down to the different workplace settings and you conducted some interviews as you told us. In my view this is a hard work for a secondary school teacher who must cover 28 contact hours of teaching load per week.

Felix (A): But it depends, if it should be seen as a necessity for the curriculum programme development, I think it can be possible. I believe that where there is a will there is a way. Why not?

Bana (B): Honestly for me it is hard at the same time to conduct research and teach in our school situation. But mostly current school mathematics books were produced via several research projects, I think whenever it should be a need for teaching and learning, the stakeholders could organise a project for that.

A separate interview with groups C and D gave very similar answers.

Interviewer: OK now I have the last question: How do you think if you are asked to include that kind of materials in your teaching programme?

Manzi(C): In my point of view it is interesting to explain and teach mathematics in the real living context, but this

requires much time and I think it is almost impossible for a teacher to teach and collect the data from his/her living environment.

Bugingo(D): Speaking of teaching mathematics in the real context is very nice, for instance I did not know the builder has several hints for earning a job, so you see you get more information. But what I am wondering is that what my classmate said; teaching in secondary school is very tough so that you cannot at the same time get time to cover a teaching load of 28 hours per week and conduct what I may call a small research.

Joys(C): Actually it is nice but difficult to implement.

Martin (D): We never know, maybe today it is difficult, but tomorrow it will be possible.

Although the students can see the value of contextualized tasks they also realize that it would be time consuming to construct them. Some of them reject the idea and suggest that others could deliver such tasks if necessary. However, both groups express that constructing such tasks might be possible in the future.

5.5 Activities of experiencing to solve and transpose mathematical tasks

For Leont'ev (1981), activity is a motive driven process, which consists of goal driven actions depending on operations driven by certain conditions. The following analysis shows an interpretation of how mathematics is visible in that process in the current study of student teachers.

Activity	Mathematics teacher education
Motive	Gaining mathematics knowledge for future profession
Action:	Solving prescribed mathematical task, and posing new tasks
Goal:	To provide data for mathematics education research
Operations:	Reading, talking, writing and thinking
Conditions:	Limited time, access to necessary tools (own knowledge, calculators, paper and pencils, and researcher's clarifications)

According to Engeström (1987), a human activity has subject(s), instrument(s), object(s) that lead to outcome(s), division of labour, community and rules as its essential elements. From the findings of student teachers, those elements are illustrated as follows

- Subject:** Four groups of future mathematics teachers
Object: Solving prescribed mathematical tasks and posing new tasks
Tools: Language, pens pencils, papers, calculator, own mathematical knowledge
Rules: It is possible to work in collaboration, use any tool or ask for information if necessary in a limited time.
Community Student teachers, researcher
Division of labour: Role of the students, role of researcher
Outcome: Set up answers to the tasks in written form

From the findings of the current study, the overarching activities involve Rwandan mathematics educators and researchers (*community*). Those activities are characterised by the problem solving processes of four groups of future mathematics teachers (*subjects* of the activities). The tasks were solved in a restricted time and participants were allowed to use any tool or ask information if necessary (*rules*). While dealing with solving and posing three contextualised mathematical tasks (*object* of the activities) related to three different workplace settings, participants used various means and *tools* (language, pens, papers, calculator and mathematical knowledge) to set up answers and design tasks for secondary school students (*outcome* of the activities). While solving tasks, collaboration within the members of their respective groups and asking clarification about the tasks from the researcher (*division of labour*) took place.

Within these problem solving processes, the common feature was that at the beginning, the participants of each group started by individual reading which was followed by collaborative work through the group members discussions. Their discussion was mainly dominated by questions and suggestions about how to solve the tasks. When consensus about how to proceed is reached the model was

constructed, solved and evaluated to make sure that the answers to the tasks are valid. For instance, while solving the task related to taxi driving workplace, the discussion of the morning groups shows how the constructed model was a results of long discussion.

6 MATHEMATICAL ACTIVITIES AT A SECONDARY SCHOOL SETTING

In this chapter, I start to analyse my transpositions of the future mathematics teachers' tasks by relating them to the original task I gave them and the data I got from the workplaces. After that, I analyse the nature of mathematical organisations that were carried out by the secondary school students when dealing with the tasks designed for them. Next, the students' reflections on the tasks are analysed. Finally, I analyse the processes in line with activity theory.

6.1 Reconstructing the transposed mathematical tasks

It was imperative to make the tasks constructed by the student teachers more understandable so that the reading could be clear for the secondary school students. In the following analyses I first consider my transpositions related to the original task formulated in French for the student teachers. Secondly I analyse my transpositions related to the tasks constructed by the student teachers, which also were formulated in French. Below is my translated formulation of the taxi driving task which is similar to the one formulated by group B.

Task related to the taxi driving workplace setting

One day, a taxi driver based in Kigali city transported a customer towards Nyamata centre (Eastern Province) and they turned back together to Kigali. Having arrived at Kigali, the same taxi driver had the chance to transport a second customer towards Butare and they turned back together to Kigali. The rough distance between Kigali and Nyamata is 50 km, whereas the rough distance between Kigali and Butare is 120 km. According to the information provided by the driver, the cost of the used petrol was estimated at 50 Frw per one kilometre for both trips (50 Frw/km). Knowing that the cost of the petrol for the first trip is worth $\frac{1}{3}$ of the total amount of money that the first customer paid, and the cost of petrol for the second trip is worth 40% of the total amount of money that the second customer paid, you are asked to find the driver's balance after payment of the petrol for both trips.

The task formulated by group B contains the same information as my original task. My transposition consists of presenting the task in a narrative form and I stated the concrete price of the petrol. Hence, I did not include their warning to the students.

Using the information from the house constructing context I formulated a task in line with my original task but without specifying the factors which influence the estimation of the total costs of building a house. Also, I did not mention the concept 'profit margin'. Moreover, details about the reflections the builder made on strategies to get a job are not mentioned.

Task related to the house construction workplace setting

To earn the job market of constructing a house with cement blocks, a builder (based in South Province) acted in the following way: He considered one m^3 to be the unit of quantities of materials. In one m^3 of cement blocks, there are 50 blocks. The cost of one block is estimated at 400 Frw. To elevate a wall of 50 blocks in one working day, the builder envisages one bricklayer and two assistant bricklayers. For that reason he needed 0,65 m^3 of sand, 0,3 m^3 of water and 90 kg of cement. The cost of one m^3 of sand was 8000 Frw, the cost of one m^3 of water was 2500 Frw and the cost of one kg of cement was 150 Frw. The wage of one bricklayer was 1500 Frw whereas the wage for an assistant bricklayer was 600 Frw a working day. Having calculated the cost of whole house, the approximated cost that allowed the builder to earn the job was fixed at 51285 Frw for elevating a wall of cement blocks contained in one m^3 . Your task consists of finding his profit for elevating that wall.

The task is built on the one suggested by group C. My transposition is that I added the information about cost for labour per day. Finally, I formulated the task in a narrative form which would be understood by the secondary students.

The task related to the restaurant management is based on my own original task for the student teachers with some additional information given by the restaurant owner.

Task related to the restaurant management setting

70 persons celebrated a party in one of the restaurants which are based in Kigali. Before the day of the party, the customer ordered one meal for each guest and different kinds of drinks. According to the information

given the restaurant owner spent 41 250 Frw for drinks and 27 550 Frw for different food products. After the party, she noticed that she earned 17 750 Frw in profit for the sale of drinks and the total amount of money acquired on the order was 115 000 Frw. Your task consists of finding the cost of one meal and the total profit the restaurant owner gained.

The task is, to some degree, built on the one suggested by group B where they stated costs for food, drinks, the total amount of money the restaurant owner got after selling the drinks, and number of guests. In my transposition, I provided the profit that the restaurant owner gained for drinks and the total amount of money she received for the reception. Then secondary students were asked to find out the price of one meal. Finally, I formulated the task in a narrative form which would be understood by the secondary students.

To summarize, the researcher's designed tasks to be solved by secondary school students can be described as condensed word problems including contextualised quantitative data framed by a contextual description with a medium degree of representativeness in taxi driving and house construction and a low degree of representativeness in restaurant management. There is strong reference to local placement on taxi driving and includes some information for critical discussion such as the level of costs for petrol and the level of profit. A weaker reference to local placement is found in the house construction and restaurant management tasks but they include information for critical discussion of the level of profit, of workers' wages in the case of house construction and of costs for drinks as compared to food in restaurant management task. The three different workplace tasks have an average word count of 153, distributed as follows:

- Taxi driving: 161 words
- House construction: 191 words
- Restaurant management: 106 words

The principles for transposing the student teachers' designed tasks were to keep their ideas but to allow some clarification and coherence in constructing sentences compared to what they proposed and to increase the degree of representativeness.

6.2 Secondary students solving contextualised tasks

6.2.1 Task related to the taxi driving workplace

This task was assigned to six groups. Four of them solved the task successfully and two groups failed due to the fact that they did not notice that a trip means to go from somewhere and come back to the starting place. In the interviews which I conducted with them, they affirm that they used equations, addition, multiplication, subtraction and fractions of numbers to solve the task.

- Interviewer: OK how about you people who solved the task related to drive a taxi?
- Kamanzi: For us it was not easy to understand the text but after the reading, we used an equation and it was fine.
- Interviewer: All of you, you used equations
- Cecile: Yes
- Sofia: No, sometimes we used addition, multiplication, subtraction and fractions of numbers.

However, looking at how all six groups proceeded to solve the task, two methods were adopted: The first one consists of calculation of the total amount spent on petrol for each trip and adding the two sums, then calculation of the money that each client paid to the taxi driver and adding the two sums. The answer to the task is then found by subtracting the two sums.

Réponse

les données

- la distance approximative entre Kigali et NYATATA est 50km
- la distance approximative entre Kigali et BUTARE est 120km
- le prix du carburant est 50Frw/km pour les 2 trajets.
- le prix du carburant pour le 1^{er} trajet vaut 1/3 du montant total que le premier client a payé
- le prix du carburant pour le 2^{es} trajet vaut 40% du montant total que le 2^{es} client a payé.

INCONNU

Le montant total resté dans la poche de ce chauffeur après avoir payé le carburant des deux trajets = ?

SOLUTION

le prix total du carburant de Kigali vers NYATATA = $(50 \text{ km} \times 50 \text{ Frw}) \times 2$
= 5000 Frw

le prix total du carburant de Kigali vers BUTARE est $(120 \text{ km} \times 50 \text{ Frw}) \times 2$
= 12000 Frw

le prix total du 1^{er} client : $5000 \times 3 = 15000 \text{ Frw}$

le prix total du 2^{es} client ; $\frac{12000 \times 5}{40} = 15000 \text{ Frw}$

le prix total du carburant de 2 trajet = $5000 + 12000 \text{ Frw}$
= 17000 Frw

le prix total de 2 client ont payé = $15000 \text{ Frw} + 15000 \text{ Frw}$
= 30000 Frw.

le montant total resté dans la poche de ce chauffeur
= $30000 \text{ Frw} - 17000 \text{ Frw} = 13000 \text{ Frw}$

In the above written work three of Engeström's steps in an expansive learning process are followed: (1) set up the data provided (données) in the task as a way of understanding the task, i.e. questioning (2) set up what must be found (inconnu) i.e. analyse what the task is directed to and (3) the solution of the task, that is, they applied a known model. In their solution processes the students did as follows:

The total cost of petrol for the first trip (Kigali-Nyamata-Kigali) is
 $(50 \times 50km) \times 2 = 5000Frw$

The total cost of petrol for the second trip (Kigali-Butare-Kigali) is
 $(50 \times 120km) \times 2 = 12000Frw$

The total amount paid by the first client is
 $5000Frw \times 3 = 15000Frw$

The total amount paid by the second client is
 $\frac{12000Frw}{40} = 30000Frw$

The total cost of the petrol for both trips is
 $5000Frw + 12000Frw = 17000Frw$

The total amount paid by both clients is
 $30000Frw + 15000Frw = 45000Frw$

The total amount remained within the driver's pocket is
 $45000Frw - 17000Frw = 28000Frw$

The second method adopted can be seen in the following written document:

Réponse :

Les données

- la distance approximative entre Kigali et Nyamata est de 50 km
- la distance approximative entre Kigali et Butare est de 120 km
- le prix du carburant à un kilomètre est de 50 FRS
- le prix du premier client pour le premier trajet est de $\frac{1}{3}$ du montant total
- le prix du carburant pour le deuxième trajet est de 4000

Inconnu

- le montant total reste dans la poche du chauffeur après avoir payé le carburant des deux trajets.

Solution

- le prix du carburant en 1 km = $\frac{50 \text{ FRS}}{\text{km}} \times (50 \text{ km} \times 2) = 5000 \text{ FRS Kigali-Nyamata}$
- le prix du carburant du 2^{ème} trajet = $\frac{50 \text{ FRS}}{\text{km}} \times (120 \text{ km} \times 2) = 12000 \text{ FRS}$

x = le montant total que le client a payé

$\frac{1}{3} x$ = l'argent que le chauffeur a payé pour acheter le carburant

y = le montant total que le deuxième client a payé

$\frac{40 \cdot y}{100}$ = le montant total ~~payé~~ du carburant que le 2^{ème} a payé

$$\frac{12000}{3} = \frac{1}{3} \cdot x = \frac{5000 \text{ Frs}}{1}$$

$$= \frac{x}{3} = 5000 \text{ Frs}$$

$$x = 15000 \text{ Frs}$$

2^{ème}

$$\frac{40 \cdot y}{100} = \frac{12000}{1}$$

$$40y = 12000 \text{ Frs} \times 100$$

$$40y = 1200000 \text{ Frs}$$

$$y = \frac{1200000 \text{ Frs}}{40}$$

$$= 30000 \text{ Frs}$$

Le montant total qui reste dans la poche du chauffeur =
 $(30000 \text{ Frs} + 15000) - (5000 + 12000)$

$$= 28000 \text{ Frs}$$

Like their classmates, the students only reached the third step in Engeström's model of expansive learning. They started by setting up the data provided (données), then they set up what must be found (inconnu) and ended up with the resolution of the task (solution). In their solution processes, they proceeded as follows:

The cost of petrol for the first trip (Kigali-Nyamata-Kigali) is

$$50 \text{ Frw}(50 \text{ km} \times 2) = 5000 \text{ Frw}$$

The cost of petrol for the second trip (Kigali-Butare-Kigali) is

$$50 \text{ Frw}(50 \text{ km} \times 2) = 12000 \text{ Frw}$$

x = the total amount that the first client has paid.

$\frac{1}{3}x$ = the money paid by the driver to cover the distance of the first trip

y = the total amount that the second client has paid

$\frac{40y}{100}$ = the total amount to cover the distance of the second trip

$$1^{\text{st}} \quad \frac{x}{3} = \frac{5000Frw}{1} = 5000Frw$$

$$x = 15000Frw$$

$$2^{\text{nd}} \quad 40y = 12000Frw \times 100$$

$$40y = 1200000Frw$$

$$y = \frac{1200000}{40}$$

$$y = 30000Frw$$

The total amount that remained within the driver's pocket is

$$(30000 + 15000) - (5000 + 12000) = 28000Frw$$

To characterise the mathematical organisations that are embedded in these task solving processes, the task for those groups was 'to find the driver's balance after the payment of the petrol for both trips'. The technique used is characterised by a combination of the use of the rule of three in mathematics and the solution of a linear equation with one unknown. For instance, to calculate the cost of petrol for both trips they proceeded as follows:

$$\frac{1km}{2 \times 50km} = \frac{50Frw}{a} \quad \text{and} \quad \frac{1km}{2 \times 120km} = \frac{50Frw}{b}$$

$$a = 5000Frw \quad \text{and} \quad b = 12000Frw$$

By using these two results, they formed two linear equations

$$\frac{x}{3} = 5000Frw \quad \text{and} \quad 40y = 1200000Frw$$

These two equations were solved in the context of algebra. From the beginning of the task solving till the end, multiplication, addition

division and subtraction operations were applied where it was necessary.

6.2.2 Task related to the house construction workplace

This task was assigned to six groups. All of them solved the task successfully. In the interviews which I conducted with them they affirm that they used addition, subtraction and multiplication.

Interviewer: OK let us then move to people who solved the task related to house construction.

Josephine: For us there was a problem of calculation of the money spent to purchase each quantity of material and the payment of bricklayer and assistants. But when we found them it was easy to find the profit through the use of addition, multiplication and subtraction of numbers

Interviewer: Did all other groups proceed in the same way?

Jeanne: Yes but the most important thing was to read the text, because it was too long

All of them adopted the same method (technique) which can be observed in the following written document.

Réponse

Les données

$$m^3 = 50 \text{ blocs ciment}$$

$$1 \text{ bloc ciment} = 400 \text{ Frw}$$

$$\text{maçon} = 1500 \text{ Fr} / \text{Jr}$$

$$\text{deux aides} = 600 \text{ Fr} \times 2 = 1200 \text{ Fr} / \text{Jr}$$

$$\text{Sables} = 0,65 m^3 \text{ tandis que } 1 m^3 = 8000 \text{ Frw}$$

$$\text{eau} = 0,3 m^3 \text{ tandis que } 1 m^3 = 2500 \text{ Frw}$$

$$\text{ciments} = 90 \text{ kg} \text{ tandis que } 1 \text{ kg} = 150 \text{ Frw}$$

$$\text{Le prix fixé} = 51285 \text{ Frw}$$

Demande

Le prix estimatif d'un constructeur = ?

calculs

$$\text{Pour 50 blocs ciment} = 400 \text{ Frw} \times 50 = 20.000 \text{ Frw}$$

$$\text{Pour sables} \Rightarrow m^3 \rightarrow 8000 \text{ Frw} \times 0,65 m^3 = 5200 \text{ Frw}$$

$0,65 m^3 \rightarrow$

$$\text{Pour l'eau} \Rightarrow m^3 \rightarrow 2500 \text{ Frw} \times 0,3 m^3 = 750 \text{ Frw}$$

$0,3 m^3 \rightarrow$

$$\text{Pour les ciments} \Rightarrow \text{kg} \rightarrow 150 \text{ Frw} \times 90 \text{ kg} = 13500 \text{ Frw}$$

$90 \text{ kg} \rightarrow$

a) Le prix estimatif de cet constructeur =

$$\Rightarrow 20.000 \text{ Frw} + 5200 \text{ Fr} + 750 \text{ Fr} + 13500 \text{ Fr} + 1200 \text{ Fr} + 1500 \text{ Fr} = \underline{\underline{42150 \text{ Frw}}}$$

b) Intérêt de cet constructeur = $51285 \text{ Frw} - 42150 \text{ Frw} = \underline{\underline{9135 \text{ Frw}}}$

Like their classmates, in the task solving processes, they also first of all set up the data as it is provided in the task. Secondly, they set up what the task is directed to and finally they proceeded to find out the answer to the task. According to the content of their written work, their calculation was done as follows:

The cost of 50 cement blocks is

$$400Frw \times 50 = 20000Frw$$

If one m^3 of sand costs 8000 Frw, then the price of $0,65m^3$ of sand is

$$8000Frw \times 0,65m^3 = 5200Frw$$

If one m^3 of water costs 2500 Frw, then the price of $0,3m^3$ of water is

$$2500Frw \times 0,3m^3 = 750Frw$$

If one kg of cement costs 150 Frw, then the price of 90 cement blocks is

$$150Frw \times 90kg = 13500Frw$$

The wage of one bricklayer and two assistants is

$$1500Frw + 600 \times 2 = 2700Frw$$

Therefore, the estimated amount of money that the builder spent is

$$20000Frw + 5200 + 750 + 13500 + 2700 = 42150Frw$$

Since the estimated price that allowed the builder to earn the job is 51285Frw, then the builder's profit is

$$51285Frw - 42150Frw = 9135Frw$$

In these mathematical task solving processes, the participants' task was to 'find the builder's profit' The technique used is characterised by the application of the rule of three in mathematics while they calculated the price of the needed quantities of each material (cement blocks, sand, water and cement). This can be seen for instance in their written work

$$\frac{1}{50} = \frac{400}{a} \Leftrightarrow a = 20000Frw$$

$$\frac{1}{0,65} = \frac{8000}{b} \Leftrightarrow b = 5200Frw$$

$$\frac{1}{0,3} = \frac{2500}{c} \Leftrightarrow c = 750Frw$$

$$\frac{1}{10} = \frac{150}{d} \Leftrightarrow d = 13500Frw$$

To find the builder's profit, multiplication, addition and subtraction operations were applied in the context of elementary arithmetic.

6.2.3 Task related to the restaurant management workplace

Like the two previous tasks, this task was also assigned to six groups. Three of them solved the task successfully while the other three groups failed to find the cost of one meal but they successfully found the total profit. In the interviews which I conducted with them, they affirm that they used addition, subtraction and division of numbers.

- Interviewer: I see your written work, please tell me how you managed to solve the third task.
- Agnes: To solve our third task /.../
- Interviewer: What kind of task did you solve?
- Agnes: The management of the restaurant task /.../ we tried to understand first the task and by using the data in the task we solved it.
- Interviewer: Please explain to me how you have done it.
- Agnes: We made some addition and subtraction of numbers.
- Interviewer: Other groups? How have you done it?
- John: Yes we also made addition, subtraction and division to find the price of one meal

While the task solving processes were going on, the method adopted by the students can be seen for instance in the following document:

Réponse

Donnée

70 personnes dépense 27500 F pour les différents produits alimentaires et 41250 F pour les boissons.

Il a gagné 17750 F d'intérêt de boissons la commande était 115000 Frw.

On demande :

- * le prix d'un repas
- * l'intérêt total

$$\text{calcul: } 27500 \text{ F} + 41250 \text{ F} = 68800 \text{ F}$$

$$\times \text{ Intérêt total} = 115000 - 68800 = 46200 \text{ F}$$

$$\times \text{ Intérêt du repas} = 46200 \text{ F} - 17750 \text{ F} = 28450 \text{ F}$$

$$\times \text{ total de la dépense du repas} = 28450 \text{ F} + 27500 = 56000 \text{ F}$$

$$\times \text{ prix d'un repas} = \frac{56000 \text{ F}}{70} = 800 \text{ Frw}$$

In the written work, similar to their classmates, they started by setting up the data. Next, they set up what is expected to be calculated, and finally they complete the calculation process. First they calculated the total amount that the restaurant owner spent to purchase all drinks and food products i.e. $27550 + 41250 = 68800 \text{ Frw}$.

Since the amount of money that the restaurant owner received after the party is known (115000 Frw), the total profit gained is $115000 - 68800 = 46200 \text{ Frw}$, and the profit from the sale of food is $46200 - 17750 = 28450 \text{ Frw}$.

In order to calculate the cost of one meal, the students first found out the total amount of money that the restaurant owner got from the sale of food, i.e. $28450 + 27550 = 56000$ Frw. Then they divided that amount by the number of customers and they found 800 Frw.

In these mathematical task solving processes, the participants' task consisted of 'finding the cost of one meal and the total profit that the restaurant owner gained'. The technique used is characterised by the use of addition, subtraction and multiplication of numbers which are applied in the context of elementary arithmetic.

In sum, in the problem solving processes the secondary school students used different mediating tools including mathematical knowledge. These problem solving processes are characterised by the use of the rule of three in mathematics where elementary arithmetic operations of numbers are applied. In addition, to solve the task related to the taxi driving workplace setting, the notion of solving a linear equation was introduced in the context of algebra. However, none of the tasks seem to be mathematically challenging enough to lead to expansive learning.

6.3 Secondary students' reflections on the contextualized task

In the interviews held, after the students at the secondary school setting had collectively accomplished their contextualized tasks, they came up with the following opinions:

Interviewer: What do you think about solving mathematical tasks which are formulated in the context of a real situation?

Beata: It is interesting because most of the exercises we solve look like strange without any real Rwandan context. They do not illustrate our real situation. At least here I got some information from the builder.

Cecile: Yes it is [interesting]. For example, I know very well the road Kigali-Butare but I did not know before that the price is linked to the price of petrol.

Interviewer: Do you think that these kinds of tasks can help to understand mathematics in a deeper way?

- Daniel: I think that it depends /.../ one can say yes, another no.
- Josée: Anyway, these tasks allowed me to think about mathematics that we learned previously such as solving equation.

The students' reflections show that although there was much to read they found the tasks interesting. For example, they learnt things about their community and they had to think about mathematics. This was said in contrast to other experiences of mathematics, which were 'strange'. My overall interpretation of the students' reflections on their work with the tasks is that they found it meaningful, not following the routines of other tasks. However, as they only reached the third step in Engeström's (1999) model of expansive learning the activity cannot be called expansive but rather routine in the use of mathematics. Instead, what was challenging to the students was to read and understand the task and some students seemed to appreciate the link between school mathematics and the local cultural environment in which the tasks were contextualised.

6.4 Experiencing the activity of solving contextualised tasks

When analysing the activity of solving contextualised mathematical tasks in the secondary school students' setting the analyses must be seen as tentative as they are built on my interpretation of the situation.

Elements of the activity at the secondary school setting

- Activity:** Secondary school mathematics education
Motive: Future education or profession
Action: Solving contextualised mathematical tasks
Goal: To provide data for mathematics education research
Operations: Doing mathematics
Conditions: One lesson available, restricted access to tools

From the students' perspective, an alternative interpretation of the motive of the activity is to follow a routine to solve tasks presented by an adult who acts as a teacher even if he is a researcher and has

followed the ethical rules for doing research. Or, their motive might align with the goal – to help to provide research data. There is no interview data on motives and goals but the other elements can be observed.

Engeström's model is interpreted in line with the design of the procedure and what can be observed.

- Subject:** Eighteen groups of secondary school students
- Object:** Solving contextualised tasks
- Tools:** Language, pens pencils, papers, own mathematical knowledge
- Rules:** Pair work, use available tools; ask for information if necessary; limited time.
- Community:** Secondary school students, researcher
- Division of labour:** Researcher introduces tasks and provides clarifications, Students discuss and solve tasks,
- Outcome:** Students provide answers to the tasks in written form. Students experience new types of tasks

7 DISCUSSION AND CONCLUSIONS

In the Rwandan society as elsewhere in the world, the utility of mathematics is recognized through several activities. Those activities are seen, on the one hand, in academic institutions such as schools and universities, where mathematics is used and learned for the purpose of developing knowledge about the subject per se. On the other hand, at different workplaces, mathematics is used as a mediating tool to facilitate production within the workplace. The present study is partly an answer to policy departments' demands for a more contextualized mathematics education with a move away from using pseudo-problems to more culturally adapted problems.

The overarching aim is to investigate how workplace mathematics can be contextualised and connected to university and school mathematics classroom practices so that mathematics becomes significant to the beneficiaries in terms of its content as well as of its context. Another aim is to meet a theoretical challenge that attempts to combine the second and the third generations of activity theory with Chevallard's anthropological theory of didactics to find out what types of knowledge could be generated. The latter makes it possible to analyse the observed mathematical organisations of workplaces and educational settings that deepen the understanding of the purpose and function of the use of mathematics.

First, common aspects as well as differences between the activity systems in the three settings are discussed. Secondly, I continue to compare mathematical organisations, what transpositions took place in the different settings and what influenced them. I conclude by looking forward toward possible pedagogical implications and needs for further research.

7.1 Mathematics related to activity systems

The evolution of human activity theory is recognised in three main generations. In brief, Vygotsky (1978) proposes the model of "a complex mediated act" including subject, object and mediating artefacts. In the second generation, Leont'ev (1981) uses Vygotsky's concepts when he relates processes at various levels, with respect to

the objects to which these processes are oriented. In this section of the discussion I will relate the major findings from the fieldwork to Leont'ev's key notions. Thereafter, I will make use of the third generation of human activity developed to a system by Engeström (1987). The approach to employ two different generations of activity theory is made with an attempt to firstly, make a process oriented analysis by using Leont'ev's concepts and in a second phase, deepen and differentiate the analyses in line with Engeström's expanded system.

In this study the significant common aspects in all activity systems are the use of language and mathematical knowledge. For instance, at the workplace settings, language is used to facilitate negotiation with customers or colleagues while estimating prices, which in turn concerns mathematics. In an analysis of the activity using Leont'ev's (1981) terminology common motives for running small-scale enterprises are to generate income for earning a good living. The action is to estimate and negotiate prices with the goal to earn a good income under the conditions that impact income generation.

In contrast, in the educational settings language is used when the students' are reading, talking and writing about mathematics. Again, drawing on Leont'ev's key notions, the analyses of the activity in the educational settings reflect a different content compared to the workplaces. Hence, activity is either profession oriented (mathematics teacher education) or education oriented (secondary school). The motive is to achieve a future profession or pave the way for further studies respectively. The action is to solve and construct mathematical tasks and to solve prescribed tasks respectively. These actions were realised through operations, for example,, by reading and reflecting on the tasks and negotiate mathematical strategies. According to activity theory, to reach their goal the individual has to follow existing conditions, like amount of time to be spent and access to necessary tools (own knowledge, calculators or pen and paper). These conditions varied between the educational settings. Also, the researcher had different roles. In the workplace setting he acted as an inquiring listener but in the educational settings he organized the activity and gave clarifications when requested.

Consequently, Leont'ev's model of analysis shows that workplace activity has strong focus on present reality and the outcome of the

activity has immediate impact on living conditions. In contrast, the work in the educational settings is future oriented and the efforts might impact future professional life or lead to better life chances.

Depending on the participants' motives, major findings show that all five activity systems related to the respective fieldwork settings are quite different in terms of subjects, objects, rules, community and division of labour which is further elaborated in the analysis according to Engeström's model (Table 5). In this table the researcher is made visible as his presence influenced the workplace and he was the one organizing the classroom activities. This is shown in the components of community and division of labour in the educational settings.

Table 5: Analysis of activity at workplaces and educational settings guided by concepts from Engeström (1987).

Settings Components	Workplace settings	University	Secondary school
Subject	Three entrepreneurs and one employed worker	Mathematics student teachers at university level	Third year secondary school students
Object	To use mathematics to reduce the risk of loss. Gain the ultimate profit	Solve prescribed contextualized tasks and pose new tasks	Solve prescribed contextualized tasks
Instrument	Accessible tools in terms of routine calculations, experiences of estimation, and sometimes pen and paper or calculators	Access to tools (own knowledge, calculators, paper and pencils), interaction with fellow students Group work	Restricted access to tools (own knowledge, paper and pencil only) Pair work
Rules	Follow routine practices of their group. Use individual strategies	Help the researcher Find correct answers Finish as quickly as possible	Help the researcher Find correct answers
Community	Workers in the same profession, their customers and society at large	University students and researcher	Secondary school students and researcher
Division of labour	Specific roles that belong to partners involved in the activity or contribute with goods or services for the respective activities	The role of the students and the role of the researcher	The role of the students and the role of the researcher
Outcome	Gaining profit in terms of money	Learn mathematics and related didactics for future profession	Set up answers to the tasks in written form Learn mathematics

Differences between the settings can be identified from Table 5. For example, in the workplace settings the *object* is to use mathematics with the goal to reduce the risk of loss and to gain ultimate profit. The way in which elementary arithmetic is applied should be understood in the context of continuous control of changing situational and cultural factors, which make up a fundamental basis for the workers' success. In contrast, at the educational settings the *object* is not to use mathematics to make a living for the time being, but to solve prescribed mathematical tasks, to pose new tasks or as at the secondary school level to solve contextualized tasks.

Generally, the above examples can be discussed in the light of the third generation of activity theory, where cultural diversity of applications, different perspectives or traditions, become increasingly serious challenges (Engeström, 1996). Moreover, the whole activity process, including my part in it, can be further analysed in Engeström's terminology. The *object* is to situate mathematics in a culturally relevant context.

The desired *outcome* is to reach better understanding and greater interest among secondary school students and start to inspire student teachers to explore new ways to formulate tasks. In the new types of tasks, the instruments and the object can be shared with curriculum developers and teachers. Hence, *division of labour* becomes vertical (workplace, university, secondary school and research) and horizontal in that it involved groups of people.

Considering the instrument component, the participants present some common aspects of using language but in contrast to the investigated educational settings, the workers were alone with the responsibility for their calculations. All participants used mathematical knowledge but in the workplace settings, particularly in the taxi driving and restaurant managing contexts, the calculations are often mental and routine built on estimations related to previous experiences. In the educational contexts the students also followed routines but they looked different, for example, to set up problems, calculate and give a fixed answer. Restricted access to calculators was only noticed in the secondary school. However, as could be seen in the restaurant manager report, calculators can be misleading if you are not careful in what you enter.

In this study, I have attempted to create interactions via mathematics tasks between participants of three cultures with the purpose to develop knowledge about how mathematics approaches can be understood and tried out. In my interpretation, this utilisation of activity theory for getting knowledge of culturally embedded activities forms a contextualisation for the sections below where I will discuss mathematical organisations and the didactic transpositions related to mathematical tasks.

7.2 Mathematical organisations at the fieldwork settings

In the investigations of mathematics use at the workplace settings, a pre-determined common object for the four workers was to avoid any risk of loss while generating their income. The observed techniques used by the subjects build on basic arithmetic related to addition and subtraction and simple multiplication. Taken-for-granted cultural knowledge is seen in the example when the drivers request a higher profit for the distance Kigali – Butare as most local people travel this distance by frequently running minibuses. Taxis are for those who can pay. For community members the return fee to Kigali is subject to negotiation. The same situation should be observed in the case of the restaurant owner where the level of profit is about 50%.

The observed mathematical work was analyzed as a mathematical organisation characterised by techniques that are functional to the problems at hand, the cultural constraints and the educational background of the workers. As long as they are pragmatic for the goals of the activity, no further justification of the techniques is needed, resulting in a mathematical organisation with undeveloped logos. This is reflected in the evident fact that the workers' goal is not to develop knowledge in the discipline of mathematics. What is functional at workplaces may in some cases be less functional in an educational context, where levels of justification often play an important role.

In the task solving and task posing processes, student teachers used different mediating tools including mathematical knowledge. In the task solving processes, two conclusions can be drawn. First, according to their explanations, all four groups of participants affirm that they solved the task related to taxi driving by using the same mathematical

technique, i.e. a system of two linear equations. This fact may most likely be due to the context in which the task was formulated (Säljö & Wyndhamn, 1990) where the participants were not asked to solve the task by using different techniques. However, if considering how the afternoon groups solved the task, the mathematical reasoning behind the solving is that there was a double use of one linear equation with two different notations of unknowns (x and y). In this case the actual mathematical organisation can be said to be specific because the morning groups used the technique of solving a system of two linear equations whereas the afternoon groups proceeded through the double use of a linear equation. For that type of tasks, the technique (solving linear equations, technology and theory (algebra) remained the same during the problem solving process.

Secondly, both the house construction and restaurant management tasks were solved through the use of the same technique (the rule of three). This means that although the tasks are different and were solved by independent groups, they belong to the same type of tasks. In this case the corresponding mathematical organisation is specific (ponctuelle). Like in the task solving process related to taxi driving workplace, the technique and the technology/theory remained the same

7.3 Didactic transpositions of contextualised tasks

Current Rwandan policy aims to develop a knowledge based economy. This includes an emphasis on teaching and learning science and technology. However, these two subject areas cannot be taught or learned without knowledge in mathematics. In this context, mathematics must be understood on a practical as well as on a theoretical level. Students, at different levels of education, will need to understand both how mathematics works and how to use it in different domains of daily activities and professional life. As in Rwandan mathematics education connections between workplace mathematical activities and classroom work has been weak the present study has put its focus on ways to strengthen this connection. To achieve this, information about Rwandan workplace mathematical practices was collected with the aim, in a first step, to construct educational material for prospective mathematics teachers during their training that draw on

the workplace mathematics, and in a second step build on their work with this material, to provide them with the experience of constructing similar material for secondary school students. In a third step, the researcher elaborated this material for the secondary school students and analysed how groups of such students solved and reflected on the tasks presented to them. In this process the participants stayed within their respective institution, working with mathematics within the institutional conditions and constraints. Therefore, the educational material constructed had to be aligned with the specific norms and rules guiding the community in focus.

The work done during such adaptation processes has been analysed in this thesis by employing the anthropological theory of didactics, as elaborated by Chevallard (see section 2.2), using the notions of didactic transposition and mathematical organisation. In this study, the transpositions are performed in a network where all participants in the study are involved. Already at the workplaces, the workers contribute with a meta-analysis, in oral or written form, of how they calculate to avoid loss and earn as much as possible.

The first mathematical transposition consists of transforming the data from the three workplaces into tasks to be solved by the university students. Within that transposition process, the researcher re-arranged parts of the information from each workplace and adapted it to a level of mathematical knowledge taught in specific classrooms, while at the same time aiming to maintain a high degree of representativeness of the tasks in relation to the workplace context upon which they were based. In this case the researcher drew directly on the information that was given to him by the workers. The second transposition consisted of student teachers' activity of designing tasks for secondary school students. During the process, the student teachers tried to use some of the data provided in their original tasks and re-formulated them to the secondary school students' level. However, this was somehow different to the first transposition, because although they were advised to refer to the authentic data in their tasks, some of them used own imaginary data. They could also draw on the information from the workplaces only indirectly via the tasks given to them. Other differences refer to the fact that they worked in small groups, in contrast to the researcher who designed the tasks for the student

teachers autonomously, and designed the tasks for target groups with different prior knowledge.

In their reasoning, the principles that guided the student teachers' transpositions, as regards the mathematical aspects of the tasks, were based on their views of secondary students' prior knowledge and their own knowledge about the secondary school mathematics curriculum. The whole activity was also influenced by the prevalent community rule to be helpful to the researcher, but at the same time they did not want to take too much time from other commitments. This is shown in how they used division of labour which appeared in the dialogues. Another interpretation of the transposition process is that the students did not realize the difficulties involved in formulating tasks from workplaces as it was a new experience for them. This set of constraints is the background for the third didactic transpositions that was performed by the researcher.

That the student teachers formulated their tasks for the secondary school students with very few words, as compared not only to the tasks given to them by the researcher but also to the final tasks given by the researcher to the secondary school students, may be explained by the time constraint of the activity itself, the view of the student teachers on what was appropriate for the target group, as well as the way the problem posing task was presented to them: "using the given data in your task, you are asked to formulate a task which could be solved by your future students" (see p. 75). In fact, the expression 'using the given data' can be interpreted as to referring to the quantitative data more than to the contextual descriptions framing those quantitative data. Therefore, the student teachers may not have seen it as a main objective to include the local cultural context in their task formulation. The same reason may lie behind the low degree of comprehensiveness in terms of weak local placement in their task formulations.

It can also be observed how the work of the 'transposers' is depending on and determined by their prior knowledge and conceptions of the different conditions and constraints of the institutions related to the didactic transposition process.

The analysis thus points to some of the pertinent issues involved also in a 'micro' transposition process like the one carried out for the purpose of the present study. Apart from the preliminary responses and reflections reported from the secondary school students in chapter six

to the contextualised tasks finally presented to them, further research will be needed to evaluate the educational outcomes of such implementations of contextualised tasks. It is evident from the analysis presented here that maintaining a reasonable degree of representativeness, when using tasks formulated as contextualised word problems, is difficult to achieve with such short texts as are commonly used in textbook word problems. In addition, working with contextualised tasks, with the purpose of connecting school mathematics to local workplace settings, cannot be restricted to solving the given problems but must also involve other forms of activities related to the contextualised tasks to increase the degree of representativeness. The data presented here shows that students, at the university as well as at the secondary school, value such local cultural relevance provided by the contextualisation of the given tasks.

7.4 Expansive learning

According to Engeström (1996; 1999), expansive learning takes place collaboratively through dialogues in the discourse of specific activities. In the present study, this is visible in the student teachers' use of mathematical knowledge and related concepts and tools. From the student teachers' perspective, one challenge that could be seen in the dialogues was to construct tasks which were built on workplace data where they also had to consider both mathematical knowledge and curricula when formulating the tasks. However, none of the students said that the construction of the tasks per se was difficult even if the analyses show that many tasks lack inner coherence.

Reflections from the student teachers and the secondary school students show that they value the work with locally contextualised tasks. However, the student teachers' claimed that it would be too time-consuming to teach and make investigations at workplaces at the same time due to a heavy teaching load. Instead, they suggested that if it is necessary to construct such tasks, a better solution could be to establish a project that could deal with such issues. One member in each group expressed future potentials of such tasks. Nobody suggested that they might be able to ask their secondary students to find data and formulate tasks for each other.

Expansive learning suggests that internal contradictions lead to creative thinking and innovation. A great challenge was to solve and

construct a task when the researcher on purpose had left out some information related to taxi driving (see Cole & Scribner, 1978). As shown in the findings, in those dialogues Engeström's seven steps in the expansive learning process - questioning, analysing, modelling, examining the model, implementing, reflection and evaluating - became particularly visible. The student teachers had to 'think as mathematicians' and use a logic employed at the university, not in secondary schools.

7.5 Critical reflections

The motive to perform this study was to introduce workplace mathematics in school mathematics to increase the students' understanding of the types of mathematics that workers can be involved in, how they solve them and how such tasks can be useful within education. In that way I have made mathematics in society more visible to the students. However, the participants have only contributed with data. I would have wished to involve the student teachers more in the process and discuss my transpositions of their tasks and let them share what happened with their tasks in the secondary school setting.

In this study several didactic transpositions were involved. It would have been interesting also to investigate workplaces where more advanced mathematics is used to construct contextualised tasks from such workplaces, providing a greater challenge to university students as well as providing insights into other kinds of professions than those related to the workplaces studied in this thesis.

Theoretically, activity theory has been helpful to analyse data both from a macro perspective dealing with the contextualisation of the tasks and from a micro perspective in the analyses of the dialogues. Anthropological theory of didactics has been supportive in the micro analyses of mathematical organisation and transpositions. Also, the participants and settings are limited in number as it is a case study. Even so, the study can inspire to further research.

7.6 Pedagogical implications

In this study, mathematical organisations at the involved fieldwork settings were investigated. This kind of focus was crucial as in Rwanda mathematics as a subject in educational programmes at different

academic levels is needed for teaching and learning other subjects such as science and technology. The pedagogical substance of this study can be seen in the sense that it is related to how mathematics teaching can be extended. Indeed, the well known standard way of teaching mathematics, consists of lecturing new mathematical concepts and properties and then provide related problems to strengthen learners understanding. However, in a society such as Rwanda mathematical knowledge is not only needed to widen the learners' understanding but also to develop it in terms of grasping new mathematical applications in science and technology. In this perspective, it was imperative to take initiative to make mathematics teachers' aware of the possibility to make use of real life situations with the purpose to contextualize mathematical tasks.

In addition, the approach employed in this study allows the teachers to identify how learners demonstrate their mastering of the subject. In mathematics education, it is not enough to say that learners are able to solve mathematical tasks. It is also about being able to explain how the tasks are solved. In this study anthropological theory of didactics allowed me to identify how learners used and justified mathematical techniques. This is of great importance in a pedagogical context because through that justification one can notice that at the end of the teaching of a mathematical concept, learners are able to utilise it in an extended way such as solving contextualised tasks.

Regarding the challenges of what types of knowledge that has been generated in this thesis I assume that by demonstrating the whole mathematical process from workplaces to educational settings the examples of making use of everyday knowledge in classroom settings will create a new awareness among learners, teachers, curriculum developers and policy makers, about the challenge to move from de-contextualised to contextualised learning of mathematics. However, the fact that there is research conducted in the field does not mean that any changes will be brought about. Rather, this thesis may be a supportive tool in future educational development to facilitate new mathematical approaches to be tested, adopted and critically evaluated.

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APPENDIX

Interview guiding questions

Taxi driver

- 1 Could you explain me how the negotiation of prices is done between you driver and your customer?
- 2 On which basis do you charge the customers?
- 3 What is included in the fixed price?
- 3 Could you explain me how mathematics intervene in your dairy activities?

House builder

- 1 What is needed to start the work of construction house?
- 2 Could you explain me how you proceed to earn the job?
- 3 What do you include in your estimated price?
- 4 Could you explain me how mathematics intervene in your daily work?
- 5 Could you explain me how mathematics intervene in your daily work?
- 6 Could you provide un example of successful case of the earned job?

Restaurant owner

- 1 How do you purchase different food products?
- 2 Can you explain me how the negotiation of prices is done at the market?
- 3 What do you think of other things such as your employees, taxes and furniture?
- 4 Can you explain me how mathematics intervene in your daily activities?
- 5 Could you explain me how the price of one meal is determined?
- 6 Could you give me an example of the order you have received recently?

Interview guide for university students

- 1 Could you explain how you proceeded to solve these tasks?
- 2 How do you think about the mathematical techniques used for sub-question (b) for both tasks?
- 3 What would you say if you are asked to include that kind of mathematical material in your teaching programme?

Interview guides for secondary school students

- 1 Could you explain me how proceeded to solve those tasks?
- 2 What do you think about solving mathematical tasks that are brought from the real situation?
- 3 Do you think those tasks can help to understand mathematics deeper?

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