UNIVERSITY of RWANDA

Research and Postgraduate Studies (RPGS) Unit

Queuing Analysis of Patient Flow in Rwandan Hospitals

Case study "Rwanda Military Hospital"

By

GOD M. MUTABARUKA

Ref Number: 218014801

A dissertation submitted in partial fulfilment of the requirements for the degree of

MSc in Information and Communication Technologies (Option: Operational Communication)

In the University of Rwanda, College of Science and Technology, School of ICT

Supervisor: Dr. Charles KABIRI

Co-Supervisor: Dr. Didacienne MUKANYILIGIRA

30 November 2020

DECLARATION

I, GOD M. MUTABARUKA with Reg No: 218014801 do hereby declare that all the work presented in this dissertation is my original work unless otherwise acknowledged. It has never been presented either in part or in full for publication or award of a degree in any university. I, therefore, present it for the award of Master of Science in Information and Communication Technologies (Option: Operational Communication) of the University of Rwanda, College of Science and Technology, School of ICT.

Signature……………..
GOD M. MUTABARUKA
Date: 30 November 2020

Supervisor's signature……………..
Dr. Charles KABIRI
Date: 30 November 2020

Co-Supervisor's signature……………..
Dr. Didacienne MUKANYILIGIRA
Date: 30 November 2020

DEDICATION

I dedicate this work to my lovely family members for the continued prayers, love, and encouragements - Sister Phinnah Wibabara amongst, my serious girlfriend Cyuzuzo KH for contributing to the project running over time due to her special love and care.

ACKNOWLEDGMENT

I wish to acknowledge with profound gratitude my supervisors, Dr. Charles Kabiri and Dr. Didacienne Mukanyiligira for their invaluable skilful assistance and tireless support solidified towards my studies and particularly during the preparation and execution of this work.

My sincere acknowledgment extends to the Government of Rwanda for sponsorship and the entire leadership of the University of Rwanda, College of Science and Technology for having natured me to carry out this thesis.

I am also very grateful to the staff of the Emergency and Accidents department at Rwanda Military Hospital who took part in this study.

I can't thank enough Lt Col Francis Ngabo who in one way or the other supported during this work execution.

I would like to most sincerely, thank my classmates for the moral support and encouragement that they accorded to me during the program.

Finally, I am indebted to my family for their love, patience, and support rendered during the entire time of my studies.

May God bless you all!

LIST OF ACRONYMS AND ABBREVIATIONS

EAD Emergency and Accident Department

PHC Primary Health Care

RMH Rwanda Military Hospital

QT Queuing Theory

M/M/S Multiple, Markovian, Server

AM Ante Meridiem

PM Post Meridiem

LIST OF SYMBOLS

λ Lamda

μ Mu

∞ Infinity

ρ Rho

Z Zed Score

E Error Margin

ABSTRACT

The increasing population and health-need due to adverse environmental conditions have led to escalating waiting times and congestion in hospitals, especially in the Emergency and Accident Departments. It is universally acknowledged that a hospital should treat its patients, especially those in need of critical care promptly. Queueing theory is a mathematical approach in Operations Research applied to the analysis of waiting lines. Rwanda Military Hospital is one of many hospitals that are presenting long waiting times for patients looking for services in all departments. This research project focuses on both analytical techniques and simulations to design an algorithm based on queuing theory of multi servers queuing network model of outpatient flow that will improve the healthcare services in the department using MATLAB/Simulink tool to model and simulate the current and designed models. Finally, the research project will compare the results to see the improvement and the performance of the designed model. The research thesis examined first, the waiting time of patients in the emergency and accident department by using queuing model after calculating the mean number of arrivals per hour and the mean number of patients served per hour. Further results from data collected for all four blocks were analysed to know the factors that cause long waiting for queues in the department. The results showed that the system traffic intensity is greater than one. This means that the queue grew endlessly. There were a big number of patients waiting in the queue and they waited for a long time before being serviced.

Key Words: Queuing Theory, Queuing Network Model, Waiting Queue, Patient Flow

LIST OF FIGURES

LIST OF TABLES

TABLE OF CONTENTS

CHAPITER 1: INTRODUCTION

1.1. General Introduction

Customer satisfaction has become a serious concern in the service sector. In the Healthcare industry, several initiatives have been introduced to enhance customer satisfaction. The healthcare industry providers globally are experiencing increasing pressure to concurrently reduce cost and improve the access and quality of care they deliver. Many healthcare institutions are confronted with long waiting times, delays, and queues of patients. Long waiting time in any hospital is considered as an indicator of poor quality and needs improvement. Managing waiting lines create a great dilemma for managers seeking to improve the return on investment of their operations.

Customers also dislike waiting for a long time. If the waiting time and service time are high, customers may leave the queue prematurely and this, in turn, results in customer dissatisfaction. This will reduce customer demand and eventually revenue and profit (Imahsunu, 2014).

The concept of using flow to improve medical care services has received increasing attention within healthcare, especially concerning the reduction in patient waiting times for emergency and elective care.

Awareness has been growing of the ideas, first tested in other industries, and results that organizations have generated by applying flow thinking to their organizations (Health Foundation, 2013). Health care systems in general and hospitals in particular, constitute a very important part of the service sector. Over the years, hospitals have become increasingly successful in deploying medical and technical innovations to deliver more effective clinical treatments. However, they are still often rife or widespread with inefficiencies and delays, thus presenting a propitious ground for research in numerous scientific fields, specifically Queuing Theory (Mor Armony)

1.2. Background

Regarding to (Anbarasi, 2018), Queuing theory has its origin in research by a Danish telephone engineer Agner Krarup Erlang when he generated models to describe the Telephone Exchange of Copenhagen." The Theory of Probability and telephone conversations" is the first paper on queuing theory published in 1909. He pondered the problems of determining the number of telephone circuits necessary to provide phone services that would prevent customers from waiting too long for an available circuit. In developing a solution to this problem, he began to realize that the problem of minimizing waiting time applied to many fields and began developing the theory further. Therefore, A.K. Erlang is considered the father of queuing theory.

After that great invention, queuing theory has introduced in my fields like Hospitals, airline companies, banks, manufacturing firms, etc., try to minimize the total waiting cost, and the cost of providing service to their customers (Obamiro, 2010).

As (Supriya Burungale1, 2018) made a research on queuing theory, stating that the end of the twentieth and the initial twenty-first centuries is the period of emerging application of queuing theory in the medical field. This emerges highly specialized hospitals for serving patients. Nowadays most of the hospitals are overcrowded with patients. It may affect patients' symptoms, clinical outcomes, and satisfaction. It can also affect physicians' effectiveness, causing frustration among medical staff.

This overcrowding is due to the lack of an effective queue management system in hospitals, which is the due time required for each patient would be uneven based on how much time the doctor takes and other tasks such as scanning, pharmacy, testing, etc. This is a challenging and complicated job because every patient in the queue may come just for consultation of doctor or check-up or test etc. Each treatment task can have varying time requirements for each patient of different age groups.

1.3. Problem Statement

Long waiting queues are symptomatic of inefficiency in-hospital services. Unfortunately, this is the case in many public hospitals in Rwanda and other developing countries. Capacity management decisions in Rwanda hospitals are generally based on experience and rules of the thumb rather than with the help of strategic research model-based analysis (Sam Afrane, Frebruary 2014).

Rwanda is among the few countries to have achieved universal health coverage due to its vision of inclusiveness, equity, and comprehensive and integrated quality service delivery, with a focus on primary health care (PHC). Rwanda's health sector has made tremendous progress in improving the health status of the population. These improvements are mirrored by the improvements in access to health care services and utilization of those services. According to Rwanda annual health statistics, the PHC utilization rate increased from 0.81 to 0.94 visits per inhabitant from 2009 to 2013 (Organization(PRIMASYS), 2017)

Despite the effort provided by the government of Rwanda to make health care services more accessed, it has been remarked that there are still gaps in different public and private hospitals including Rwanda Military Hospital as a case study, where patients are still experiencing delays during the health care process. Rwanda Military Hospital receives a large number of patients every day and this generally results in long patient waiting times. In response to this challenge, our research project analyses the existed queuing system of the outpatient department of Emergency of Rwanda Military Hospital to design queuing network model algorithm based on queuing Analysis of patient flow in the Accidents and Emergency department that can help reduce the long waiting time of patients.

In the M/M/S queuing network model, there is a presence of a single line of patients, multiple servers (nurses or physicians), and one phase. All servers are performing at the same rate and patients are served on FCFS queuing discipline.

1.4 Objectives

1.4.1 General Objective

The main objective of this project is to design an algorithm based on a multi-server queuing network model of outpatient flow that will improve the healthcare services in the Emergency and Accidents department of Rwanda Military Hospital.

1.4.2. Specific Objectives

The specific objectives for our research project are:

1. To determine the mean number of arrivals per hour **(λ)** in the Emergency and Accidents outpatient department of Rwanda Military Hospital.
2. To determine the mean number of patients served per hour **(μ)** in the Emergency AND Accidents outpatient department of Rwanda Military Hospital.
3. To compare the current outpatients flow queuing model system and the designed queuing network model system.

1.5. Research Questions

1. What causes a long waiting queue in the Emergency and Accidents outpatient department of Rwanda Military Hospital?
2. How does long waiting queues be minimized in the Emergency and Accidents outpatient department?
3. What factors we could adjust to improve patient flow in the Emergency and Accidents outpatient department?

1.6. Research Hypotheses

The following hypotheses are to be tested:

1. **Null Hypothesis**: There is no significant improvement between the current existing patient flow and the developed ones.
2. **Alternative Hypothesis**: There is a considerable significant improvement between the current existing patient flow and the designed one.

1.7. Justification of the Study

This research project allows the healthcare industry to notice what impact has the improvement does on healthcare performance. Moreover, this study will show what parameters can be improved in reducing patient waiting time. This research is expected to bring new ideas and concepts to be implemented in healthcare operations in Rwanda, especially public health centres since the numbers of research about this field are limited.

1.8. Scope of the Research Project

Due to the financial and time constraints, this research project is conducted only to the patients visiting Rwanda Military Hospital, in the outpatient Department of Emergency, and Accidents for consultation by a physician or a medical doctor. A period of 54 days is covered in which five days of each week from Monday to Friday are considered because they are the working days of the week.

1.9. Research Project Structure

We have organized the research project into five chapters

CHAPTER 1: It contains a general introduction to the research and concepts of the intended work, problem statement, objectives, justification, and scope of the study research

CHAPTER 2: It contains a detailed review of the existing literature sourced related to the research, it contains theoretical frameworks and analysis models that have been previously introduced in the research area.

CHAPTER 3: It describes the methodology adopted for the research project. This chapter explains the research design, data collection methods, and sampling method to be used.

CHAPTER 4: This chapter concerns data presentation, interpretation, and discussion of the research project.

CHAPTER 5: This chapter deals with the conclusion and recommendation of the research project.

CHAPTER 2: LITERATURE REVIEW

2.1. Introduction to review

Researches show that works on the theory and applications of queuing systems have grown exponentially since the early 1950s. Queuing Theory (QT) is the mathematical study of waiting for lines or queues. QT can be applied in various fields, yet most previous studies are well documented in the literature of Probability, Operations Research, and Management Science. The theory enables mathematical analysis of several related processes, including arriving at the queue, waiting in the queue, and being served at the front of the queue. The theory permits the derivation and calculation of several performance measures including the average waiting time in the queue or the system, the expected number waiting or receiving service, and the probability of encountering the system in certain states, such as empty, full, having an available server or having to wait a certain time to be served (A.H. Nor Aziati, March 6-8, 2018)

This paper, (ALEXANDER KOMASHIE1, 2015), confirms that Patients' satisfaction is generally accepted as a key indicator of the quality of care. Patients' waiting time is considered as one measure of access to healthcare.

Ershi et al, presented in their paper (Ershi, 2015), that the crowded phenomenon in the health care of most countries in the world happens more seriously and prevalently, deserving more immediate attention. In recent years, the queuing theory was introduced into the field of medical service and combined with the information system, which become the practical solution to tackle the queuing problem and relieve the crowd in health care. The queuing theory has its advantage in producing simple models especially for the application of less random data. It is easy to construct a queuing model for a large general hospital. The queuing theory is also applied for the rapid evaluation and the comparison among various alternatives. Consequently, the queuing model becomes a powerful tool for outpatient research.

Multiple channel queuing system, in which two or more servers or channels are available to handle arriving passengers (Mohammad Shyfur Rahman chowdhury*, 2013)

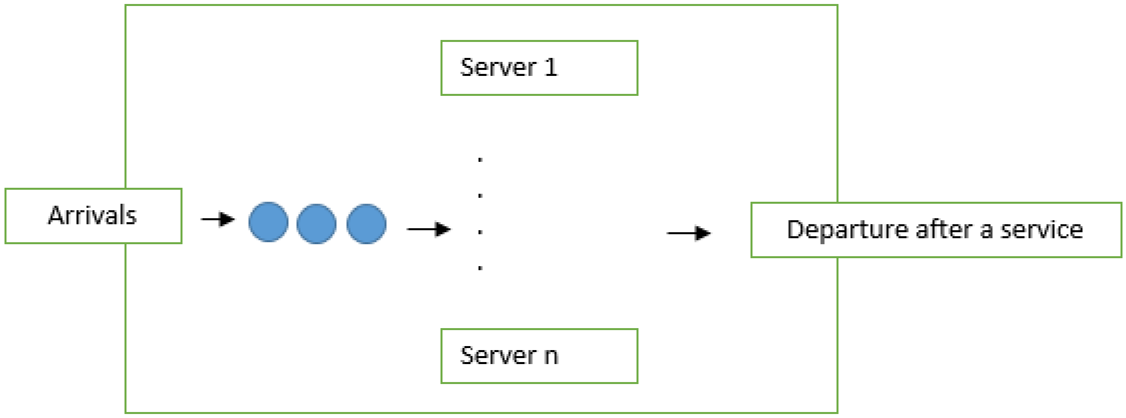

Figure 1: Simple Diagram of M/M/S Queuing Model

As you see in the above diagram, Service is first-come, first-served, and all servers are assumed to perform at the same rate.

2.2. Characteristics of Queue System

A queue system is made of arrivals, which are inputs to the system, waiting for line, and the service facility.

2.2.1. Arrival Characteristics

The input source that generates arrivals or customers for a service system has three major characteristics

1. Size of the arrival population.
2. Behaviour of arrivals.
3. Pattern of arrivals (statistical distribution)

2.2.2. Waiting Line Characteristics

A waiting line is the second component of a queuing system. It is characterised by its length that can be either limited or unlimited. A queue is limited when it cannot, either by law or because of physical restrictions, increase to an infinite length.

A second waiting line characteristic deals with queue discipline. This refers to the rule by which customers in the line are to receive service. Commonly used queue disciplines are:

1. FCFS - Customers are served on a first-come, first-served basis.

2. LIFO - Customers are served in a last-in-first-out manner.

3. Priority - Customers are served in order of their importance based on their service requirements

2.2.3. Service facility characteristics

The third part of any queuing system is the service facility characteristics. Two basic properties are important:

1. Design of the service system

2. Distribution of service times.

2.3. Measuring the Queue's Performance

Queuing models help managers make decisions tha89t balance service costs with waiting line costs. Queuing analysis can obtain many measures of a waiting line system's performance, including the following:

1. Average time that each patient spends in the queue
2. Average queue length.
3. Average time that each patient spends in the system
4. An Average number of patients in the system.
5. Probability that the service facility will be idle.
6. Utilization factor for the system.
7. Probability of a specific number of patients in the system

2.4. Description of Different Types of Queuing System

Basic Queuing System Designs of Service systems are usually classified in terms of their number of channels or servers and number of phases (number of service stops that must be made). We distinguish five major types of the queuing system and different combinations of the same can be adopted for complex networks.

2.4.1 Single Server, Single Phase System

In this queuing system, patients form a single line and one service facility is subject to serve the patient one after the other. This queuing system is also called **M/M/1 Queuing model**

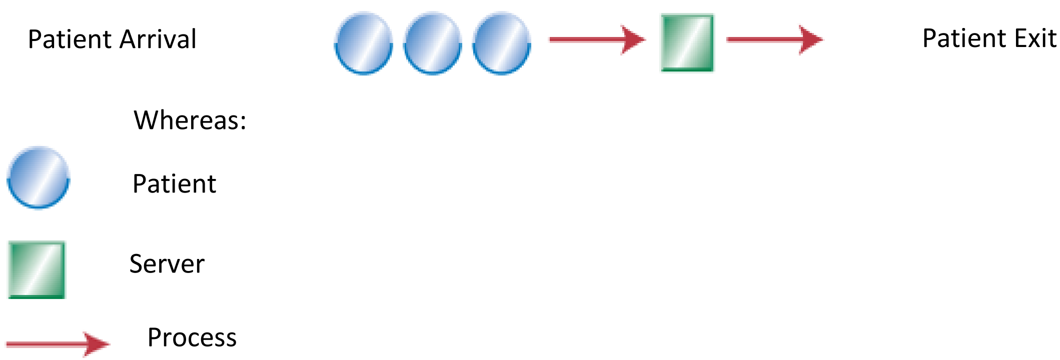

Figure 2: Single Server, Single Phase System

2.4.2. Single server, multiple phases System

For this queuing system, there's still a single queue but patients receive more than one kind of service before departing the queuing system. For instance, patients first arrive at the reception, get the registration done, and then wait in a queue to see a nurse for services before being seen by the doctor. Patients have to join the queue at each phase of the system.

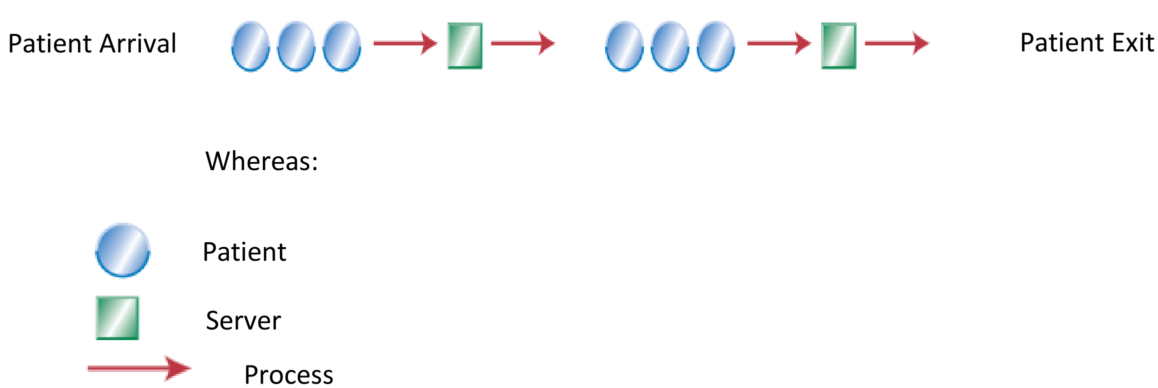

Figure 3: Single Server, Multiple Phases System

2.4.3. Multiple servers, Single phase System

For multiple servers, single-phase system, there is a presence of more than one service facility providing identical service and patients form a single queuing line. This system is also called the Multi-Channel Queuing model and it's denoted as M/M/S Queuing model. According to

(Green), M/M/S is the most commonly used queuing model and it assumes a single queue with an unlimited waiting room that feeds into s identical servers. Customers arrive according to a Poisson process with a constant rate and service duration.

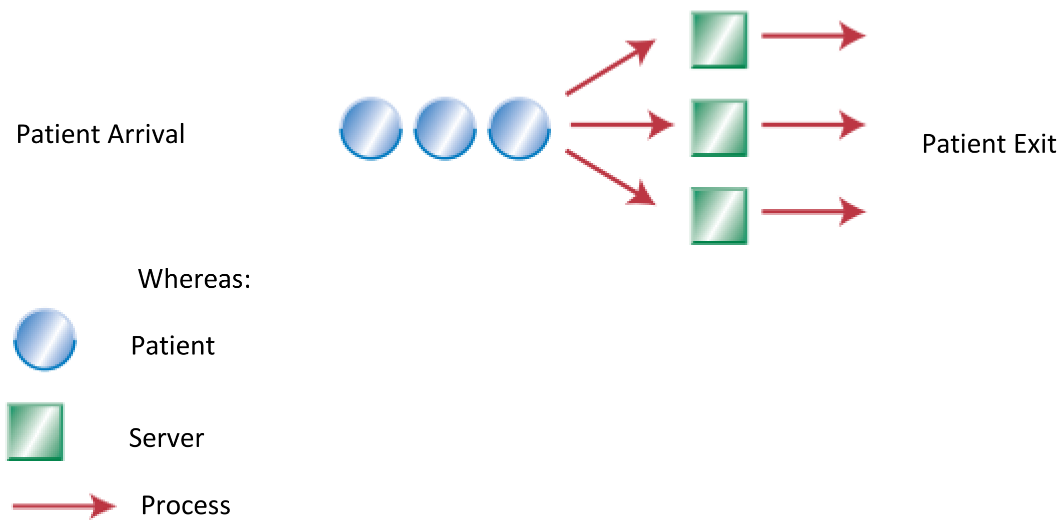

Figure 4: Multiple Servers, Single Phase System

2.4.4. Multiple servers, multiple phases System.

This type of service is typically seen in a hospital setting, in multi-specialty outpatient clinics. The patient first form the queue for registration, and then he/she is triaged for assessment, then for diagnostics, review, treatment, intervention, or prescription, and finally exits from the system or triage to a different provider.

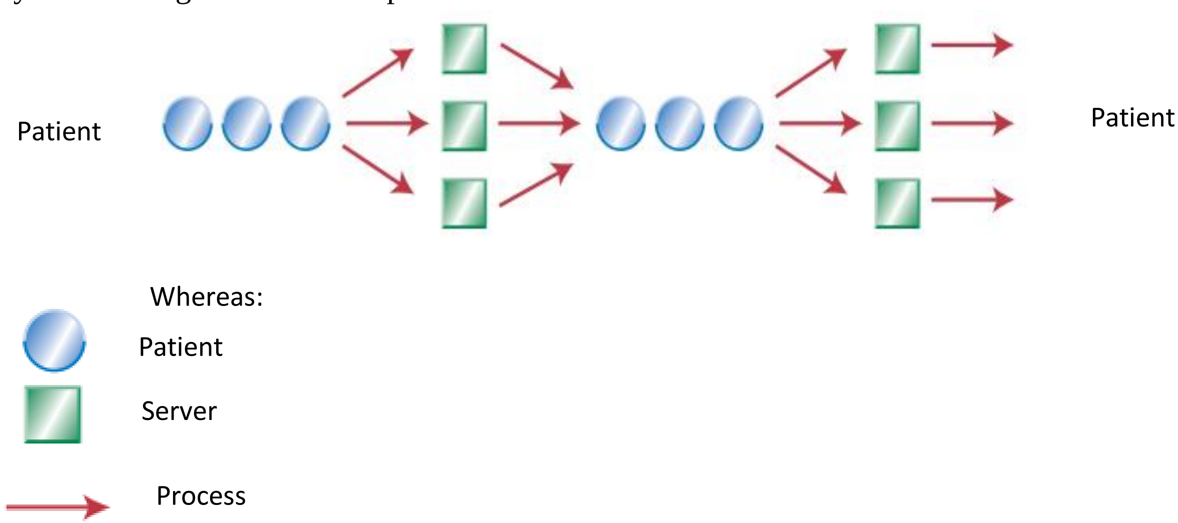

Figure 5: Multiple Servers, Multiple Phases System

2.4.5. Multiple Servers, Multiline Single Phase

The patients form multiple waiting lines according to active servers. The arrival patient chooses the shortest line to receive service quickly. This queuing model formed with multiple lines and multiple servers with a single phase.

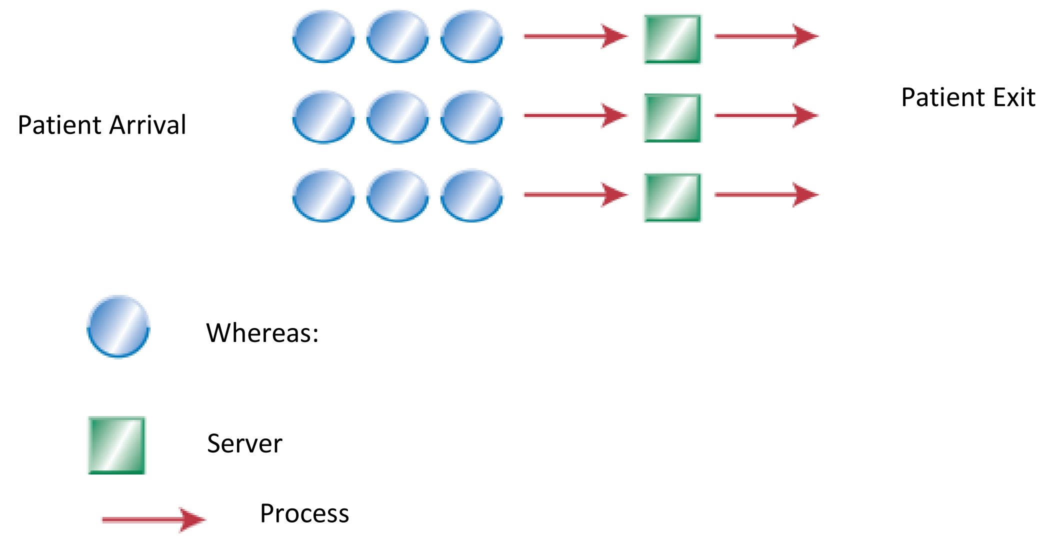

Figure 6: Multiple Servers, Multiline Single Phase

2.5 Multi servers queuing network

Among Chinese researches on outpatient service problems with the queuing theory, a part of the researches looked upon the outpatient process as a tandem queuing network that the patients entered each service node orderly within the outpatient, not considering the state that some patients visit some nodes repeatedly (Ershi Z. M., 2016)

When patients share and use multiple resources, a queuing network usually arises. Consider, for example, a patient that visits the Orthopedic outpatient clinic and then needs to have an X-ray at Radiology; or the surgical patient who is operated in the OR then cared for at the Intensive Care Unit (ICU) and subsequently cared for in a nursing ward. Thanda Aung and Lin Lin Naing, in their paper titled “Comparative Study on Different Queuing. Models to Reduce Waiting Time in Brahmaso Clinic,” attempted to compare the parameters of queuing theory in a local clinic, and the calculations performed in the paper is based upon the actual observed data collected from Brahmaso. Humanitarian Aid Organization charity special clinic for 14 days, which is located in Mandalay city, Myanmar. The required data and information are collected from “pamphlets”, “direct observation”, “daily and monthly records”, “yearly reviewed report” and “interviews”. Multiple servers M/M/2 to M/M/3 and M/M/4 to M/M/5 queuing models have been considered to reduce waiting time and also analyze and compare queuing parameters and performance measures of the system.

CHAPTER 3: RESEARCH METHODOLOGY

3.1. Introduction

C.R. Kothari, in his book titled "Research Methodology, Methods &Techniques" (Kothari, 2004) defines Research methodology as a way to systematically solve the research problem. It may be understood as a science of studying how research is done scientifically. In it, we study the various steps that are generally adopted by a researcher in studying his research problem along with the logic behind them. The researcher must know not only the research methods/techniques but also the methodology. To analyse well our research project, the method used by (Thanda Aung*, 2019) was adopted. This chapter outlines the research area, research design, research population, sample size and sampling technique, instruments used, and data collection procedures.

3.2. Research area

The research project is conducted at Rwanda military hospital that works in a multidisciplinary environment where it receives and refers both Military and Civilian patients where necessary from and to different specialists such as Orthopaedic Surgery, General Surgery, Neurosurgery, Gynaecology and Obstetrics, Internal Medicine, Paediatrics, Dermatology and so on. We mainly focus only on the outpatient department of Emergency and Accident of Rwanda Military Hospital.

3.3. Study scheme

A research design is a systematic plan to study a scientific problem. As the main objective of this research we design an algorithm based on M/M/S: FCFS/∞/∞ queuing model patient's flow that will improve the healthcare services in the outpatient department of Emergency and Accident of Rwanda Military Hospital, the patients are coming from infinite population and the system was enough to receive all the patients coming in the department.

3.2.1. M/M/S: FCFS/∞/∞ queuing Model Description

This research study adopts a network of queuing models in form of M/M/S: FCFS/∞/∞. Where the first and the second M indicate a Markovian arrival and service times respectively. The ∞ indicates infinity servers and first S represents multiple servers (S>1), which implies that the capacity of the system is limited to the number of servers. In summary

M: Markovian (or poison) arrivals.

M: Markovian Exponential service time.

S= Multi-Server (Physicians/ Nurses, cashier, pharmacist).

FCFS = First Come, First Served.

∞ = Infinite system limit.

∞ = Infinite source limit.

3.2.2. Rwanda Military Hospital Current Patient's Flow

According to Rwanda Military Hospital's human resource department, the current patient flow is based on a mixed arrangement and M/M/1 queuing model.

When a patient enters the system and at a time if the system is free, his/her service time starts at once and when the system is not free, the patient joins the queue and wait for their turn/number for service. After completion of services, the patient is free from the queue if there is not any further extended service facility. If the server is busy then the arriving patients go to orbit and become of repeated calls. This pool of sources of repeated calls may be viewed as a sort of queue. The time it takes to service every patient is an exponential random variable with parameter-µ.

In the literature, there is a representation figure that shows well a single server queuing system, (M/M/1) in which patients are standing in a queue, waiting for the server to be free for providing service. In the situation of congestion of patients in a health care system, there is very less probability for the patients arriving in the end to get treatment as there is a single server rendering services. In the worst situation, patients may leave the system without being served.

The M/M/1 queuing model parameters

- ✓ The average number of patients in the system (the number in waiting line plus the number being served) noted as L_s

$$L_s = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} \qquad (1)$$

λ: The mean number of patients arrive at RMH per day (arrival rate)

μ: The mean number of patients served per day (service rate)

- ✓ The average time a patient spends in the system (the time spent in waiting line plus the time spent being served) noted as W_s

$$W_s = \frac{1}{\mu-\lambda} \qquad (2)$$

- ✓ The average number of patients in the queue (average queue length), L_q

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} \qquad (3)$$

- ✓ The average time a patient spends waiting in the queue, W_q

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} \qquad (4)$$

The utilisation factor for the system (the probability that the service facility is being used) $\boldsymbol{\rho}$**.** It is also called Traffic intensity

$$\rho = \frac{\lambda}{\mu} \qquad (5)$$

- ✓ The present idle time (the probability that is no patient in the system) $\boldsymbol{P_0}$

$$P_0 = 1 - \frac{\lambda}{\mu} \qquad (6)$$

3.2.3. RMH designed Patient's Flow-based on multi-servers queuing network model

Now, let's turn to the designed patient's flow based on a multi-servers queuing network model in which two or more servers or channels are available to handle arriving customers on a first-come, first-served discipline with infinity patients. We assume that customers await service forms one single line and then proceed to the first available server.

The figure below shows well the designed algorithmic queuing network process model of all blocks in the Emergency and Accidents outpatient department of Rwanda Military Hospital. Patients arrive at the reception, form a single line. A patient who is in front proceeds to the available nurse. After receiving service from the reception, a patient must form a queue line to receive a service from an available physician in the consultation room. After receiving service from the consultation, a patient must form a queue line to receive a service from the available cashier for payment. Finally a patient after receiving service from billing, he/she must form a queue line to receive a service from the available pharmacist for drugs and medicine.

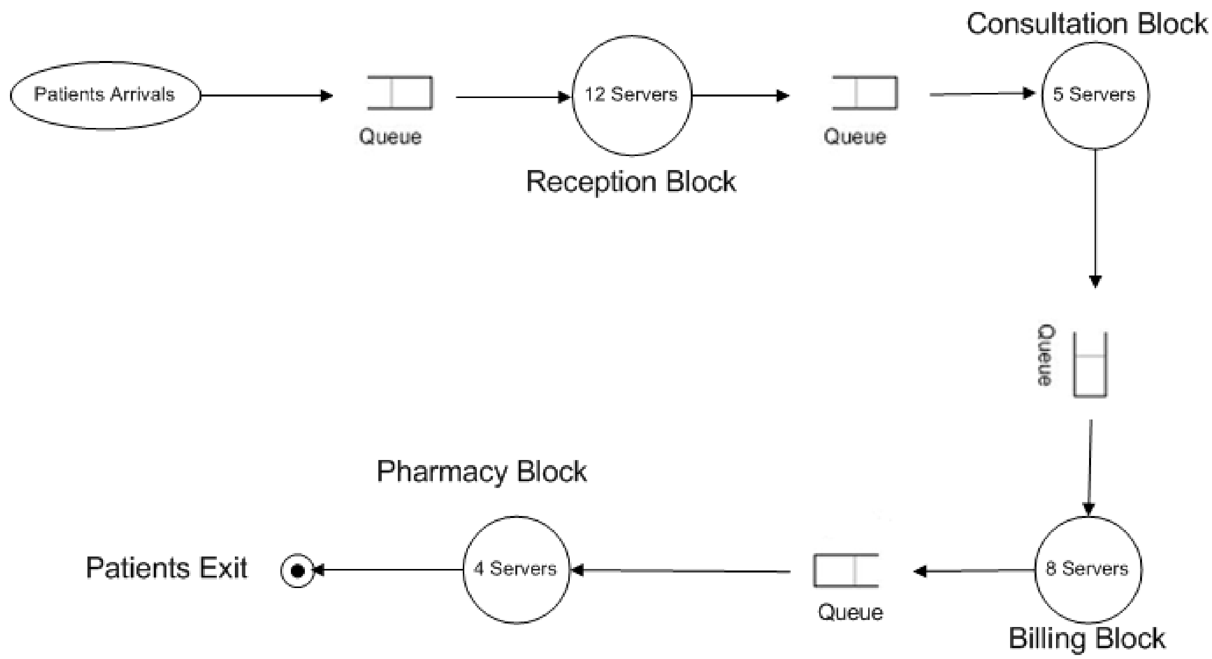

Figure 7: Designed RMH Patient's Flow of Emergency and Accident department

The M/M/S queuing model parameters

The queuing parameters for M/M/S Model are stated below with their equations. These equations are more complex than those used in the single-channel model, yet they are used in the same fashion and provide the same type of information as the simpler model.

- ✓ The average number of patients in the system (the number in the waiting line plus the The number being served) noted as $\boldsymbol{L_s}$

$$L_s = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^S}{(S-1)!(S\mu-\lambda)^2} P_0 + \frac{\lambda}{\mu} \tag{7}$$

With S number of servers

- ✓ The average time a patient spends in the system (the time spent in waiting line plus the time spent being served) noted as $\boldsymbol{W_s}$

$$W_s = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^S}{(S-1)!(S\mu-\lambda)^2} P_0 + \frac{1}{\mu} = \frac{L_s}{\lambda} \tag{8}$$

✓ The average number of patients in the queue (average queue length), $\boldsymbol{L_q}$

$$L_q = L_s - \frac{\lambda}{\mu} \tag{9}$$

✓ The average time a patient spends waiting in the queue, W_q

$$W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda} \tag{10}$$

✓ Traffic Intensity or Utilization factor ρ

$$\rho = \frac{\lambda}{S\mu} \tag{11}$$

✓ The probability that there are zero patients in the system is formulated as follow

$$P_0 = \frac{1}{\left[\sum_{n=0}^{S-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{S!}\left(\frac{\lambda}{\mu}\right)^S \frac{S\mu}{S\mu - \lambda}} \tag{12}$$

✓ The probability that there **n** patients in the system

$$P_n = \begin{cases} \dfrac{\left(\frac{\lambda}{\mu}\right)^n}{n!} \times P_0 \quad if\ 0 < n < S \\ \dfrac{\lambda^n}{S^{n-S}\, S!\, \mu^n} \times P_0 \quad if\ S \le n \end{cases} \tag{13}$$

From the Simulation results done by (P.Banumathi2, 2017), when comparing with single and multi-servers, the results of the waiting time of the customers are marginally reduced in the multi servers rather than the single server. Therefore, preferably multi channels or multi servers would be the right choice in place. This research study is about modelling a network of multiple servers system to reduce waiting time in the Emergency and Accident Department of Rwanda Military Hospital.

The main objective of this research study was achieved by applying queuing theory to study the queue behavior analysis of patients arriving in the department. The study is conducted for 54 days. A network of the multiple-servers queuing model (M/M/S) is applied to analyse the time spent by patients for the overall services in the hospital. There are 4 blocks that are identified in the department services i.e. B1 to B4.

Different blocks are shown below:

Block 1: Reception

Block 2: Consultation

Block 3: Billing

Block 4: Pharmacy.

Patients first are received and registered, continue to the consultation room, and depending on doctor's advice undergoes some laboratory and diagnostic tests at stage two. Finally, they purchase medicines from cashiers (Block 3) and go to take them to the pharmacy (Block 4) and leave the hospital, if there is no need to be admitted.

For modelling, the arrivals (n) are the outpatients. As each reaches the hospital, he/she books for service. If service is rendered immediately, he/she leaves the hospital or otherwise joins the queue. The physicians or nurses are the servers (S), the arrival rate, service time, and a number of servers were the data used for the study that has been collected using the observation method. The data collection covered a period of 54 days in which five days of each week from Monday to Friday were considered because they are the working days of the week.

3.3. Research Population.

In any research, the researcher has to identify the population under study. As with almost all decisions in the planning stage, this is determined by the research question.

The population is the group of interest and for whom the results will be applicable. The population needs to be defined in fairly formal and precise terms so that it is clear who falls within your definition and who falls outside of it (Antony Arthur, Beverley Hancock, 2009)

Particularly for comparative surveys, it is vital that a clear understanding of the target the population is reached well in advance of commencing survey fieldwork. Surveys are complex, challenging, and expensive activities. Without a clear target population, resources will likely be wasted (Murphy, 2016) for that reason, this research study is focused only on patients that coming, outgoing, and staff working in the Department of Emergency and Accident of Rwanda military hospital.

3.4. Sampling Design

To find the inter-arrival times and service times at each stage, a sample of 9540 outpatients are selected randomly from a population of 15,787 outpatients who visited RMH during 54days. Although, given that the patients arrive randomly and we are most interested in the time two successive patients spend both on queue and in the system to be serviced, we have taken our sample as several hours in a month to collect arrival and service data.

For recording the arrival and exit time of patients in the Emergency and Accidents outpatient department, observation, and quantitative methods are used in data collection.

3.5. Sampling Technique

The sample is a portion of a population or universe. However, by population, many often consider to people only. The population does not necessarily mean a number of people. It can also refer to the total quantity of the things or cases which are the subject of the research (Ilker Etikan, 2016). Probability sampling specifies to the researcher that each segment of a known population will be represented in the sample. Probability samples lend themselves to rigorous analysis to determine the likelihood and possibility of bias and error. Random selection is the process of choosing the components of a sample that ensures each member of a population stands the same chance of selection. The characteristics of the sample are assumed to be similar to the characteristics of the total population it is drawn from (Adwok, 2015). Regarding our case study, we have used a simple random sampling technique to choose the days in a month in which we conducted a data collection from the Emergency and Accidents outpatients department of Rwanda Military Hospital.

3.6. Sample Size

Cochran has developed a formula used to calculate sample size for the proportion of populations that are large (Singh, 2014). We used the Cochran formula to get the sample size of our research study,

$$n = \frac{z^2 p(1-p)}{E^2} \tag{14}$$

Where**:** $\boldsymbol{n}$ is the sample size desired

$\boldsymbol{z}$ Zed Score, which is equal to 1.96, at 95% confidence, level desired

$\boldsymbol{p}$ Assumed proportion of patients experiencing longer waiting time 0.76 in RMH

$\boldsymbol{1-p}$ Assumed target population not to have waited long in RMH

$\boldsymbol{E}$ Error Margin; in our case, we have decided to use 0.04

It is usual, when using statistics, to plan data size at the same time as planning data collection. This enables you to have a high level of confidence as well as a small margin of error

Thus $$n = \frac{(1.96)^2 \times 0.76(1-0.76)}{(0.04)^2} \tag{15}$$

$n =$ 438 hours

Officially, the working hour per day is 8 hours, our sample size is 438 hours, which corresponds to 54 days if we consider 8 working hours.

3.7. Instruments.

Books, reports, and papers are documents used in our research project. All these tools help us to make the conceptual and theoretical framework of our work as well as to analyse the data and interpret the results. For the collection of data, the observation technique is used.

3.8. Data Collection techniques

Quantitative research methods are characterised by the collection of information that can be analysed numerically, the results of which are typically presented using statistics, tables and graphs. During data collection for our research project, the observation technique has been used to find the inter-interval and service timings for patients visiting the outpatients department. The timings are noted by using a stopwatch. Observation is an effective method because it is straightforward and efficient. It doesn't typically require extensive training on the part of the data collector, and he or she is generally not dependent on other participants.

We registered the time every patient enters in Emergency and Accident outpatient department and a time when he/she gets out of the department. This helped to draw a table used in estimating the average number of patients entered in the system and the average number of patients served in one hour. From this, we have estimated the remaining performance parameters of the system. These data have been collected for a period of 54 days from Monday to Friday, from 08:00 A.M to 05:00 P.M. The data collected are recorded in an excel spreadsheet for being analysed and simulated into MATLAB/Simulink.

3.9. Data Processing and Analysis.

For analysis of our data and interpretation of the results, different computer tools are used especially Microsoft Excel and MATLAB/Simulink. The data collected using observation technique is recorded in Excel spreadsheet and after imported in MATLAB/Simulink environment for analysis where descriptive statistics and significance test is considered as well as estimation of different performance parameters describing the behaviour of patient flow at Rwanda Military Hospital.

The graphs are presented in scope-predefined objectives to make data meaningful and come out with conclusions and recommendations.

The performance parameters for both current and designed network queuing models used in this research study were defined as follows:

λ: Arrival rate of patients at outpatient department per hour

μ: Service rate of patients at outpatient department per hour

S: Number of servers working in the outpatient department.

ρ: Outpatient system utilization factor = $\lambda/s\mu$

Lq: Average number of patients at outpatient department in the queue.

Ls: Average number of patients in the system

Wq: Waiting time of patients in the queue

Ws: Waiting time of patients in the system

Po: Possibility of 0 patients existing in the system.

Pn: the probability of n outpatients existing in the system

We calculate the value of each parameter in both model systems in every block and make a comparison of the performance by plotting graphs to see if there is a considerable improvement in patient flow in the Emergency and Accident department of the hospital.

CHAPTER 4: RESEARCH RESULTS AND DISCUSSIONS

4.1 Introduction.

In this chapter, we describe how data analysis and simulation are done for both current and deigned network queuing model systems and the findings have been presented. The main results are presented and finally, the model is built that is used to respond to the research questions of this study and address the main objectives. The general objective of this project was to design an algorithm based on queuing network model for healthcare services in Rwanda Military Hospital. In this study, we have used multiple servers queuing network model M/M/S: FCFS/∞/∞ where we have multiple servers (nurses, physicians, cashiers, pharmacists), infinite system limit, and infinite source limit of patients

4.2. Data Analysis

4.2.1 Introduction

As the data collected for the currently existing system model, show that there is always a long waiting queue in the emergency and accident department, we increased the number of servers to minimize the waiting time and increase the service rate. In this section, we present how data were analysed for both the current system model and the designed model system.

4.2.2. The mean number of arrivals per hour (λ)

The arrival interval time is a period between the arriving patient time on queue and the next arriving patient time. From the data collected during 54 days at Rwanda Military Hospital for the arrival interval time, we have the following data of arrivals for 4 blocks of the department. The data are presented in Table 1, Table 2, Table3, and Table 4 corresponding to block 1, block 2, block 3 and block 4 respectively.

Parameter	N	Min	Max	Mean
Interval arrival time	9540	0	67	4.14

Table 1: The mean arrivals per hour for the Reception block

According to the recorded above data into table and calculations, we have remarked that every 4.14 minutes one patient joined the queue of reception block. This corresponds to

λ_1=14.49 patients arrived per hour for the current model system. As we have considered the arrival rate values as the same as for both the current system and designed model system, this means that λ_1 is also 14.49 for the designed system model because patients will continue to come as usual for a health service.

Parameter	N	Min	Max	Mean
Interval arrival time	9540	0	54	2.65

Table 2: The mean arrivals per hour for Consultation block

The above table is drawn from the interval arrival time recorded from the consultation block of the hospital for the current model system. Moreover, the results obtained show us that, every 2.65 minutes, a patient joins a waiting line. This means that in one hour, λ_2= 22.64 patients join a queue of consultation blocks. Thus λ_2 for the designed system model is 22.64.

Parameter	N	Min	Max	Mean
Interval arrival time	9540	0	89	6.26

Table 3: The mean arrivals per hour for Billing block

For billing block also, the data recorded there, resulted that, in every 6.26 minutes, at least one patient joins a queue for making payment of service and drugs/medicines. This tells us, that, λ_3= 9.58 patients arrive in one hour at the billing block. This value of arrival rate for the current model system is the same as the designed one.

Parameter	N	Min	Max	Mean
Interval arrival time	9540	0	59	3.25

Table 4: The mean arrivals per hour for Pharmacy block

The mean arrivals data recorded at the pharmacy block of the current system model helped us to remark that every 3.25 minutes one patient joined the queue to take drugs and medicines. This corresponds to λ_4=18.46 patients arriving per hour. Thus the mean number of arrivals per hour for the designed model system is 18.46.

4.2.3. The mean number of patients served per hour (μ)

For calculating the mean number of patients serviced during of period of one hour at each block from block 1 to block 4, we have recorded a time, each patient received (registered time), and Exit time at each block. The data are presented in below Table 4, Table 5, Table 6, and Table 7 corresponding to block 1, block 2, block 3 and block 4 respectively.

Parameter	N	Min	Max	Mean
Patients served	9540	0	78	8.61

Table 5: Mean number of patients served per hour for Reception block

The data recorded and presented in the above table, we found that every 8.61 minutes there was one patient served at the reception block. This corresponds to μ_1 = 2.32 patients served per hour. In this block, the service rate will be improved from 2.32 up to 27.84 for our designed system model because we have used 12 servers

Parameter	n	Min	Max	Mean
Patients served	9540	0	57	5.4

Table 6: The mean number of patients served per hour for Consultation block

The above table was drawn to present data recorded for a number of patients serviced at the consultation block. The results show that every 5.4 minutes, at least one patient was served. Thus μ_2**=** 11.11 patients were served in one hour at the consultation block. We have used 5 servers in this block to serve patients. This will increase the service rate from 11.11 to 55.55.

Parameter	n	Min	Max	Mean
Patients served	9540	0	61	16

Table 7: The mean number of patients served hourly for billing block

The results presented in the table above, help us to find that at least one patient was serviced at the billing block every 16 passing minutes. Thus $\mu_3 = 3.75$ patients were served in one hour at the billing block. For the designed system model, 8servers are used to rise the service rate to 30.

Parameter	n	Min	Max	Mean
Patients served	9540	0	86	5.87

Table 8: The mean number of patients served hourly for pharmacy block

The data presented in the above table, are recorded at the pharmacy block of the hospital and they show us, that, every 5.87 minutes, one patient was serviced. And the average number of patients serviced hourly is $\mu_4 = 10.22$ patients. In the pharmacy block for the designed system model, the service rate will be improved from 10.22 to 40.88 because we used 4 servers.

4.2.4 Traffic Intensity or System Utilization Factor (ρ).

The system utilization factor is the ratio between the mean number of arrivals (λ) and the mean number of patients served (μ) per period. The factor also can give an idea of the system performance and show that there is a probability that a queue can be formed or not. This factor has been calculated for all blocks of the hospital. It has been calculated using the equation (5)

$$\rho_1 = \frac{14.49}{2.32} = 6.24 \tag{16}$$

$$\rho_2 = \frac{22.64}{11.11} = 2.03 \tag{17}$$

$$\rho_3 = \frac{9.58}{3.75} = 2.55 \tag{18}$$

$$\rho_4 = \frac{18.46}{10.22} = 1.80 \tag{19}$$

The terms (16), (17), (18), and (19) are traffic intensities for the current system model. Whereas the terms below (20), (21), (22), (23) are traffic intensities for the designed model system.

$$\rho_1 = \frac{14.49}{12\times(2.32)} = 0.52 \tag{20}$$

$$\rho_2 = \frac{22.64}{5\times(11.11)} = 0.40 \tag{21}$$

$$\rho_3 = \frac{9.58}{8\times(3.75)} = 0.31 \tag{22}$$

$$\rho_4 = \frac{18.46}{4\times(10.22)} = 0.45 \tag{23}$$

Traffic intensity must be lower than 1 because 1 represents 100% usage of system capacity. There exists a general rule in practice that system utilization shouldn't be higher than 0.8. Higher values of utilization mean nothing more than the increased time of service, long queues, and how long the patient has to wait in a queue before comes his/her time to be served. Some systems can even deny or drop incoming customers if the system utilization becomes too high (Mikoláš, 2016). As the traffic intensity values of our designed system are less than 1 and all are under practical value(0.8), we have a stable system model for all blocks of the department to serve the patients effectively and the queue will be shorter compared to the current system. The obtained traffic intensity values are greater than 1 for the current system model, then the queue will grow endlessly in all four blocks of the department. In addition, we can conclude that the system is always overloaded and the patients are not happy with the service rate. The table below represents the comparison between the current model system and the designed system model. We have used to put more servers in the reception block because it is where we see long waiting for queues compared to other blocks of the department.

Blocks / Models	Reception Block			Consultation Bloc			Billing Block			Pharmacy Block		
	λ_1	μ_1	ρ_1	λ_2	μ_2	ρ_2	λ_3	μ_3	ρ_3	λ_4	μ_4	ρ_4
Current System	14.49	2.32	6.24	22.64	11.11	2.03	9.58	3.75	2.55	18.46	10.22	1.80
Designed System	14.49	27.84	0.52	22.64	55.55	0.40	9.58	30	0.31	18.46	40.88	0.45

Table 9: Current and Designed System Model Comparison

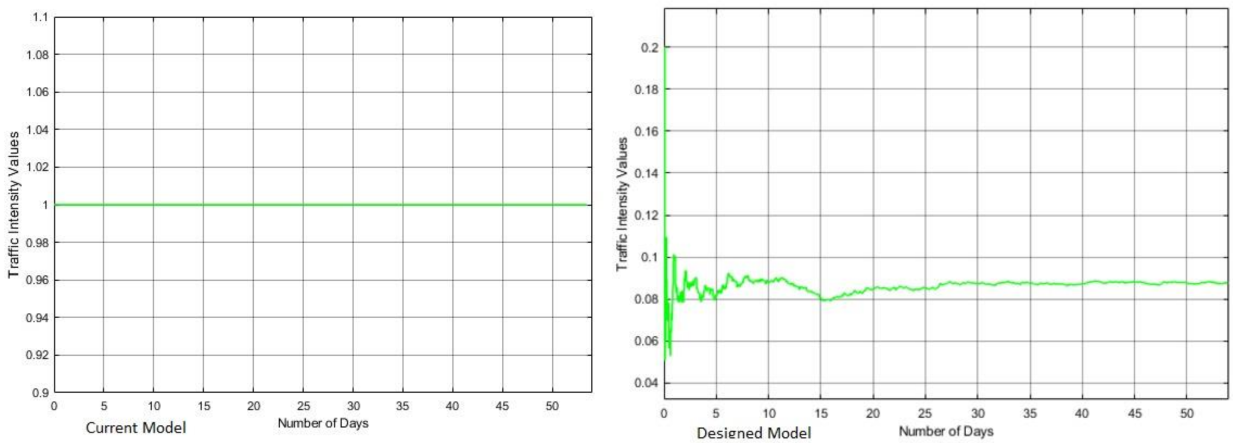

Figure 8: Comparison of Traffic Intensity RMH Simulation Results for a current and designed Model

The graph above shows the comparison of current and designed model traffic intensity. In the current existed model, the traffic intensity value is 1 whereas, for the designed system model, it is below 1, which means that in practice, our designed model system has high stability. The designed system can receive many patients to render the best health service rate.

4.2.5 Building and Simulation of System Models

Generally, a model intended for a simulation study is a mathematical model developed with the help of simulation software. Mathematical model classifications include deterministic (input and output variables are fixed values) or stochastic (at least one of the input or output variables is probabilistic); static (time is not taken into account) or dynamic (time-varying interactions among variables are taken into account). Typically, simulation models are stochastic and dynamic (Maria, 1997).

For simulating our designed network queuing model, we have used MATLAB Environment. Simulations are done using Simulink with SimEvents discrete-event engine. SimEvents provides libraries of predefined blocks, such as queues, servers, and switches, which we can use to accurately represent analysed systems and customize routing, processing delays, prioritization, and other operations.

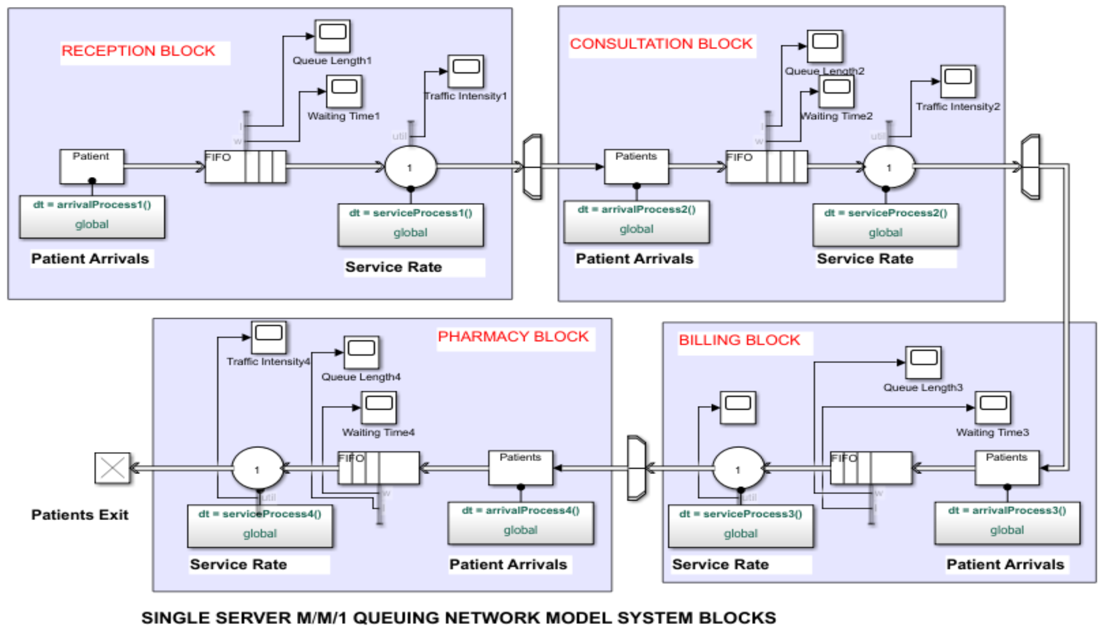

Figure 9: RMH Current Network Model System

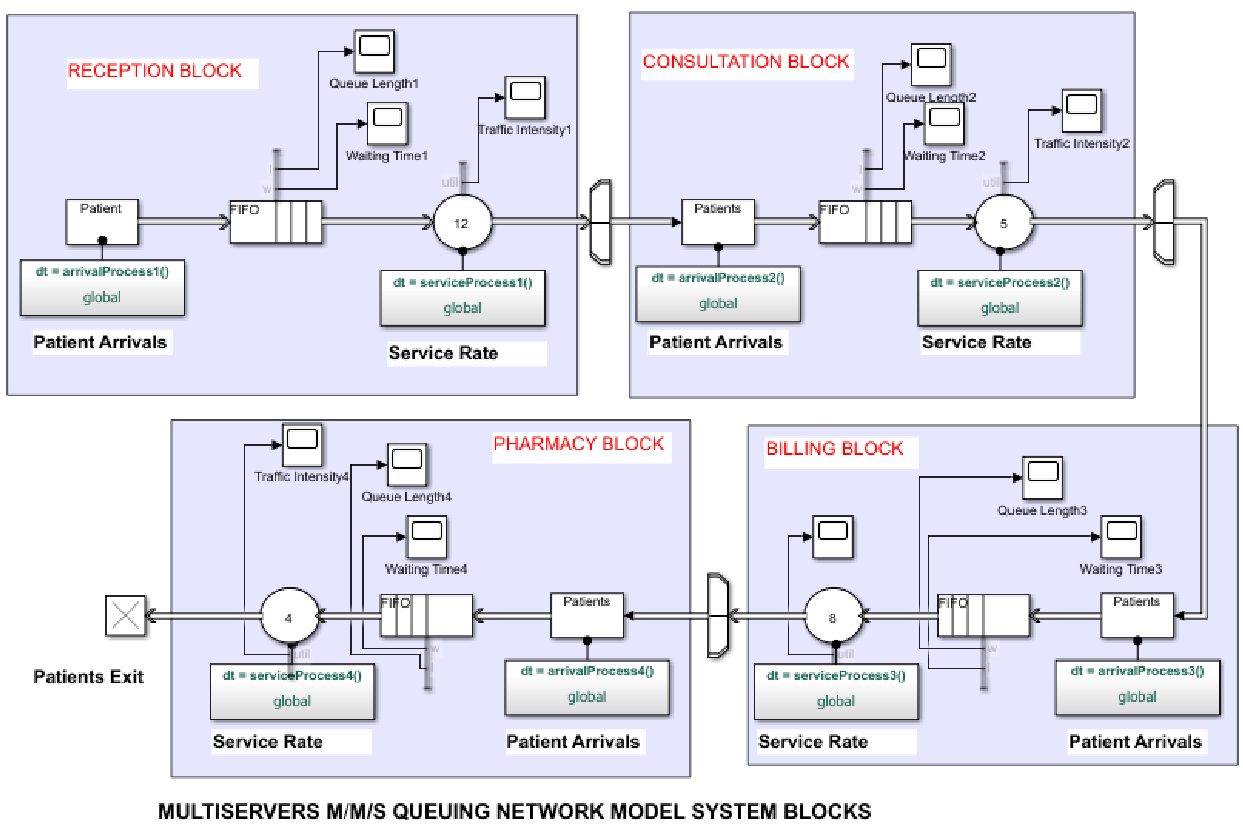

Figure 10: RMH Designed Queuing network Model System Built-in MATLAB/Simulink

4.2.6 Probability that there is no patient P_0

This parameter for the designed system is calculated by using the term (12). Moreover, we calculate it for all blocks of the Emergency and Accident department of the hospital.

Reception block

$$P_0 = \left[\left[\sum_0^{11} \frac{1}{0!}\left(\frac{14.49}{2.32}\right)^0\right] + \frac{1}{12!}\left(\frac{14.49}{2.32}\right)^{12} \frac{12(2.32)}{12(2.32) - 14.49}\right]^{-1} \tag{24}$$

After all calculations we find that P_0= 0.01949

Consultation block

$$P_0 = \left[\left[\sum_0^{5} \frac{1}{0!}\left(\frac{22.64}{11.11}\right)^0\right] + \frac{1}{5!}\left(\frac{22.64}{11.11}\right)^{5} \frac{5(11.11)}{5(11.11) - 22.64}\right]^{-1} \tag{25}$$

After all calculations, we get P_0= 0.1581

Billing block

$$P_0 = \left[\left[\sum_0^{8} \frac{1}{0!}\left(\frac{9.58}{3.75}\right)^0\right] + \frac{1}{8!}\left(\frac{9.58}{3.75}\right)^{8} \frac{8(3.75)}{8(3.75) - 9.58}\right]^{-1} \tag{26}$$

After all calculations, we get P_0= 0.07783

Pharmacy block

$$P_0 = \left[\left[\sum_0^{4} \frac{1}{0!}\left(\frac{18.46}{10.22}\right)^0\right] + \frac{1}{4!}\left(\frac{18.46}{10.22}\right)^{4} \frac{4(10.22)}{4(10.22) - 18.46}\right]^{-1} \tag{27}$$

After all calculations, we get P_0= 0.150

4.2.7 The average number of patients in the system L_s

The average number of patients in the system is obtained by using the equation (7). This parameter was calculated for each block in the department.

Reception block

$$L_s = \frac{14.49\times 27.84\left(\frac{14.49}{27.84}\right)^{12}}{(12-1)!(12\times 2.32-2.32)^2} \times 0.01949 + \frac{14.49}{2.32} = 6.27 \text{ patients} \tag{28}$$

Consultation block

$$L_s = \frac{22.64\times 11.11\left(\frac{22.64}{11.11}\right)^{5}}{(5-1)!(5\times 11.11-11.11)^2} \times 0.1581 + \frac{22.64}{11.11} = 2.05 \text{ patients} \tag{29}$$

Billing block

$$L_s = \frac{9.58 \times 3.75 \left(\frac{9.58}{3.75}\right)^8}{(8-1)!(8 \times 3.75 - 3.75)^2} \times 0.07783 + \frac{9.58}{3.75} = 2.55 \text{ patients} \tag{30}$$

Pharmacy block

$$L_s = \frac{18.46 \times 10.22 \left(\frac{18.46}{10.22}\right)^4}{(4-1)!(4 \times 10.22 - 10.22)^2} \times 0.150 + \frac{18.46}{10.22} = 1.852 \text{ patients} \tag{31}$$

4.2.8 The average time a patient spends in the system W_s

From equation (8), we have calculated the average time a patient spends in the system. This value also is calculated for all four blocks.

$$W_s = \frac{6.27}{14.49} = 0.43 \text{ min} \tag{32}$$

$$W_s = \frac{2.05}{22.64} = 0.09 \text{ min} \tag{33}$$

$$W_s = \frac{2.551}{9.58} = 0.26 \text{ min} \tag{34}$$

$$W_s = \frac{1.852}{18.46} = 0.1 \text{ min} \tag{35}$$

The above terms (32), (33), (34), (35) are average waiting times a patient spends in the system in pharmacy, consultation, and billing and pharmacy blocks respectively.

4.2.9 The average number of patients in the queue L_q

The average number of patients in the queue is calculated from equation (9) and we have four values that correspond to four blocks.

$$L_1 = 6.27 - \frac{14.49}{27.84} = 5.75 \text{ patients} \tag{36}$$

$$L_2 = 2.05 - \frac{22.64}{11.11} = 0.02 \text{ patients} \tag{37}$$

$$L_3 = 2.55 - \frac{9.58}{3.75} = 0 \text{ patients} \tag{38}$$

$$L_4 = 1.852 - \frac{18.46}{10.22} = 0.052 \text{ patients} \tag{39}$$

The above terms (36), (37), (38), and (39) are the average numbers of patients waiting in queue for health service for reception, consultation, billing, and pharmacy blocks respectively.

4.2.10 The average time a patient spends waiting in the queue W_q

The average time a patient spends waiting in the queue comes from the equation (10), we have got the following four terms (40), (41), (42), and (43) for reception, consultation, billing, and pharmacy blocks respectively

$$W_1 = W_S - \frac{1}{\mu} = \frac{5.75}{14.49} = 0.3 \text{ min} \tag{40}$$

$$W_2 = W_S - \frac{1}{\mu} = \frac{0.02}{22.64} = 0.00088 \text{ min} \tag{41}$$

$$W_3 = W_S - \frac{1}{\mu} = \frac{0}{9.58} = 0 \text{ min} \tag{42}$$

$$W_4 = W_S - \frac{1}{\mu} = \frac{0.052}{18.46} = 0.0028 \text{ min} \tag{43}$$

4.2.11 Simulation results of Queue Length and Average Waiting time in queue

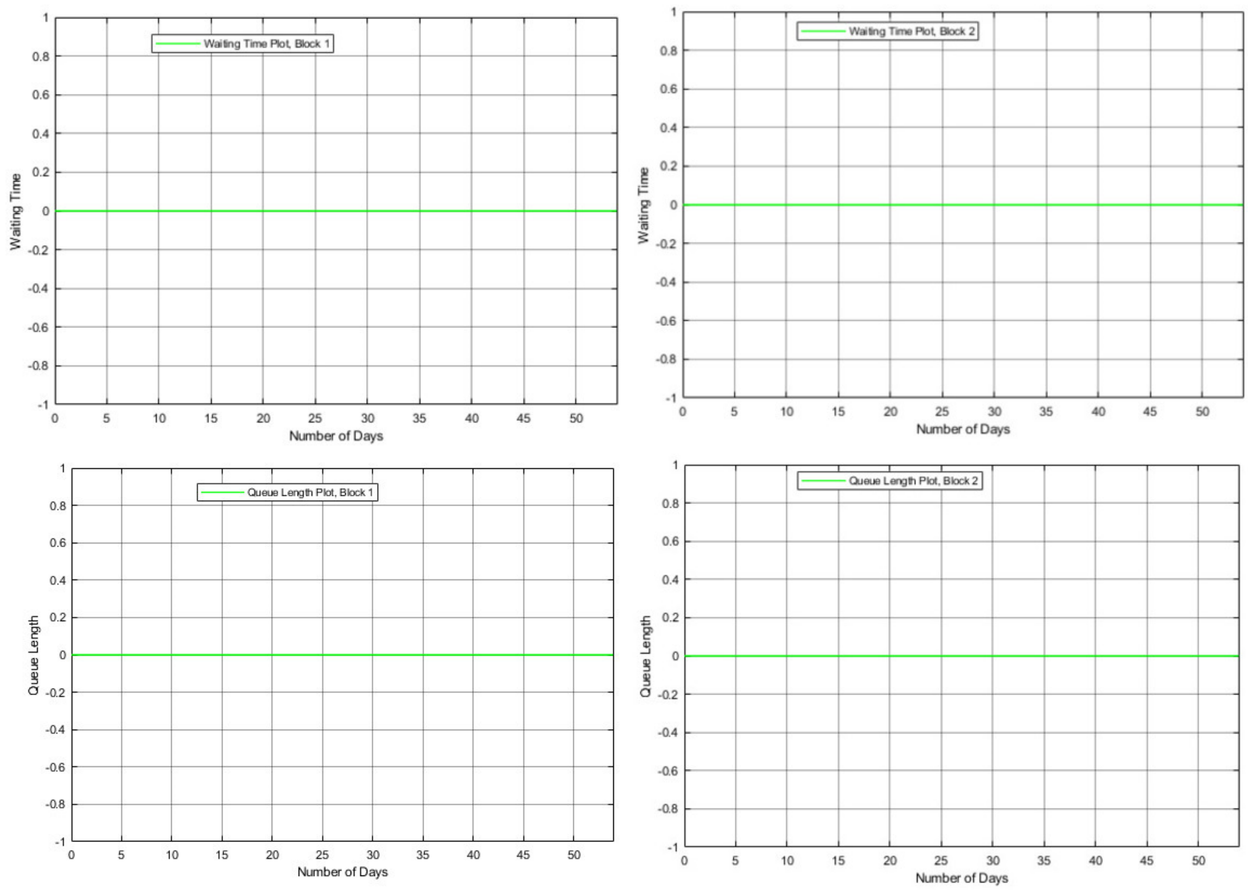

Figure 11: Queue Length and Average waiting time for Reception and Consultation blocks

The theoretically calculated values of queue length from the terms (36), (37) for reception, consultation blocks are 5.75 patients and 0.02 patients but from simulation results, we have 0 patients in queue. This means that each patient who arrived in the department is served without waiting a certain time.

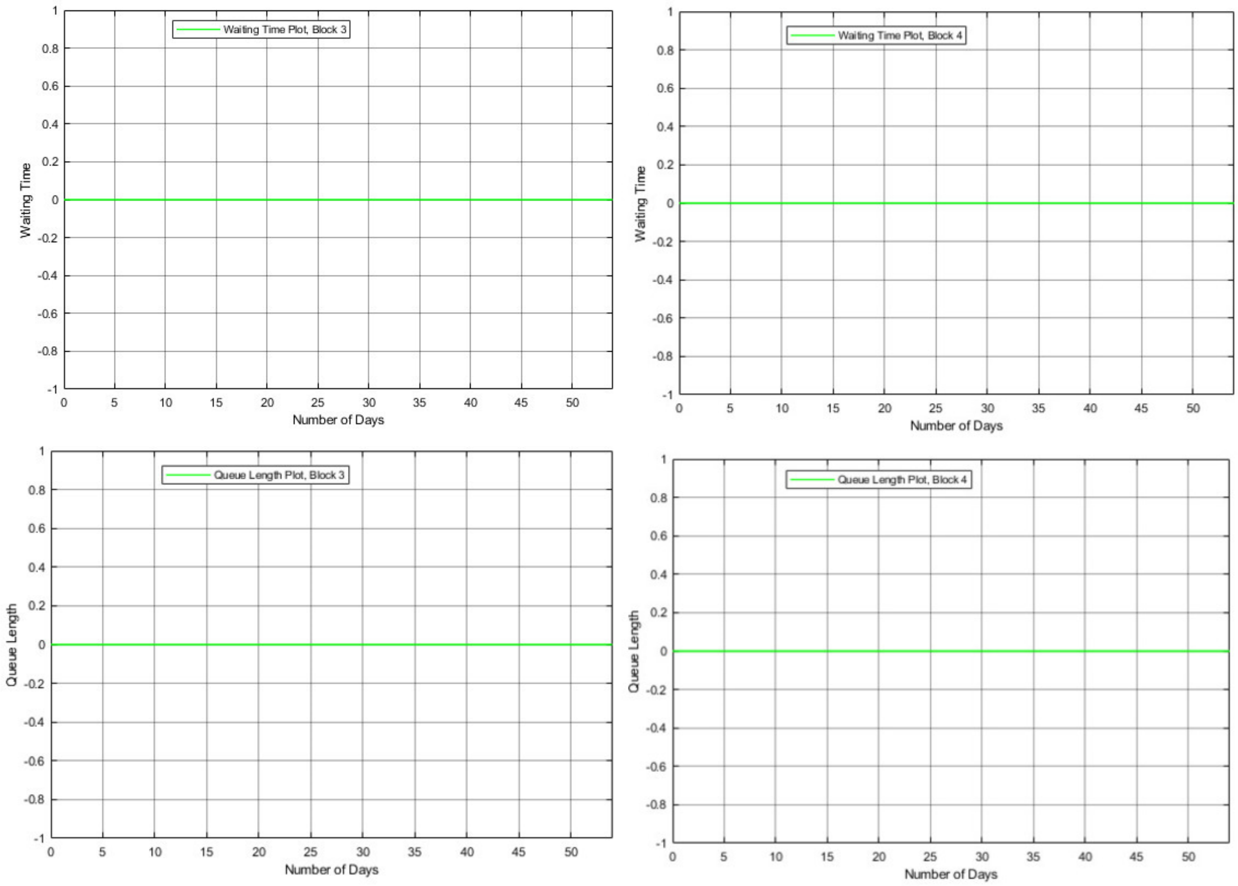

Figure 12: Average waiting time and Queue Length of Billing and Pharmacy blocks

According to the terms (38) and (39) for billing and pharmacy blocks respectively are 0 patients and 0.052 patients. But from simulation results, we have 0 patients in the queue in all blocks. This will help us to say that we have a reduction queue length.

By considering the results from the figures (13), (14), and (15) we conclude that, in practice, our designed system model will improve the service rate as we desire or as we expect. And also the average waiting time in the designed model system is reduced compared to the current model system in Rwanda Military Hospital.

4.3. Performance Analysis of RMH Designed Queuing Network Model System

As we have mentioned earlier, in the research design of this project, refer to figure 7, the designed queuing model system has 4 blocks (reception, consultation, billing, and pharmacy). We have simulated all blocks and presented their graph plots. The time a patient spends in the whole system to get serviced is equal to the summation of time he/she spends at each block. Below is the table that shows the summary of performance analysis of each block that composed the designed M/M/S queuing network model system at Rwanda Military Hospital, in the Emergency and Accidents department.

Blocks / Parameters	Reception M/M/12	Consultation M/M/5	Billing M/M/8	Pharmacy M/M/4
Traffic Intensity (ρ)	0.52	0.40	0.31	0.45
Queue Length (L_q)	5.75	0.02	0	0.05
Waiting Time (W_q)	0.3	0.00088	0	0.0028
The probability that there is no patient (P_0)	0.01949	0.1581	0.07783	0.150

Table 10: Performance analysis of RMH M/M/S Queuing Network Designed Model

We have chosen to put more servers on reception and billing blocks of the department because it is where we see a long queue of patients than other blocks.

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

Our research project's main objective was to design a patient flow based on queuing algorithmic model that will improve the service rate at Rwanda Military Hospital, department of Emergency and Accidents, we finally conclude that the current patient's flow in Rwanda Military Hospital, in the department of Emergency and Accident. Has a traffic intensity or system utilization factor in all blocks (reception, consultation, billing, and pharmacy) which is greater than one, which means all blocks are always overloaded with patients looking for a health service. Moreover, both Queue length and waiting time in the current RMH patient's flow queuing model is growing limitlessly in all blocks.

But by applying our designed queuing network model system based M/M/S: FCFS/∞/∞ algorithm for patient's flow, the health service rate will be increased from 2.32 to 27.84 at reception block, 11.11 to 55.55 at consultation block, from 3.75 to 30 at billing block and from 10.22 up to 40.88 at pharmacy block. And both queue length and waiting time are reduced considerably in all blocks of Rwanda Military Hospital in the department of Emergency and Accident reflects the study's alternative hypothesis.

5.2 Recommendations

5.2.1 Rwanda Military Hospital

As the problem of long waiting lines of patients found in this hospital, in the department of Emergency and Accident, I would like to recommend the hospital to adopt this new designed queuing model algorithm in this project research to increase the service rate and patients satisfaction.

5.2.2 Future Researches

Our research study was conducted only in the department of Emergency and Accident; other researchers should further study on other remaining departments of the hospital or on. Other health care on the choice to estimate the total time a patient can spend in the hospital considering the patient flow.

REFERENCES

1. A.H. Nor Aziati, N. S. (March 6-8, 2018). Application Of Queuing Theory Model And Simulation Patient Flow At The Outpatient Department, Bandung, Indonesia. *Proceedings of the International Conference on Industrial Engineering and Operations Management*, 3017.

2. Adwok, J. (2015). Probability Sampling - A Guideline for Quantitative Health Care Research. *The ANNALS of AFRICAN SURGERY*, 95.

3. Afsoon Aeenparast1, F. F. (2019). Patient Flow Analysis in General Hospitals: How Clinical Disciplines Affect Outpatient Wait Times. *Hospital Practices and Research*, 130.

4. ALEXANDER KOMASHIE1, 2. A. (2015). An Integrated Model of Patient and Staff Satisfaction Using Queuing Theory. *IEEE Journal of Translation Engineering in Health and Medicine*.

5. Anbarasi, R. (2018). A study on M/M/3 queuing model on waiting for time reduction in a local health care center. *International Journal of Statistics and Applied Mathematics*.

6. Antony Arthur, Beverley Hancock. (2009). *Introduction to the Research Process.*

7. Ershi, Z. M. (2015). A Multi-Type Queuing Network Analysis Method for Controlling Server Number in the Outpatient. *The Open Automation and Control Systems Journal*.

8. Ershi, Z. M. (2016). A Multi-Type Queuing Network Analysis Method for Controlling Server Number in the Outpatient. *The Open Automation and Control Systems Journal*.

9. Green, L. (n.d.). QUEUEING THEORY AND MODELING. In L. Green. Graduate School of Business, Columbia University, New York, New York 10027: Linda Green.

10. Hajnal Vass, Z. K. (2015). *Application of Queuing Model to Patient Flow in Emergency Department.* Mures County: Elsevier.

11. Health Foundation, 9. L. (2013). *Improving Patient Flow.* Health Foundation, 90 Long Acre, London WC2E 9RA.

12. Ilker Etikan, S. A. (2016). Comparison of Convenience Sampling and Purposive Sampling. *American Journal of Theoretical and Applied Statistics*.

13. Imahsunu, A. F. (2014). QUEUING THEORY FOR HEALTHCARE OPERATIONS MANAGEMENT. In A. F. Imahsunu.

14. Kothari, C. (2004). *Research Methodology, Methods & Techniques(SECOND REVISED EDITION).* Jaipur(India): NEW AGE INTERNATIONAL(P) LIMITED PUBLISHER.

15. Maria, A. (1997). INTRODUCTION TO MODELING AND SIMULATION. *Proceedings of the 1997 Winter Simulation Conference ed. S Andradottir, K. J. Healy, D. H. Withers, and B. L. Nelson*, 7.

16. Mikoláš, M. (2016). *Markov Models Application for Calculations of the Call Centre Traffic Parameters.* Ostrava.

17. Mohammad Shyfur Rahman chowdhury*, M. T. (2013). Solving Of Waiting Lines Models in the Bank Using Queuing Theory Model the Practice Case: Islami Bank Bangladesh. *IOSR Journal of Business and Management (IOSR-JBM),* 25.

18. Mor Armony, S. I.-T. (n.d.). *On Patient Flow in Hospitals: A Data-Based Queueing-Science Perspective. An Extended Version.*

19. Murphy, M. (2016). *Population definitions for comparative surveys in education.* Australia.

20. Obamiro. (2010). Queuing Theory and Patient Satisfaction: An Overview of Terminology and Application in AnteNatal Care Unit.

21. Organization(PRIMASYS), W. H. (2017). *Primary Health Care Systems.*

22. P.Banumathi2, S. a. (2017). A comparative study on M/M/1 and M/M/C Queueing Models Using Monte Carlo Simulation. *Global Journal of Pure and Applied Mathematics.,* ISSN 0973-1768 Volume 13, Number 11 (2017), pp. 7843-7853.

23. Sam Afrane, A. A. (Frebruary 2014). Queuing theory and the management of Waiting-time in Hospitals. *International Journal of Academic Research in Business and Social Sciences*, 35.

24. Singh, A. S. (2014). SAMPLING TECHNIQUES & DETERMINATION OF SAMPLE SIZE IN APPLIED STATISTICS RESEARCH: AN OVERVIEW. *International Journal of Economics, Commerce, and Management*, 14.

25. Supriya Burungale1, K. K. (2018). Patient Queue Management System. *International Journal of Engineering Science Invention (IJESI)*, 48.

26. Thanda Aung*, L. L. (2019). Comparative Study on Different Queuing Models to Reduce Waiting Time in Brahmaso Clinic. *International Journal of Scientific and Research Publications, Volume 9, Issue 6, June 2019, ISSN 2250-3153*.