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**NEW LIGHT NEUTRAL PARTICLES IN MESON  
DECAYS**

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## Declaration

I, **Fidele TWAGIRAYEZU**, hereby declare that this thesis entitled **NEW LIGHT NEUTRAL PARTICLES IN MESON DECAYS** is my own work and has not been submitted anywhere for the award of any degree. Wherever I extracted a sort of statement, I acknowledged the owner. This thesis was directed and supervised by **Prof. Yassaman Farzan**.

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# Abstract

The elementary fermions that so far have been discovered can be divide into two subgroups: leptons and quarks. Quarks,unlike leptons , can have strong interactions. As a result, they cannot be found in nature as free particles but they are always confined in bound states called hadrons. The two well-established groups of hadrons are mesons ,which are composed of valence quark and anti-quark and baryons which are composed of of three valence quarks.In recent years, some exotic hadrons consisting of four valence quarks (tetraquarks) and five valence quarks (pentaquarks) have been registered and the study of their properties is under progress. In this thesis, we focus on mesons, especially on pions, Kaons and B-mesons. We enumerate different species of mesons, categorized based on their quark content (i.e., strangeness, beauty and charm) and their spin. We discuss their well established properties such as lifetime and decay modes within the standard model. We then review CP-violation in neutral Kaon and B-meson systems and how the parameters of the CKM quark mixing can be derived from experiments. We then discuss the recent observations in the B-meson decay modes that deviate from the standard model predictions. These deviations are known as B-anomalies. We review some classes of beyond standard model explanations for these anomalies. We follow with a short discussion of the recently reported KOTO anomaly and its possible explanations.In the end, we show how studying the different decay modes of Kaons and pions can help us to search for new physics beyond the standard model of the elementary particles. In particular we show that if there are new neutral particles with a mass of less than 50 MeV coupled to the neutrinos, a deviation from the standard model prediction for the energy spectrum of the final particles in charged meson decays is expected. We study how such a coupling can be constrained by studying meson decay modes.

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# Chapter 1

## Introduction and Motivation

The "Standard Model" (SM) theory, is one of the greatest successful theory of physics. It does a great job of describing the fundamental particles and their interactions. The Standard Model has been enormously successful in reproducing data and predicting various experimental results collected from collider physics experiments such as at Large Electron-Positron Collider (LEP) and at the Large Hadron Collider (LHC). LEP accelerator started operating in 1989 before LHC and finishes taking data in 2001.

It was searching for physics Beyond Standard Model(BSM) but it has failed. Instead, it has measured the SM parameters with good precision. Afterwards, LHC (or proton-proton collider) through its CMS and ATLAS detectors, searched for New Physics (NP) unfortunately it has not found any evidence for BSM. On 4<sup>th</sup> July 2012, Compact Muon Solenoid (CMS) and A Toroidal LHC Apparatus (ATLAS) collaborations reported the discovery of Higgs particle which was the last missing particle of SM. The presence of Higgs particle was predicted in 1964 ( about four decades ago ). The properties of the particle that was discovered by CMS and ATLAS coincided with particle predicted by Higgs.

Despite the triumphs of the SM in prediction of the experimental results in colliders, there are still some shortcomings not yet explained. We do not expect SM to be the final and the most complete theory. We expect that researchers will find pieces of evidence for NP. The most important evidence that the SM of particle physics is incomplete comprises neutrino masses and mixing, and the presence of dark matter (DM) in the Universe [6]. Here, we can simply mention: hierarchy problem, unification of forces, charge quantization and the anomalous magnetic moment of muon and electron,  $(g - 2)_{\mu,e}$ . We know that 95% of the universe's matter content is in form of DM but we do not know what is the nature of it and it cannot be presented in



any of the known particles within SM. None of the known particles in SM can play the role of DM. This discussion shows that there is something BSM which we should, as physicists who are curious, go and investigate and propose strategies to find that. There are two conventional approaches to search for NP and new particles.

- **The energy frontier** in Collider such as LHC where we increase the energy of the centre of mass (CM) hoping that we find the heavier and heavier particles which couple to SM particles assuming the couplings are not very small.
- **The intensity frontier** where instead of increasing energy, one prefers to increase the number of colliding particles (intensity of colliding to each other) like for example at Belle and BaBar experiment, and NA62 experiment. So far, LHCb is the best intensity frontier which has a big intensity and luminosity and is considered as the highest energy frontier experiment

Therefore, LHCb is suitable for searching for NP because of two reasons. First, the idea for searching for relatively light particles with very small coupling. Suppose that we have a new particle BSM which has a very tiny coupling. Such a particle has not been observed and discovered so far, not because it is very heavy but because it has very small coupling. Then this intensity frontier experiments by increasing luminosity or intensity eventually will be able to discover these particles.

Moreover, by measuring scattering features from experimental interaction, other properties, when measured with higher precision can be seen. As result, it might be possible to discover Beyond Standard Models with high particles in the hierarchy. Of course , these high particles will not be directly produced at lower energy intensity frontier experiments, but their effects are indirectly identified. Once there is evidence with high precision that you do not see directly such particles, then this indicates that there are possibilities to search for NP. Therefore, high energy intensity frontier experiment is necessary to identify what kind of physics that leads us to such effects. Particle physics makes progress in high energy frontier, and high intensity Frontier. The recent constructed B-decays by high intensity frontier experiment using a data sample corresponding to integrated luminosity of around  $7fb^{-1}$  , but at relatively moderated center of mass energy ( $\sqrt{s}= 2TeV$ ), gave accurate results at B-factories. The anomalies encountered in SM that deviate from SM predictions are mainly found in B-meson decays. The search for Lepton Flavour Univers (LFU) in B decays, is a big motivation to experimenters in B-physics. One of the most interesting phenomena reported by particle physics experiments in the last few years are the numerous hints

of (LFU) violations observed in semi-leptonic B decays. The very recent LHCb results on the LFU ratios  $R_{K^{(*)}}^{\mu e}$  and  $R_{D^{(*)}}^{\tau l}$ . For example, the deviations from  $\frac{\tau}{\mu}$  (and  $\frac{\tau}{e}$ ) universality in  $b \rightarrow cl\bar{\nu}$  charged currents and deviations from  $\frac{\mu}{e}$  universality in  $b \rightarrow s\bar{l}$  neutral currents. Furthermore, a strong evidence for a deviation from the SM prediction has been observed by LHCb in the angular distribution of the  $B^0 \rightarrow K^{*0}\mu^+\mu^-$  decay, which is consistent with the deviations from LFU in neutral-current B decays [7].

Neutrinos have played a crucial role from earlier in the 20<sup>th</sup> century in the construction of the SM. All along this period, the direct detection of their masses is not an easier task [8]. Despite, solar and atmospheric neutrino anomalies, occur in neutrino sector. The Liquid Scintillator Neutrino Detector (LSND) experiment observed the appearance of neutrino anomalies in a two-neutrino oscillation model,  $\nu_\mu \rightarrow \nu_e$ . The same observation was replicated by MiniBooNE experiments. A two-neutrino oscillation interpretation of the data, would require at least four neutrino types (3+1 model) and imply the physics beyond the three neutrino paradigm [9]. The additional neutrinos might be **sterile neutrinos**. Sterile neutrinos (or inert neutrinos) are hypothetical particles that interact only via gravity and do not interact via any of the fundamental interactions of the SM.

There are some hints that neutrinos may be coupling to some new light particles. This is not established but some discrepancies can be explained. The cosmological observations set two tensions. Firstly, from the Cosmic Microwave Background CMB observation as well as other observable find an upper bound on the sum of neutrino masses ( $\sum m_i < 0.12eV$ ). If any neutrino with mass heavier than 1eV is produced in the early universe then it violates this bound. And secondly, cosmology tells us that in the early universe only three neutrinos could be produced. If there were sterile neutrinos that mixed with active neutrinos such that the recent results might be explained, both bound, some of the neutrino masses and the number of neutrinos could be violated. So introducing new interactions for neutrinos is a technique to prevent the sterile neutrinos being produced in the early universe.

The thesis is organised in five chapters as follows. In chapter 1, we give the introduction of the thesis and the reason why we have chosen to work on it. In chapter 2, we will review mesons and their main decays and the kaon physics. In chapter 3, we will discuss deviation from SM predictions presented in B-decays and try to find the explanations within is BSM framework. In chapter 4, we will discuss the new neutral light gauge boson resulted in from the charged meson ( $\pi^+$ ,  $K^+$ ) and how it couples to neutrinos. Finally, the summaries of the conclusion of the work are presented in

## Chapter 5.

## Chapter 2

# REVIEW OF MESONS

The Standard Model (SM) of particles provides understanding of the principal natural interactions (forces) which govern the matter-antimatter content of the universe. In particle physics domain, a fundamental particle is a subatomic particle with no sub-structure; i.e., it has no composition of other particles. Elementary particles include fermions (leptons, quarks) and bosons (gauge bosons, Higgs boson). For ordinary charged matter particles, there are corresponding anti-matter particles as was shown in Quantum Field Theory (QFT) in Dirac formalism. Particles and antiparticles look the same way, only differ in all charges. Fermions and bosons are distinguished by their intrinsic spin. Fermions are those particles which are described by the Fermi-Dirac statistics and generally have half odd integer spin ( $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \dots$ ). These particles obey the Pauli exclusion principle where two or more identical particles cannot occupy the same quantum energy state. Besides, they change sign whenever their position becomes reversed. For instance, the operation of parity operator on their state functions change sign. While Bosons are integer spin particles and obey Bose-Einstein statistics. Here a collection of non-interacting and indistinguishable particles can occupy a set of available discrete energy states. In this chapter we will discuss the general overview of some elementary particles within the SM framework. We will mainly focus on mesons, their decays modes but not all of them, we will be interested especially on pions, kaons, D-mesons, and B-meson in which we will be restricted on principal decay modes. We will show that the study of neutral mesons (K and B-meson) and their oscillations provides constraints on the values of the elements of the CKM matrix and allow CP violation to be studied in the quark sector.

## 2.1 Fermions

In particle physics, as mentioned previously, fermions have half-integer spin and have quantum numbers described by the Pauli-exclusion principle. They include leptons and quarks. Leptons are primary particles, constituents of matter, which undergo electromagnetic and weak (electroweak) interactions. The charged leptons only feel such interaction while neutral ones and neutrinos undergo only weak interaction. Furthermore, quarks are fundamental sub-atomic particles, constituents of hadrons. In addition to electroweak undergone by leptons, they also feel strong interaction. The strong forces found in quarks are mostly due to the presence of color charge carried by each quark. There exist three types of color charges: red(r), green(g), and blue (b) with their corresponding anti-colors. In quantum chromodynamic (QCD), also called color field theory, color-charged particles interact via gluon exchanges, in the same way, charged particles interact by exchanging photon (for electromagnetic interaction) or  $W^\pm$ ,  $Z^0$  are gauge bosons (in weak interaction). Quarks are not found as an isolated particle in nature, but they always in bound states (quark confinement) and when combined, they form hadrons. They are twelve (12) fermionic particles (six leptons and six quarks) and twelve corresponding anti-fermionic particles. Since the whole subatomic structure is made of fermions (leptons (electrons) and quarks (protons and neutron)), one can conclude that the fermions are the basic building blocks of matter fields mediated by gauge bosons.

Fermions are classified in three generations according to how their masses increase from one generation to another, see the tables (Tab 2.1 and Tab 2.2) below:

**Table 2.1:** Leptons [10]

Generation	Particle	Mass (GeV)	charge (Q)	interaction
First	electron ( $e^-$ )	0.000511	-1	electroweak
	$\nu_e$	$< 6.224 \times 10^{-9}$	0	weak
Second	muon ( $\mu^-$ )	0.106	-1	electroweak
	$\nu_\mu$	$< 89.78 \times 10^{-9}$	0	weak
Third	tau ( $\tau^-$ )	1.78	-1	electroweak
	$\nu_\tau$	$< 368.7 \times 10^{-9}$	0	weak

**Table 2.2:** quarks [11]

Generation	Particle	Mass(GeV)	charge(Q)	interaction
First	down (d)	0.003	$-\frac{1}{3}$	strong and electroweak
	up (u)	0.005	$\frac{2}{3}$	strong and electroweak
Second	strange (s)	0.1	$-\frac{1}{3}$	electroweak and strong
	charm(s)	1.3	$\frac{2}{3}$	strong and electroweak
Third	top (t)	174	$\frac{2}{3}$	strong and electroweak
	bottom(b)	4.5	$-\frac{1}{3}$	electroweak and strong

## 2.2 Bosons

As we have seen before, a boson is one type of the fundamental particles. Bosons are divided into two groups: gauge bosons and scalar boson (see Tab2.3). The gauge bosons have the spin of 1 and mediate the interactions between quarks and leptons. The electromagnetic interaction is mediated by the exchange of virtual photons, the weak charged-current interactions by the charged  $W^\pm$  bosons, and weak neutral-current interactions by  $Z^0$  bosons. The strong force is carried by eight different massless and electrically neutral gluons. ( see Tab2.3 below :

**Table 2.3:** Forces and their mediators

Force	Boson	Symbol	Mass(GeV)	Spin
Strong	gluon	g	-	1
Electromagnetic	photon	$\gamma$	-	1
Weak	Z-boson	$Z^0$	91.2	1
Weak	W-boson	$W^\pm$	80.4	1

In 2012, the discovery of a scalar field with mass equal 125 GeV and zero spins was announced. The Higgs boson is the scalar boson, which has no intrinsic spin and plays an essential role in the SM by explaining the origin of mass of all particles. The SM is characterized by the gauge symmetry group which is a combination of the  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ , where:

the  $U(1)$  which corresponds to the photon is not the  $U(1)_Y$ , but it is the combination which is left after the electroweak symmetry breaking, i.e.  $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}$  symmetry;

$SU(2)_L$ : two-Component special unitary group, which describes the weak interactions. This group has three generators of gauge symmetry,  $\frac{1}{2}\sigma_a$ , (a= 1,2,3) which are represented as Pauli spin matrices,  $\sigma_a$ ,

$SU(3)$ : three-dimensional special unitary group, which describes the strong interactions.

The symmetry has eight generators, which are associated with eight types of gluons and represented as the Gell–Mann matrices,  $\frac{1}{2}\lambda_a$ .

The conserved charge associated with this symmetry is known as "color charge",  $C$ . The SM prediction is that fermions and gauge bosons ( $W^\pm, Z^0$ ) have to be massless to preserve the electroweak gauge symmetry. However, the experimental results demonstrated that measured masses for them are found to be non-zero. Therefore, the electroweak gauge symmetry has to be broken involving the mechanism of spontaneous symmetry breaking, for them to acquire masses. The Higgs boson generates the masses of the weak gauge bosons and fermions through the Higgs mechanism, in which the local gauge symmetry of the  $SU(2)_L \otimes U(1)_Y$  gets spontaneously broken into the  $U(1)_{em}$  symmetry of QED, while the  $U(1)_{em}$  and  $SU(3)$  symmetries remain unaffected after the symmetry is broken.

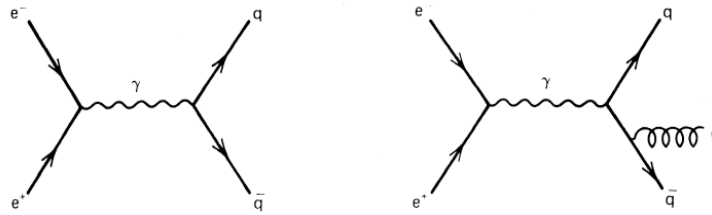
## 2.3 Mesons

Mesons are hadronic particles composed of quark anti-quark bound states ( $q\bar{q}$ ). The neutrons and protons talk to each other by exchanging mesons, which guarantee the two to live in together without basting due to the repulsion forces. Mesons feel strong, weak, and electromagnetic forces. From the fact that a quark/antiquark has a color/anti-color, they cancel out forming a neutral meson. On the other hand, three colored quarks together form a baryon (for example proton and neutron, composed by uud and ddu ,respectively ). In the constituent of the quark model, we treat a meson as a quark-antiquark pair ( $q\bar{q}$ ), and then draw the same analogy to the electron-positron (positronium system)  $e^-e^+$  to understand the picture of meson total spin. Since both  $q$  and  $\bar{q}$  have spin  $\frac{1}{2}$ , they combine to either total spin  $s = 0$  or total spin  $s = 1$ . Therefore, mesons are integer-spin particles and do not follow the Pauli exclusion principle, the argument strongly supporting them to be bosons rather than fermions. In addition to the total spin, we can have orbital angular momentum

L between  $q\bar{q}$  pair. Then S and L can combine to total angular momentum J, with  $J = L \oplus S$  and J takes the values  $|L - S|, |L - S + 1|, \dots, |L + S|$  [12].

### 2.3.1 Production modes

Mesons, like other particles can be produced by different means based on the kind of process taken into consideration. High-energy nuclear collisions frequently yield charged and neutral mesons as unstable reaction products. Charged pions ( $\pi^\pm$ ) decay into positive and negative muons that decay in turn into relativistic electrons and positrons. Neutral pions ( $\pi^0$ ) decay almost into two photons. The mesons are produced in the proton-anti-proton reaction :  $p + \bar{p} \rightarrow K^+ + \bar{K}^0 + \pi^-$  and in the decay process of some heavy particles like  $\Sigma \rightarrow n + \pi$ . As other processes that can result in meson generation, for example electron-proton(ep) deep scattering and electron-positron ( $e^+e^-$ ) annihilation ( $e^+e^- \rightarrow q\bar{q}$ ) or ( $e^+e^- \rightarrow q\bar{q} g$ ), (see Fig:2.1) below.



**Figure 2.1:** Electron-positron annihilation [1]

Besides this, the earth's atmosphere we are living in, is full of particles. For that reason, neutrinos from the atmosphere (cosmic rays) are extraterrestrial neutrinos which come from the collisions of the primary proton components of cosmic rays with the earth's atmosphere, giving rise to mesons. In turn, the mesons decay into muon and a neutrinos.

The muons are the penetrating and energetic charged leptons of the cosmic rays spectrum that reach the earth. The neutrinos, even more penetrating particles, are muon leptonic birth partners and decay products [13].

Nowadays, all the mesons can be copiously produced in high-energy collisions at accelerators by strong interaction processes such as  $p + p \rightarrow p + n + \pi^+$



### 2.3.2 Classification and Decay modes

There exist a huge number of mesons in nature, roughly 140 types of mesons have been observed. They are classified according to their characteristic properties, such as: masses, (heavier or lighter), strangeness, spin, parity, beauty, mode of production and type of interaction. Before making a classification, let us define the following important terms:

**Strangeness** is a quantum number assigned the value “-1” for quark and “+1” for anti-quark. In general, ( $s = -1$ ) is given to all particles and ( $\bar{s} = +1$ ) to all antiparticles. The zero value ( $s=0$ ) is attributed to all other quarks which do not contain the strange quark ( $s$ ) in their content. It was introduced to a better understanding of the quark structure of matter. It is a property describing a large group of strongly interacting strange particles by taking into account to conservation law. A law of conservation of strangeness states that the sum of the strangeness quantum numbers between reacting particles or the quantum number of a decaying particle should be equal to the sum of strangeness numbers of the reaction products or decay products. A violation of the law might explain different decay rates of the products compared to their rates of production. Note that the electromagnetic and strong interactions conserve the strangeness and weak does not. It is symbolized by  $S$  and is given by:  $s = n_s + n_{\bar{s}}$ , where  $n_s$  is the number of strange quarks ( $s$ ) and  $n_{\bar{s}}$ , the number of strange anti-quarks ( $\bar{s}$ ).

**Bottomness or beauty** is a quantum number reflecting particles containing bottom quarks ( $n_b$ ) and bottom anti-quark ( $n_{\bar{b}}$ ) in their quark content.  $b' = n_b + n_{\bar{b}}$ . The value “-1” is given to the bottom quark and “+1” to the bottom antiquark.

**Charmness** is a flavor quantum number (symbol  $c$ ) representing the number of charm quarks  $n_c$  and minus (“-”) charm anti-quark  $n_{\bar{c}}$  that are present in a particle. Similarly as the strangeness and beauty, the value “-1” is assigned to charm quark ( $c = -1$ ) and ( $\bar{c} = +1$ ) to its conjugate. By convention the sign of flavor quantum numbers agree with the sign of the electric charge carried by the quarks of corresponding flavors.

**Branching ratio (or Decay fraction)** we define the branching ratio (BR) as the fraction of particles which decay by an individual decay mode with respect to the sum of all decay channels, or simply the ratio of the partial decay constant to the overall decay constant, ( $BR = \frac{\Gamma_i}{\Gamma_{tot}}$ ). A single particle can decay into many different products with different values  $BR(A \rightarrow B + C + D + \dots) = \frac{\Gamma(A \rightarrow B + C + D + \dots)}{\Gamma_{tot}(A)}$ . This means that the value of any decay process determines how much probable such process can take place. The bigger BR is, the high probability of a decay process

to occur among others in nature. We remember that the decay constant  $\Gamma$  or decay width is the inverse of the average lifetime ( $\Gamma = \frac{1}{\tau}$ ) and the total partial decays sum up to one ( $\sum \Gamma_i = 1$ ).

### 2.3.3 Light mesons: Pions

Pions are the lightest mesons, sometimes called unflavored mesons, are the lightest bound states of all hadrons. The pion can be considered as one of the particles that mediate the interaction between two nucleons (proton-neutron) within nucleus. The interaction is attractive, i.e., it pulls the nucleons together and is sometimes known as non relativistic Yukawa interaction. Here proton and neutron communicate to each other by exchanging pi meson for them to avoid the nucleus collapsing. In their quark contents, pions do not have strange, charm and bottom quark ( $s = c = b = 0$ ) and they are formed by the 1<sup>st</sup> generation of quarks. Their quark contents are up(u) and down (d) quarks as well as their corresponding anti-quarks. There exist 3 kinds of pions (pi-meson):  $\pi^\pm$  and  $\pi^0$  (two charged  $\pi^\pm$  and their neutral one  $\pi^0$ ).

The characteristic features of pions are:

Mass :  $m_{\pi^\pm} = (139.57061 \pm 0.00024)$  MeV

$m_{\pi^0} = (134.9770 \pm 0.0005)$  MeV

Electric charge:  $Q_{\pi^\pm} = \pm 1e$

$Q_{\pi^0} = 0$  (neutral pion, not charged)

color charge:  $Q_c = 0$  (they do not decay via strong interaction)

Spin:  $S = 0$  (zero spin for both  $\pi^\pm$  and  $\pi^0$ )

parity:  $(j^p) = 0^-$  (the intrinsic party of pi mesons is -1)

Mean lifetime:  $\tau_{\pi^\pm} = (2.6033 \pm 0.0005) \times 10^{-8} s$

$\tau_{\pi^0} = (8.55 \pm 0.18) \times 10^{-17} s$

Quark content:  $\pi^- = u\bar{d}$ ,  $\pi^+ = d\bar{u}$ , and  $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$

All mesons are unstable because of their short life time, they immediately decay into other light particles just after being produced.

### 2.3.4 Decay modes of pions

In general, particle decay is a spontaneous disintegration of a subatomic particle with the emission of energetic particles or radiation, such as electrons and neutrinos or gamma-ray photons. In this process, the unstable particle transforms into numerous other particles with masses, for each, less than the mother's particle. The total mass of the system must be conserved.

A particle is unstable if there is at least one allowed final state that it can decay into. Unstable particles often have multiple ways of decaying, each with its associated probability. Decays can be mediated by one or several gauge bosons. The particles in the final state may themselves be unstable and subject to further decay. It may happen that the end products are not pieces of the starting particle, but new particles. For the lightest mesons, which have zero orbital angular momentum ( $J = 0$ ), the total angular momentum  $J$  is determined by the spin state alone. Consequently the lightest mesons divide into the  $J = 0$  pseudoscalar mesons (These include: pions, kaons, and eta) and the  $J = 1$  (the vector mesons: rho, star-kaons( $K^*$ ), omega( $\omega$ ), and phi ( $\phi$ ), respectively with  $s = 0$  and  $s = 1$ ). In this section, we discuss different decay modes of mesons.

### 2.3.5 Decays of charged pions

The  $\pi^\pm$ -pions decay via weak interaction and they are mediated by  $W^\pm$  gauge bosons. By looking at the measured lifetime of  $\pi^\pm$ -pions, stated above, it is obvious that they live longer than the neutral pion. Because mesons (pi-mesons) are the lightest among all the hadrons, they decay mostly into leptons as final bound states. The reason why their decay modes are referred to as leptonic decay. If a pion decays by emitting photons, this phenomenon is called radiative decay. The following are the main decay modes of charged pions with their BR :

$$\begin{aligned} \pi^\pm &\rightarrow \mu^\pm \nu_\mu, \text{BR}(\pi^\pm \rightarrow \mu^\pm \nu_\mu) = 99.9877\% \\ \pi^\pm &\rightarrow e^\pm \nu_e, \text{BR}(\pi^\pm \rightarrow e^\pm \nu_e) = 1.23 \times 10^{-4} \\ \pi^\pm &\rightarrow \mu^\pm \nu_\mu \gamma, \text{BR}(\pi^\pm \rightarrow \mu^\pm \nu_\mu \gamma) = 2.00 \times 10^{-4} \\ \pi^\pm &\rightarrow e^\pm \nu_e \gamma \\ \pi^\pm &\rightarrow \pi^0 e^\pm \nu_e \\ \pi^\pm &\rightarrow e^\pm \nu_e \bar{\nu} \nu. \end{aligned}$$

[source:[Http://pdg.lbl.gov](http://pdg.lbl.gov)].

From these results of branching ratio, it was seen that the  $\pi^\pm$  mostly decay into muon, muon neutrino rather than into electron and electron-neutrino. The pion has spin zero; therefore the lepton and the antineutrino must be emitted with opposite spins in order to preserve net zero spin and conserve the total angular momentum. However, we know that neutrinos are always right-handed particles, their corresponding antineutrinos must be left handed. Hence the lepton will be emitted with spin in the same direction as its momentum (rightward). Further more, for any decaying process, the dominant decay is identified because of how massive the decay products

are. This could be the reason why  $\text{BR}(\pi^\pm \rightarrow \mu^\pm \nu_\mu) = 99.9877\%$  is much greater than  $\text{BR}(\pi^\pm \rightarrow e^\pm \nu_e) = 1.23 \times 10^{-4}$ .

Again it is obvious that the branching ratio of a radiative decay modes of pions are comparatively small. This is because if we have more particles in final state, then we will have suppression by the coupling of new particles. In this case the new particle is photon and the coupling is the electron (e). Hence, at mathematical point of view, the decay rate is proportional to  $\frac{e^2 G_F^2}{64\pi^2}$ , with  $G_F$  is a Fermi coupling constant and  $64\pi$ , a phase constant.

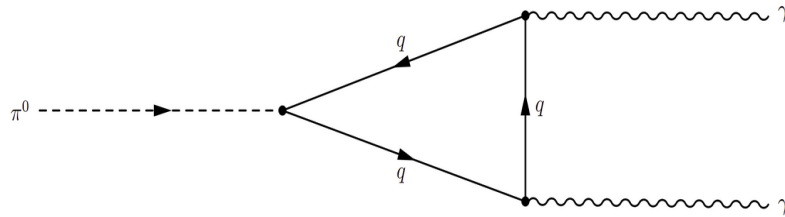
### 2.3.6 Neutral pion

The neutral pion ( $\pi^0$ ) decays via electromagnetic interaction (one-loop decay) and decays mainly into photons, it is totally radiative decay.

$$\pi^0 \rightarrow 2\gamma, \text{BR}(\pi^0 \rightarrow 2\gamma) = (98.823 \pm 0.034)\%$$

$$\pi^0 \rightarrow \gamma + e^+ + e^-, \text{BR}(\pi^0 \rightarrow \gamma + e^+ + e^-) = (1.174 \pm 0.035)\%$$

The Feynmann diagram associated to the neutral pion decay (the first channel above) (is shown in Fig:2.2 bellow):



**Figure 2.2:** Neutral pion decay

By comparing the measured mean lifetimes of  $\pi^\pm$  and  $\pi^0$ , we see that the  $\pi^\pm$  live much longer than  $\pi^0$ ,  $\tau_{\pi^\pm} > \tau_{\pi^0}$ . This is the reason why the  $\pi^\pm$  decays via weak interaction while  $\pi^0$  decays via electromagnetic interaction. The mass splitting between these mesons is less than 5%.

### 2.3.7 Isospin

The isospin (I), is a quantum number that is assigned to particles which share almost common properties (like the same mass, feel the interaction) and differ in some others, like charges. Its description is quite similar to that of usual spin, except that it does not couple with ordinary angular momenta as spin does. Here, we introduce

the concept of isospin because of the nearly identical masses of  $\pi^\pm$  and  $\pi^0$ . The involved symmetry is SU(2) flavour symmetry or isospin. The three pions belong to the triplet representation of SU(2), two doublet of  $\pi^\pm$  and singlet of  $\pi^0$ . The isospin operator  $I$ , like the ordinary spin operator, is a vector quantity with three components  $I_x, I_y, I_z$ , which are its coordinates in the three-dimensional vector space in which the 3-representation acts. Two quantum numbers  $I$  (for total spin) and  $I_3$  are used to characterise flavor states. The second quantum number  $I_3$  is an eigenvalue of the  $I_z$  projection. The  $I_z$  projection is not arbitrary as in the case for ordinary spin but a projection for which the flavor states as eigenstates i.e., each  $I_3$  state describes a specific flavor of the multiplet. The third coordinate ( $z$ ), corresponding to the 3 subscript, is chosen due to notational convention which connects bases in 2 and 3 representation spaces. As we have seen,  $I_3 = \frac{1}{2}\sigma_3$ . The operation of  $I_3$  on quark-antiquark gives:  $I_3(u) = \frac{1}{2}$ ,  $I_3(\bar{u}) = -\frac{1}{2}$ ,  $I_3(d) = \frac{1}{2}$ ,  $I_3(\bar{d}) = -\frac{1}{2}$ . Since  $I_3$  is additive quantum number (in analogy with angular momentum),  $I_3(\pi^+) = 1$ ,  $I_3(\pi^-) = -1$ , and  $I_3(\pi^0) = 0$ .

### 2.3.8 Rho mesons

A rho meson is a short-lived hadronic particle that is an isospin triplet whose three states are denoted as  $\rho^\pm$  and  $\rho^0$ . After the pions and kaons, the rho mesons are the lightest strongly interacting particle, with a mass of  $(775.26 \pm 0.25)$  MeV for all the three states ( $\rho^\pm$  and  $\rho^0$ ). Sometimes it is denoted as  $\rho(770)$  to imply its mass. The rho meson has the same quark composition as the pion.

In the description of rho mesons, De Rujula–Georgi–Glashow, defined the rho meson as a bound state of anti-quark and quark and an excited state of the pion. It is like the pion, except that the rho meson has spin  $s = 1$ . It is a vector meson, meaning that its spins are aligned/parallel) and very massive compared to pion (about five and half times). This difference in between the pions and rho mesons is attributed to a large hyperfine interaction (the energy levels of atoms, molecules, and ions, due to interaction between the state of the nucleus and the state of the electron clouds) between the anti-quark and quark. It is known that the hadron masses depend upon the dynamics inside the particle, and not just upon the quarks contained. The rho mesons have a very short lifetime ( $\tau = 4.5 \times 10^{-24}$  sec) and their decay width is about 149 MeV which means that it is very faster. The primary decay modes of the rho mesons is to a pair of pions ( $\rho \rightarrow \pi\pi$ ) with BR of 99.9%. It is colorless and decays

via strong interaction [14].

The other decays are listed below: [source:[Http://pdg.lbl.gov](http://pdg.lbl.gov)].

$$\rho^\pm \rightarrow \pi^\pm \gamma, \text{BR}(\rho^\pm \rightarrow \pi^\pm \gamma) = (4.5 \pm 0.5) \times 10^{-4}$$

$$\rho^\pm \rightarrow \pi^\pm \eta, \text{BR}(\rho^\pm \rightarrow \pi^\pm \eta) = (6 \times 10^{-3})$$

$$\rho^0 \rightarrow \pi^\pm \gamma, \text{BR}(\rho^0 \rightarrow \pi^\pm \gamma) = (9.9 \pm 1.6) \times 10^{-3}$$

$$\rho^0 \rightarrow \pi^0 \gamma, \text{BR}(\rho^0 \rightarrow \pi^0 \gamma) = (4.7 \pm 0.6) \times 10^{-4}$$

$$\rho^0 \rightarrow \mu^\pm, \text{BR}(\rho^0 \rightarrow \mu^\pm) = (4.55 \pm 0.28) \times 10^{-5}$$

### 2.3.9 Eta meson

The ( $\eta$ )-meson is an iso-singlet that is made of a mixture of up, down and strange quarks and their antiquarks,  $\eta = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - s\bar{s})$ . It is a neutral pseudo-scalar (zero spin, negative parity, zero electric charge, zero strangeness, and zero hyper charge). The mass of eta meson is,  $m_\eta = 547.862 \pm 0.017$  MeV and the mean life time of about  $(5.0 \pm 0.3) \times 10^{-19}$  sec. The full width  $\Gamma = 1.31 \pm 0.05$  KeV. The eta meson undergoes all interaction forces, but decays mainly via weak or electromagnetic interaction. The following are the main decays of eta meson: [15], [16].

$$\eta \rightarrow 2\gamma, \text{BR}(\eta \rightarrow 2\gamma) = (39.5 \pm 0.2)\%$$

$$\eta \rightarrow 3\pi^0, \text{BR}(\eta \rightarrow 3\pi^0) = (32.24 \pm 0.29)\%$$

$$\eta \rightarrow \pi^\pm \pi^0, \text{BR}(\eta \rightarrow \pi^\pm \pi^0) = (22.92 \pm 0.28)\%$$

$$\eta \rightarrow \pi^\pm \gamma, \text{BR}(\eta \rightarrow \pi^\pm \gamma) = (4.22 \pm 0.8)\%$$

$$\eta \rightarrow \pi^0 \gamma \gamma$$

$$\eta \rightarrow \pi^0 \pi^0 \gamma \gamma$$

$$\eta \rightarrow 4\gamma.$$

### 2.3.10 Strange mesons

The strange mesons, also known as K-mesons, are formed by the 2nd generation of quarks and contain a strangeness ( $s = \pm 1$ ) as their characteristic quantum number. One can wonder why these are called strange particles.

To answer this question, we can have a look at the properties of the strange particles. So far, strange particles present some unusual properties. It has been observed that most of the time, Kaons are produced in pairs via strong interaction. Scientists have suggested that they carry some new charge, strangeness. When they decay into other light mesons which do not have strangeness, they decay weakly and violate the strangeness but electromagnetic and strong interactions do not (see section 1.3.2).

The following are examples of reactions involving kaons :

$\pi^- \rightarrow K^0 + \Lambda^0$  and  $p + \bar{p} \rightarrow K^+ + \bar{K}^0 + \pi^-$  are allowed because they conserve the strangeness number but  $\pi^- + p \rightarrow K^0 + n$  is not allowed [11].

Where lambda ( $\Lambda$ ) is a strange particle while proton p, neutron n, and pion ( $\pi^-$ ) are not strange. The production of strange particles, in general, is not a result of the interaction of pions with protons and neutrons in natural cosmic processes (in the atmosphere) as it happens for other particles, but they were created and decayed through an artificial mechanism (reactors accelerator or/ and colliders). Like pions, there are three (3) types of K-mesons, two charged ( $K^\pm$ ) and a neutral one ( $K^0$ ).

The characteristic features of kaons are:

mass  $m = (493.6770 \pm 0.016) MeV$

Electric charge  $Q = \pm 1e$

spin = 0

stranges =  $\pm 1$

parity: ( $j^p$ ) =  $0^-$

Mean lifetime: =  $(1.23800 \pm 0.0020) \times 10^{-8} s$

Quark content:  $s\bar{u}, u\bar{s}$

For neutral kaons,  $K^0$  (not charged), there exist two types according to their mean lifetime, short- and long-lived kaons ( $K_S, K_L$ ). The ratio of their lifetimes is about 571 ( $\frac{\tau_L}{\tau_S} \approx 571$ ) Concerning the characteristics of neutron kaons, they share the same strangeness, spin, parity with the charged ones, and differ in lifetime, mass and quark content:

$\tau_{K_L^0} = (5.15 \pm 0.14) \times 10^{-8} s$

$\tau_{K_S^0} = (8.934 \pm 0.02) \times 10^{-11} s$

mass  $m = (497.440 \pm 0.05) MeV$

Quark content:  $s\bar{d}, d\bar{s}$

### 2.3.11 Decay modes of kaons

#### Charged kaons ( $K^\pm$ )

Kaons are the lightest hadrons that have a strange quantum number. They are unstable and decay weakly. The main decay products of charged kaons are pions, leptons, and /or photons. Hence, we will consider: leptonic decay, semi-leptonic decay, and hadronic decays or decays with  $l\bar{l}$  pair (l-lepton).

## Leptonic and semi-leptonic modes

In these decay processes, the decays products are either pure leptons (leptonic) or mixture of leptons and hadrons (semi-leptonic) [source:[Http://pdg.lbl.gov](http://pdg.lbl.gov)] and some other decays can be found in the ref [17].

We have:

$$K^\pm \rightarrow e^\pm \nu_e, \bar{\nu}_e, \text{BR}(K^\pm \rightarrow e^\pm \nu_e, \bar{\nu}_e) = 1.582 \times 10^{-5}$$

$$K^\pm \rightarrow \mu^\pm \nu_\mu, \bar{\nu}_\mu, \text{BR}(K^\pm \rightarrow \mu^\pm \nu_\mu, \bar{\nu}_\mu) = 63.56\%$$

$$K^\pm \rightarrow \pi^0 \mu^\pm \nu_\mu, \bar{\nu}_\mu, \text{BR}(K^\pm \rightarrow \pi^0 \mu^\pm \nu_\mu, \bar{\nu}_\mu) = 3.352\%$$

$$K^\pm \rightarrow \pi^0 e^\pm \nu_e, \bar{\nu}_e, \text{BR}(K^\pm \rightarrow \pi^0 e^\pm \nu_e, \bar{\nu}_e) = 5.07\%$$

$$K^\pm \rightarrow \pi^\pm \mu^\pm \nu_\mu, \bar{\nu}_\mu$$

$$K^\pm \rightarrow \pi^\pm e^\pm \nu_e, \bar{\nu}_e$$

## Hadronic modes

[17]

$$K^\pm \rightarrow \pi^\pm \pi^0, \text{BR}(K^\pm \rightarrow \pi^\pm \pi^0) = 20.67\%$$

$$K^\pm \rightarrow \pi^+ \pi^+ \pi^-, \pi^- \pi^- \pi^+, \text{BR}(K^\pm \rightarrow \pi^+ \pi^+ \pi^-) = 1.76\%, \text{BR}(K^\pm \rightarrow \pi^- \pi^- \pi^+) = 5.583\%$$

## Leptonic and Semi-leptonic modes with photons

$$K^\pm \rightarrow \mu^\pm \nu_\mu \gamma, \text{BR}(K^\pm \rightarrow \mu^\pm \nu_\mu \gamma) = 6.2 \times 10^{-3}$$

$$K^\pm \rightarrow \pi^0 e^\pm \nu_e \gamma, \text{BR}(K^\pm \rightarrow \pi^0 e^\pm \nu_e \gamma) = 2.56 \times 10^{-4}$$

$$K^\pm \rightarrow \pi^0 \mu^\pm \nu_\mu \gamma$$

## Hadronic modes with photons

$$K^\pm \rightarrow \pi^\pm \pi^0 \gamma, \text{BR}(K^\pm \rightarrow \pi^\pm \pi^0 \gamma) = 6.0 \pm 0.4 \times 10^{-6}$$

$$K^\pm \rightarrow \pi^\pm \gamma \gamma, \text{BR}(K^\pm \rightarrow \pi^\pm \gamma \gamma) = 1.010 \pm 0.6 \times 10^{-6}$$

$$K^\pm \rightarrow \pi^\pm \pi^0 m \pi^0 \gamma$$

$$K^\pm \rightarrow \pi^+ \pi^+ \pi^- \gamma, \pi^- \pi^- \pi^+ \gamma$$

## Leptonic modes with $l\bar{l}$ pairs

$$K^\pm \rightarrow e^\pm \nu_e \nu \bar{\nu}, \text{BR}(K^\pm \rightarrow e^\pm \nu_e \nu \bar{\nu}) = 6 \times 10^{-5}$$

$$K^\pm \rightarrow \mu^\pm \nu_\mu \nu \bar{\nu}$$

$$K^\pm \rightarrow e^\pm \mu^+ \mu^-$$

$$K^\pm \rightarrow \mu^\pm \nu_\mu e^+ e^-$$



## 2.3.12 Neutral kaons

[source:[Http://pdg.lbl.gov](http://pdg.lbl.gov)].

They were essential in establishing the foundations of the Standard Model of particle physics, such as the quark model of hadrons and the theory of quark mixing. Kaons have played a distinguished role in our understanding of fundamental conservation laws: CP violation, a phenomenon generating the observed matter-antimatter asymmetry of the universe, was discovered in the kaon system in 1964.

For every neutral kaon ( $K^0$ ), there is its conjugate kaon ( $\bar{K}^0$ ). The two neutral kaons are distinguished by their modes of production while short( $K_S$ ) and long( $K_L$ ) kaons are distinguished by their decay modes.  $K_S$  primarily decays in two pions, while  $K_L$  decays primarily into three pions as it is shown in the decays below of this section. Like charged kaons, neutral kaons also decay weakly with leptons or pions as their decay products. Their decays may be accompanied with photon or pairs as we did for charged K-mesons.

### Short-neutral kaon( $K_S^0$ )

Different neutral types of neutral (short) kaon decays are listed below with branching ratios **Hadronic modes**

$$K_S^0 \rightarrow \pi^0\pi^0, \text{BR}(K_S^0 \rightarrow \pi^0\pi^0) = (30.69 \pm 0.05)\%$$

$$K_S^0 \rightarrow \pi^\pm, \text{BR}(K_S^0 \rightarrow \pi^\pm) = (69.20 \pm 0.05)\%$$

$$K_S^0 \rightarrow \pi^\pm\pi^0, \text{BR}(K_S^0 \rightarrow \pi^\pm\pi^0) = (3.5 \pm 0.1) \times 10^{-7}$$

### Hadronic modes with photons

$$K_S^0 \rightarrow \pi^\pm\gamma, \text{BR}(K_S^0 \rightarrow \pi^\pm\pi^0), \text{BR}(K_S^0 \rightarrow \pi^\pm\gamma) = (1.79 \pm 0.05) \times 10^{-5}$$

$$K_S^0 \rightarrow \pi^0\gamma\gamma$$

$$K_S^0 \rightarrow \pi^\pm e^+ e^-$$

$$K_S^0 \rightarrow \pi^\pm e^\pm \nu_e$$

### Long-neutral kaon( $K_L^0$ )

Similarly to short neutral kaons, also different neutral types of neutral (long) kaon decays are listed below with branching ratios **Hadronic modes**

$$K_L^0 \rightarrow \pi^0\pi^0\pi^0, \text{BR}(K_L^0 \rightarrow \pi^0\pi^0\pi^0) = (19.52 \pm 0.12) \%$$

$$K_L^0 \rightarrow \pi^\pm\pi^0, \text{BR}(K_L^0 \rightarrow \pi^\pm\pi^0) = (12.54 \pm 0.05)\%$$

### semi-leptonic mode and /or with a photon

$$K_L^0 \rightarrow \pi^\pm e^\pm \nu_e, \gamma, \text{BR}(K_L^0 \rightarrow \pi^\pm e^\pm \nu_e) = (40.55 \pm 0.11)\%$$

$$K_L^0 \rightarrow \pi^\pm \mu^\pm \nu_\mu, \gamma, \text{BR}(K_L^0 \rightarrow \pi^\pm \mu^\pm \nu_\mu) = (27.04 \pm 0.07) \%$$

### Hadronic modes and /or with a photon

$$K_L^0 \rightarrow \pi^0 \pi^0 \pi^0 \gamma$$

$$K_L^0 \rightarrow \pi^\pm \gamma$$

$$K_L^0 \rightarrow \pi^0 2\gamma$$

$$K_L^0 \rightarrow \pi^0 \gamma e^+ e^-$$

## 2.3.13 Charmed mesons

The charmed mesons, also denoted D-meson, are the lightest particles containing charm quarks ( $c = \pm$ ). If they have the strange quark in their composition, then they are called the strange D-mesons ( $D_s$  with  $S = \pm 1$ ) and they are always charged. They feel all forces (strong, weak, and electromagnetic interactions), they are included in the family of heavy quarks because they contain c-quark. The symbolical list of D-mesons and their quark-content are given bellow:

$$D^\pm = c\bar{d}/d\bar{c}, D^0/\bar{D}^0 = u\bar{c}/c\bar{c}, D_s^\pm = c\bar{s}/s\bar{c}$$

they are massive and short-lived compared to kaons as shown by the following experimental values of rest masses and lifetimes:

$$m = (1869.4 \pm 0.5) MeV \text{ and } \tau = (1040 \pm 7) \times 10^{-15} s \text{ (for charged D-mesons)}$$

$$m = (1864.6 \pm 0.5) MeV \text{ and } \tau = (410.3 \pm 1.5) \times 10^{-15} s \text{ (for neutral D-mesons)}$$

$$m = (1968.47 \pm 0.30) MeV \text{ and } \tau = (490 \pm 9) \times 10^{-15} s \text{ (for strange D-mesons)}$$

## 2.3.14 Decays modes

Since D-mesons are heavier compared to pions and kaons, they decay into either leptons or hadrons. Therefore, in D-mesons, one can have leptonic, hadronic, pionic, or semi-leptonic decays. Each D-meson decay differently from one another. Here is the list of decays:

### Charged D-mesons

#### leptonic and semi-leptonic decay

$$D^\pm \rightarrow \mu^\pm \nu_\mu, \text{BR}(D^\pm \rightarrow \mu^\pm \nu_\mu) = (3.74 \pm 0.17) \times 10^{-4}$$

$$D^\pm \rightarrow \bar{K}^0 e^\pm \nu_e, \text{BR}(D^\pm \rightarrow \bar{K}^0 e^\pm \nu_e) = (8.76 \pm 0.17) \%$$

$$D^\pm \rightarrow K^\pm \pi^\mp e^\pm \nu_e, \text{BR}(D^\pm \rightarrow K^\pm \pi^\mp e^\pm \nu_e) = (4.02 \pm 0.18) \times 10^{-4}$$

$$D^\pm \rightarrow K^\pm \pi^\pm e^\mp \nu_e$$

$$D^\pm \rightarrow K^\pm \pi^\mp \mu^\pm \nu_\mu$$

### hadronic decay

$$D^\pm \rightarrow \bar{K}^0 \pi^\pm, \text{BR}(D^\pm \rightarrow \bar{K}^0 \pi^\pm) = (1.562 \pm 0.031)\%$$

$$D^\pm \rightarrow K^\pm \pi^\pm \pi^\pm, \text{BR}(D^\pm \rightarrow K^\pm \pi^\pm \pi^\pm) = (9.32 \pm 0.016)\%$$

$$D^\pm \rightarrow \bar{K}^0 K^\pm K^\pm \pi^\pm$$

### pionic decay

$$D^\pm \rightarrow \pi^\pm \pi^0, \text{BR}(D^\pm \rightarrow \pi^\pm \pi^0) = (1.247 \pm 0.033) \times 10^{-3}$$

$$D^\pm \rightarrow \pi^\pm \pi^\pm \pi^\mp, \text{BR}(D^\pm \rightarrow \pi^\pm \pi^\pm \pi^\mp) = (3.27 \pm 0.18) 10^{-3}$$

$$D^\pm \rightarrow \rho^0 \pi^\pm$$

### Neutral D-mesons

#### Semi-leptonic modes

$$D^0 \rightarrow K^\mp \pi^\pm e^\pm \nu_e, \text{BR}(D^0 \rightarrow K^\mp \pi^\pm e^\pm \nu_e) = (3.542 \pm 0.035)\%$$

$$D^0 \rightarrow K^\mp \pi^\pm \mu^\pm \nu_\mu, \text{BR}(D^0 \rightarrow K^\mp \pi^\pm \mu^\pm \nu_\mu) = (3.41 \pm 0.04)\%$$

$$D^0 \rightarrow K^\mp \pi^0 e^\pm \nu_e$$

$$D^0 \rightarrow K^\mp \pi^\pm e^\pm \nu_e$$

#### hadronic modes

$$D^0 \rightarrow K^\mp \pi^\pm, \text{BR}(D^0 \rightarrow K^\mp \pi^\pm) = (3.950 \pm 0.031) \%$$

$$D^\pm \rightarrow \bar{K}_S^0 \pi^0, \text{BR}(D^\pm \rightarrow \bar{K}_S^0 \pi^0) = (1.240 \pm 0.022) \%$$

$$D^\pm \rightarrow \bar{K}^0 \pi^\pm$$

$$D^0 \rightarrow K^\mp \pi^\pm \pi^0$$

#### pionic modes

[source: [Http://pdg.lbl.gov](http://pdg.lbl.gov)].

$$D^0 \rightarrow 2\pi^\pm, \text{BR}(D^0 \rightarrow 2\pi^\pm) = (1.455 \pm 0.024) \times 10^{-3}$$

$$D^0 \rightarrow 2\pi^0$$

$$D^0 \rightarrow \rho^\pm p i^\pm, \text{BR}(D^0 \rightarrow \rho^\pm p i^\pm) = (1.01 \pm 0.04) \%$$

$$D^0 \rightarrow \rho^0 p i^0, \text{BR}(D^0 \rightarrow \rho^0 p i^0) = (3.86 \pm 0.023) 10^{-3}$$

$$D^0 \rightarrow 2\pi^\pm \pi^0$$

#### radiative modes

$$D^0 \rightarrow \rho^0 \gamma, \text{BR}(D^0 \rightarrow \rho^0 \gamma) = (1.82 \pm 0.32) \times 10^{-5}$$

$$D^0 \rightarrow \omega \gamma$$

$$D^0 \rightarrow \phi \gamma$$

$$D_s^\pm\text{-Mesons}$$

The main decays of charmed-strange mesons are:

#### leptonic modes

$$D_s^\pm \rightarrow \mu^\pm \nu_\mu, \text{BR}(D_s^\pm \rightarrow \mu^\pm \nu_\mu) = (5.49 \mp 0.16) \times 10^{-3}$$

$$D_s^\pm \rightarrow \tau^\pm \nu_\tau, \text{BR}(D_s^\pm \rightarrow \tau^\pm \nu_\tau) = (5.48 \pm 0.23)\%$$

#### hadronic decays

$D_s^\pm \rightarrow K^\pm \bar{K}^0_S$  , BR( $D_s^\pm \rightarrow K^\pm \bar{K}^0_S$ ) = (1.46 ± 0.04)%  
 $D_s^\pm \rightarrow K^\pm K^\mp \pi^\pm$  , BR( $D_s^\pm \rightarrow K^\pm K^\mp \pi^\pm$ ) = ( 5.39 ± 0.15)%  
 $D_s^\pm \rightarrow \rho^\pm \phi$  , BR( $D_s^\pm \rightarrow \rho^\pm \phi$ ) = (8.4 ± 1.9)%  
 $D_s^\pm \rightarrow 2\pi^\pm \pi^\mp$   
 $D_s^\pm \rightarrow \rho^\pm \pi^\mp$   
 [source: [Http://pdg.lbl.gov](http://pdg.lbl.gov)].

### 2.3.15 Bottom mesons : B-Mesons

The  $B$  mesons are bound states of a bottom quark ( $b = \pm 1$ ) and a light antiquark, either an up, down or strange quark. If they contain a strange quark, they are called bottom-strange mesons ( $s = \pm 1$ ) or charmed B-mesons. If they contain charm quark ( $c = \pm 1$ ). B mesons are the only mesons containing quarks of the third generation and thus their decays provide a unique opportunity to measure the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and the  $B - \bar{B}$  mixing phenomenon as will be discussed in the last section of this chapter. According to their quark content, we have 4 types of B-mesons, (see Tab:2.4)

**Table 2.4:** characteristics of B mesons

Particle	Rest mass (GeV)	Lifetime ( $\times 10^{-12}s$ )	Charge (e)	Parity	Quark content
$B^\mp$	$5.279 \pm 0.0005$	$1.671 \pm 0.0181$	$\pm 1$	-1	$ub/\bar{b}\bar{u}$
$B^0/or \bar{B}^0$	$5.2794 \pm 0.00005$	$1.536 \pm 0.014$	0	-1	$d\bar{b}/b\bar{d}$
$B_s^0/or \bar{B}_s^0$	$5.3696 \pm 0.0024$	$1.461 \pm 0.057$	0	-1	$s\bar{b}/b\bar{s}$
$B_c^\mp$	$6.4 \pm 0.4$	$0.46 \pm 0.18$	$\pm 1$	-1	$c\bar{b}/b\bar{c}$

### Decay Modes of B-Mesons

Because B-meson has a heavier quark which belongs to the third generation, their final state products may include all other quarks. The B-mesons can only decay via the weak interaction. The B-meson decays play an important role in particle physics because they allow the experimental determination of the CKM parameters, the study of CP violation as well as the transition flavor of quarks that leads to the New Physics beyond the standard model. We classify the main B-decay modes as follow:

### leptonic and semi-leptonic modes

$$B^\mp \rightarrow l^\pm \nu_l, \text{BR}(B^\mp \rightarrow l^\pm \nu_l) = (10.99 \pm 0.28)\% \text{ with } l \text{ any lepton}$$

$$B^\mp \rightarrow \pi^0 e^\pm \nu_e$$

$$B^\mp \rightarrow e^\pm \nu_e, \text{BR}(B^\mp \rightarrow e^\pm \nu_e) < 9.8 \times 10^{-7}$$

$$B^\mp \rightarrow \mu^\pm \nu_\mu, \text{BR}(B^\mp \rightarrow \mu^\pm \nu_\mu) < 1.0 \times 10^{-6}$$

$$B^\mp \rightarrow \mu^\pm \nu_\mu \gamma, \text{BR}(B^\mp \rightarrow \mu^\pm \nu_\mu \gamma) < 3.4 \times 10^{-6}$$

### D-modes

$$B^\mp \rightarrow \bar{D}\pi^\pm, \text{BR}(B^\mp \rightarrow \bar{D}\pi^\pm) = (4.68 \pm 0.13) \times 10^{-3}$$

$$B^\mp \rightarrow \bar{D}\rho^\pm, \text{BR}(B^\mp \rightarrow \bar{D}\rho^\pm) = (1.34 \pm 0.18)\%$$

$$B^\mp \rightarrow \bar{D}K^\pm$$

$$B^\mp \rightarrow D^\mp \pi^\pm \pi^\mp$$

$$B^\mp \rightarrow D^*(2010)^\pm K^\pm \pi^\mp \pi^\mp \pi^\pm \pi^0, \text{BR}(B^\mp \rightarrow D^*(2010)^\pm K^\pm \pi^\mp \pi^\mp \pi^\pm \pi^0) = (1.5 \pm 0.7)\%$$

$$B^\mp \rightarrow D_s^\mp \phi, \phi(s\bar{s}) \rightarrow K^\pm \pi^\pm \pi^0$$

### Charmonium modes

$$B^\mp \rightarrow \eta_c K^\pm, \text{BR}(B^\mp \rightarrow \eta_c K^\pm) = 1.06 \pm 0.09 \times 10^3, \eta_c \rightarrow K_s^0 \pi^\pm \pi^\pm$$

$$B^\mp \rightarrow \psi \pi^\mp, \text{BR}(B^\mp \rightarrow \psi \pi^\mp) = 1.006 \pm 0.027 \times 10^{-3}, \psi(c\bar{c}) = J/\psi \rightarrow \mu^\pm, e^\pm$$

$$B^\mp \rightarrow \psi K^\mp$$

$$B^\mp \rightarrow \psi K^0 \pi^\mp$$

### K-modes

$$B^\mp \rightarrow K^0 \pi^\pm, \text{BR}(B^\mp \rightarrow K^0 \pi^\pm) = 2.37 \pm 0.08 \times 10^{-5}$$

$$B^\mp \rightarrow K^\pm \pi^0, \text{BR}(B^\mp \rightarrow K^\pm \pi^0) = (1.29 \pm 0.05) \times 10^{-5}$$

$$B^\mp \rightarrow K^0 \pi^\pm$$

$$B^\mp \rightarrow \eta K^\pm$$

$$B^\mp \rightarrow K^0 \rho^0$$

$$B^\mp \rightarrow K^\pm \phi$$

$$B^\mp \rightarrow K^0 \pi^\pm \pi^0 \gamma$$

### Baryon modes

$$B^\mp \rightarrow p\bar{p}$$

$$B^\mp \rightarrow p\bar{p}\pi^\pm, \text{BR}(B^\mp \rightarrow p\bar{p}\pi^\pm) = (2.87 \pm 0.19) \times 10^{-6}$$

$$B^\mp \rightarrow p\bar{p}\pi^\pm$$

$$B^\mp \rightarrow p\bar{p}K^0$$

$$B^\mp \rightarrow p\bar{\Lambda}\pi^-$$

$$B^\mp \rightarrow p\bar{\Lambda}\bar{\Sigma}\pi^-$$

[source: [Http://pdg.lbl.gov](http://pdg.lbl.gov)].

## The $B^0/\bar{B}^0$ -Decays

### D-modes

$$B^0 \rightarrow l^\pm \nu_l x, \text{ BR}(B^0 \rightarrow l^\pm \nu_l x) = (10.33 \pm 0.28)\% \text{ with } x \text{ any element}$$

$$B^0 \rightarrow D^\pm \pi^\mp, \text{ BR}(B^0 \rightarrow D^\pm \pi^\mp) = (2.52 \pm 0.13) \times 10^{-3}$$

$$B^0 \rightarrow D^\pm \rho^\mp, \text{ BR}(B^0 \rightarrow D^\pm \rho^\mp) = (7.6 \pm 1.2) \times 10^{-3}$$

$$B^0 \rightarrow D^\pm K^\mp$$

$$B^0 \rightarrow D^\pm D_s^\mp$$

$$B^0 \rightarrow \bar{D}^0 \pi^\pm \pi^\mp$$

### K-modes

$$B^0 \rightarrow K^\pm \pi^\mp, \text{ BR}(B^0 \rightarrow K^\pm \pi^\mp) = (1.96 \pm 0.05) \times 10^{-5}$$

$$B^0 \rightarrow K^\pm K^0$$

$$B^0 \rightarrow K^0 \rho^0, \text{ BR}(B^0 \rightarrow K^0 \rho^0) = (3.4 \pm 1.1) \times 10^{-6}$$

$$B^0 \rightarrow K^*(892)^\pm \pi^\pm, \text{ BR}(B^0 \rightarrow K^*(892)^\pm \pi^\pm) = (7.5 \pm 0.4) \times 10^{-6}$$

$$B^0 \rightarrow K^\pm \pi^\mp \pi^0$$

$$B^0 \rightarrow \eta K^0$$

$$B^0 \rightarrow K^0 \bar{K}^0$$

$$B^0 \rightarrow K^0 \phi \gamma$$

$$B^0 \rightarrow K^* \gamma$$

$$B^0 \rightarrow 3K_s^0$$

### Charmonmium modes

$$B^0 \rightarrow \eta_c K^\pm, \text{ BR}(B^0 \rightarrow \eta_c K^\pm) = (1.06 \pm 0.09) \times 10^{-3}$$

$$B^0 \rightarrow \psi K^0$$

$$B^0 \rightarrow \psi K^\mp \pi^\mp$$

$$B^0 \rightarrow \psi K^\pm \pi^\mp$$

$$B^0 \rightarrow \psi \rho^0$$

$$B^0 \rightarrow \psi \omega$$

## 2.4 CP Violation in Neutral Kaon and B-meson systems

A group led by V. Fitch and J. Cronin in 1964 was the first to detect a violation of CP symmetry. They discovered this violation in the decays of ‘neutral kaon’ particles. The phenomenon of CP violation allows us to distinguish between matter and antimatter as mentioned by A. Sakharov in 1967 and also provides the necessary ingredients to explain why there is more matter than antimatter in the universe (see ref [18]). The phenomenon of CP violation is incorporated in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes the weak interactions between the quarks. The weak interactions between quarks are described by coupling constants that are functions of three real parameters and one irreducible complex phase. This last one is responsible for CP asymmetry [19].

Firstly, the CP violating effects were observed in K-meson systems where the difference in mean lifetimes of neutral short-long kaons can be thought of as the source. However, the most experimental and theoretical developments revealed its occurrence in the B-system. The  $K^0 - \bar{K}^0$  and  $B^0 - \bar{B}^0$  mixing mechanisms are the convincing evidences of CP violation. There are three types of CP violations: the CP violation in decays, in mixing, and in interference.

We remember that the discrete symmetry operators in particle physics are C, P, and T, where C stands for charge conjugation (swapping particles for their anti-particles), P for parity reflection (space reflection through the origin), and T for time reversal. The symmetry operators P and C are linear and unitary in nature, while T is anti-linear and anti-unitary.

Even though the two basic symmetries P and C which both are violated maximally in the weak interaction, there is a certain very strong reason that requires the combination of all three to be a symmetry of nature (CPT symmetry Theorem). This is such a basic requirement that it is hard to imagine any theory in particle physics which does not conform to this symmetry.

[20]

### 2.4.1 Measurements of CKM parameters

Later after the discovery of the CP violation in the K meson system within the mixing process, different strengths of coupling were identified (violation of leptonic universality

of strength coupling). The difference in coupling strengths was explained by Cabbibo hypothesis. In the Cabbibo mechanism, the weak interaction of quarks can be described in terms of unitary CKM matrix, where it relates the weak eigenstates to the mass eigenstates as follow:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (2.1)$$

with

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad (2.2)$$

a  $3 \times 3$  unitary matrix parametrized by 3 angles (Cabbibo angles) and one complex phase responsible for Cp violating phenomena in SM. The standard choice of such matrix is

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.3)$$

with  $s_{ij} = \sin\theta_{ij}$  and  $c_{ij} = \cos\theta_{ij}$ , ( $i, j = 1, 2, 3$ ). The magnitudes of all CKM elements were determined experimentally and found to be:[\[11\]](#),[\[20\]](#)

$$V_{CKM} = \begin{pmatrix} 0.97446 & 0.22452 & 0.00365 \\ 0.22438 & 0.97359 & 0.4214 \\ 0.00896 & 0.04133 & 0.99911 \end{pmatrix} \pm \begin{pmatrix} 0.0001 & 0.00044 & 0.00012 \\ 0.00044 & 0.00011 & 0.00076 \\ 0.00024 & 0.00974 & 0.00003 \end{pmatrix} \quad (2.4)$$

There is another frequently method of  $V_{CKM}$  parametrisation, known as Wolfentein Parametrisation process. In this parameterisation,  $V_{CKM}$  is written as

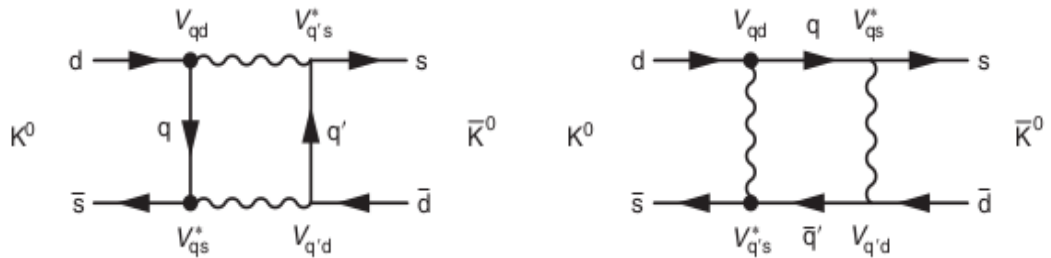
$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \quad (2.5)$$

In this case, when the value of  $\eta$  is different to zero, then the system violates CP symmetry.



## CP Violation in Neutral Kaon systems

Before the 1964 experiments of the Fitch and Cronin group all measurement has CP-symmetry including those involving the weak interaction. CP symmetry was used to explain the large difference between the lifetime of  $K_L^0$  and  $K_S^0$  particles. The 1964 experiment by Christensen, Cronin, Fitch and Turlay changes. In this section, we will discuss the mathematical proof of CP violation in the neutral kaon system. The  $K^0$  and  $\bar{K}^0$  are the eigenstates of the strong interaction and are referred to as the flavour states. Since they are the lightest hadrons containing strange quarks, only decays to final states with either leptons or pions are kinematically allowed. The weak interaction also provides a mechanism whereby the neutral kaons can mix through the  $K^0 \leftrightarrow \bar{K}^0$  box diagrams, (Fig:2.3) below:



**Figure 2.3:** K-meson box mixing diagram

The source of  $K^0 \leftrightarrow \bar{K}^0$  mixing mechanism comes from the fact that they have the common  $2\pi^0$  decay products ( $K^0 \leftrightarrow \pi^0 \leftrightarrow \bar{K}^0$ ).

In quantum mechanics, the physical states are the eigenstates of the free-particle Hamiltonian. These are the stationary states. Here because of the  $K^0 \leftrightarrow \bar{K}^0$  mixing process, a neutral kaon that is produced as a  $K^0$  will develop a  $\bar{K}^0$  component. For this reason, the  $K^0$ - $\bar{K}^0$  system has to be considered as a whole. The physical neutral kaon states are the stationary states of the combined Hamiltonian of the  $K^0$ - $\bar{K}^0$  system, including the weak interaction mixing Hamiltonian. Consequently, the neutral kaons propagate as linear combinations of the  $K^0$  and  $\bar{K}^0$ . These physical states are known as the K-long ( $K_L^0$ ) and K-short ( $K_S^0$ ) which are observed to have very similar masses, but quite different lifetimes.

The  $K_S^0$  and  $K_L^0$  mainly decay to hadronic final states of either two/three pions or to semi-leptonic final states with electrons or muons. For the hadronic decays, the  $K_S^0$  decays mostly to final states, whereas the main hadronic decays of the  $K_L^0$  are to  $\pi\pi\pi$  final states. The operation of CP on  $K_S^0$  and  $K_L^0$  give different eigenvalues,  $CP(\pi^0\pi^0) = \pm 1$  and  $CP(\pi^0\pi^0\pi^0) = -1$ . If CP were conserved, the two neutral kaons would organize themselves into CP-eigenstates; i.e., if CP was an exact symmetry of

the weak interaction, the  $K_S^0 \equiv K_1$ ,  $K_L^0 \equiv K_2$  would be cp eigenstates of the neutral kaon system, one can write

$$|K_S^0\rangle \equiv |K_1\rangle = \frac{1}{\sqrt{2}}(|K_1\rangle + |K_1\rangle) \quad (2.6)$$

$$|K_L^0\rangle \equiv |K_2\rangle = \frac{1}{\sqrt{2}}(|K_1\rangle - |K_1\rangle) \quad (2.7)$$

To understand well the physics of the neutral kaon system, it is necessary to consider the quantum mechanical time evolution of the combined  $K^0$ - $\bar{K}^0$  system . Assuming that there no mixing, the state would evolve in time as

$$|K^0(t)\rangle = |K^0\rangle e^{-i(m + \frac{i\Gamma}{2})t} \quad (2.8)$$

where  $m$  is the mass of the particle,  $\Gamma = \frac{1}{\tau}$  means that the probability density decays away exponentially. In this picture the Schrodinger's equation looks like:

$$\frac{i\partial}{\partial t}|K^0(t)\rangle = (m - \frac{i\Gamma}{2})|K^0\rangle e^{-i(m + \frac{i\Gamma}{2})t} \quad (2.9)$$

( $\hbar = c = 1$ , natural unities). Equivalently we have

$$\hat{H}|K^0(t)\rangle = (m - \frac{i\Gamma}{2})|K^0(t)\rangle \quad (2.10)$$

The mass  $m$  in the effective Hamiltonian of equation (2.10) includes contributions from the masses of the constituent quarks and from the potential energy of the system. Taking into account for the total Hamiltonian of the standard model, the mass  $m$  can be evaluated up 2nd order in perturbation theory as

$$M = m_d + m_{\bar{s}} + \langle K^0 | \hat{H}_{QCD} + \hat{H}_{EM} + \hat{H}_W | K^0 \rangle + \sum_j \frac{\langle K^0 | \hat{H}_W | j \rangle \langle j | \hat{H}_W | K^0 \rangle}{E_j - m_k} \quad (2.11)$$

where  $m_d + m_{\bar{s}} + \langle K^0 | \hat{H}_{QCD} + \hat{H}_{EM} + \hat{H}_W | K^0 \rangle \equiv M$  . The last term in the expression (2.11) comes from the small second-order contribution to the weak interaction potential from mixing. The decay rate ( $\Gamma$ ) is given by the Fermi-Golden rule

$$\Gamma = 2\pi \sum_f \langle f | \hat{H}_W | K^0 \rangle^2 \rho_f \quad (2.12)$$

with  $|f\rangle$  ,  $\rho_f$  are final states and density of states for that decay respectively.

But in generally, the neutral kaon state must include both  $K^0$  and  $\bar{K}^0$ . Therefore the general state can be written as

$$\psi(t) > = a(t)|K^0 > + b(t)|\bar{K}^0 > \quad (2.13)$$

where a and b are eigenstates satisfying the normalization condition  $|a|^2 + |b|^2 = 1$ . Now the Schrodinger's equation becomes

$$i\frac{\partial}{\partial t}\psi(t) = H\psi(t) = (M - \frac{i\Gamma}{2})\psi(t) \quad (2.14)$$

With the effective Hamiltonian given by :

$$H = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

The above effective Hamiltonian is not Hermitian because of the presence of a complex number (i) and it separately composed by two matrices, M (mass matrix) and  $\Gamma$ , decay matrix that accounts for the decay of states  $\psi(t)$  respectively

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = M + \left(\frac{-i}{2}\right)\Gamma = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

Assume the off-diagonal terms of the Hamiltonian H account for the interference between the decays of the  $|K^0 >$  and  $|\bar{K}^0 >$  components of K(t) differ to zero and the diagonal elements,  $H_{11}$  and  $H_{22}$  are real numbers. Similarly,  $M_{11}$  and  $H_{22}$  are real numbers. The CPT invariance requires that  $H_{11} = H_{11}$ , implying  $M_{11} = M_{11}$  and  $\Gamma_{12} = \Gamma_{21}$ . Noticing that both M and  $\Gamma$  being hermitian with nonzero off diagonal elements,  $\Gamma_{12} = \Gamma_{21}^*$  and  $M_{12} = M_{21}^*$ . Solving the eigenvalue equation,

$$H - \lambda I = 0 \quad (2.15)$$

and

$$H \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \lambda \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad (2.16)$$

One can find the eigenvalues and eigenstates of Hamiltonian, say  $\lambda$  and  $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$  respec-

tively. The solution of equation 2.15 for (eigenvalues) is:

$$\lambda_{\mp} = \pm \sqrt{H_{12}H_{21}} \quad (2.17)$$

Substituting equation 2.17 into 2.15, we find the eigenvectors take the form

$$\psi_{\mp} \propto \begin{pmatrix} 1 \\ \sqrt{\frac{H_{12}}{H_{21}}} \end{pmatrix} \quad (2.18)$$

. we impose a condition such that :

$$\frac{\beta}{\alpha} = \pm \xi \equiv \pm \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \quad (2.19)$$

with  $\sqrt{\frac{H_{12}}{H_{21}}} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$ .

After diagonalising the new Hamiltonian, the time dependent states look like

$$\psi(t) = \frac{1}{\sqrt{1 + |\xi|^2}} (K^0 \pm \xi \bar{K}^0) e^{-i\lambda_{\pm} t} \quad (2.20)$$

The experiments showed that  $\xi \neq 1$  which proves the CP violation in neutral K-mesons.

## CP violation in B-meson

In the previous section, we discussed the CP violation in K-meson system. Likewise, the CP violating effects have been revealed in the B-meson system. B mesons are particles component of heavy (b) quark, and a lighter (u,d, or s) antiquark. The heavy particles are produced solely in high-energy reactions and decay with a lifetime of about ( $10^{-12}$ ) seconds. The neutral particles of this family, are  $B_d^0$  and  $B_s^0$  particles, the subscript indicate the antiquark flavour and undergo mixing with their corresponding antiparticles in a way similar to that used in the neutral kaon. This mixing, coupled with possible CP violation occurring in the decay amplitude, leads to a possibility of CP asymmetry in B mesons decay. However, since they are heavier, there are many channels for B-meson decay compared to kaons. So their experimental exploration is both richer and more difficult. Three CP violation effects haven seen: decay, mixing and interference-effect.

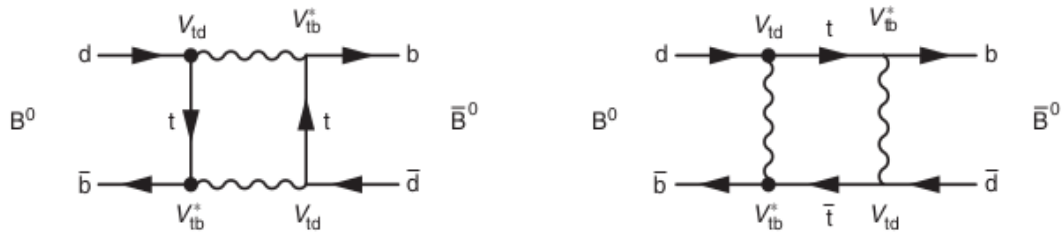
**CP violation in decay or Direct cp violation.** This kind of CP violation is said

to occurs if the decay rate of ot the B to a final state f is different from the decay rate of an anti-B to the CP-conjugated final state  $\bar{f}$  :  $\Gamma(B^0 \rightarrow K^+\pi^-)$  differs from  $\Gamma(\bar{B}^0 \rightarrow K^-\pi^+)$ ,implying that the ratio of amplitude is not unit.

**CP violation in mixing (or Indirect CP violation).** In this case the oscillation from anti-meson to meson is different from the oscillation from meson to anti-meson ,ie., Probability( $B^0 \rightarrow \bar{B}^0$ ) differs from Probability ( $\bar{B}^0 \rightarrow B^0$ );

**CP violation in interference.** This kind of CP violation is characterised by decays to a final state which is common for both the  $B^0$  and  $\bar{B}^0$  -meson.

The B-box diagram is shown (in Fig : 2.4) below:



**Figure 2.4:** B-meson box mixing diagram

The most focus at present is in the area of B-meson decay. The expectation of large CP asymmetry in the B-meson system pushed the will of B-factories construction (B-factory means an accelerator, which produces a lot of B-meson). In this regard, two B-factories, KEKB at KEK in Japan and PEP-II at SLAC in the US, were built [21]. We have already established many examples of B-meson decay modes (see section 2.3.12).

The mixing phenomenon in the neutral B-meson and its conjugate,  $B^0 - \bar{B}^0$ , decaying as: ( $B^0 \leftrightarrow \bar{B}^0 \rightarrow \psi K_S^0$ ) causes the interference between two different phase amplitudes. This provides a single CP violating angle,  $\beta$  rather than  $\delta$  in K-system (in CKM). The value of  $\sin 2\beta$  has been very accurately determined by the BaBar and Belle experiments [20], [18]. The experimental measurements showed that the measured value of  $\sin 2\beta = 0.679 \pm 0.20$ , that is  $\beta = 21^\circ$ . This is indeed a large CP violating effect evidence of the neutral B mesons. Having measurements of nonzero values of  $|\xi|$  and  $\beta$  separately implies that  $\eta \neq 0$  and hence all CKM elements are determined. Therefore the Wolfenstein parameters are determined, comparing (2.4) and (2.5), to be: [22],[11].

$$\lambda = 0.225 \pm 0.007$$

$$\rho = 0.13 \pm 0.02$$

$$A = 0.811 \pm 0.012$$

$$\eta = 0.345 \pm 0.0014$$

The experimental measurements in this section provide a strong test of the SM prediction that CKM unitary constraint is almost closed. Any deviation from this prediction would be describing physics beyond the SM. In addition the dominance of matter of the universe seems to require a new source of CP violation because it appears that CP violation of the six-quark model is too small to explain matter dominance.

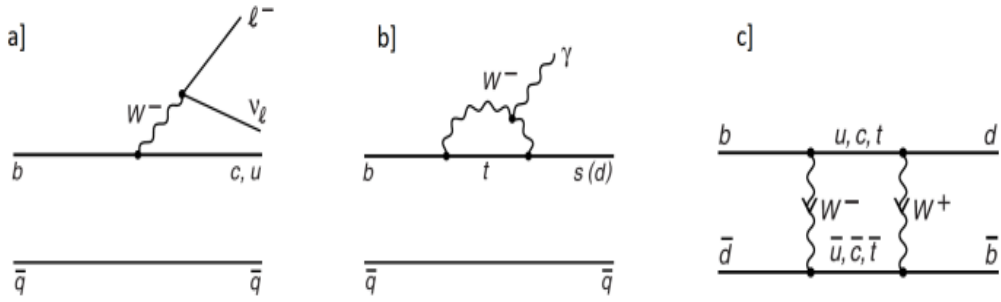
# Chapter 3

## B-MESON ANOMALIES AND THEIR EXPLANATIONS

### 3.1 Review of B-meson

In the previous chapter we discussed the B-mesons but before we go forward let us have again a short review of beauty mesons. In the ordinary pattern of three fermions and quark families, beauty (B) and top quarks belong to the third generation. Because of the top quark's mass is large, B mesons are supposed to be the only weakly decaying mesons containing quarks of the third generation [23], apart from the excited states like  $B_2^*(5747)^+$ ,  $B_2^*(5747)^0$ ,  $B_J^*(5970)^+$ ,  $B_J^*(5970)^0$  which can decay strongly, see PDG. The hadronic B meson decays occur primarily through the transition between the 3<sup>rd</sup> heavier quark and the lighter quark,  $b \rightarrow c$  (Cabibbo favoured) or  $b \rightarrow u$  (Cabibbo suppressed). This transition is governed by the CKM mechanism which describes the couplings between the two quarks. All B-meson decays that do not involve the usual transition (said above) are known as rare B decays ( $b \rightarrow s$  and  $b \rightarrow d$ ). Rare decays involve Flavour Changing Neutral Currents (FCNC) through quantum loops, allowing ( $b \rightarrow s$  and  $b \rightarrow d$ ) transitions. Rare  $b \rightarrow s, d$  quark transitions are FCNC occurring in the SM through loops via penguin or box diagrams, See the Fig:3.1. Because new particles may appear in the loops, they are excellent probes for physics beyond the SM to study such them. Examples of these decays are leptonic, semileptonic and radiative  $b \rightarrow s$  transitions, with branching ratios ranging between  $10^{-9}$  to  $10^{-5}$  see ref [2].

The search for new physics is due to the deviations from the SM predictions that were identified experimentally by different collaborative research groups. Therefore,



**Figure 3.1:** Semileptonic(a), Radiative(b) and box diagram of rare B decays(c) taken from [2]

B-meson decay provides a good opportunity to pursue such a program.

## 3.2 B-meson Anomalies

The experiments have shown some deviations in B-meson from the SM expectations. In the present section, we focus on the B-decay anomalies which deviate from the standard model predictions.

**What is anomaly** (in physical context)? It is a statistically significant experimental divergence from theoretical prediction. In the SM, deviations are presented in various areas, such as in muon (in  $(g-2)$  muon magnetic moment), in beauty quarks, neutrino oscillations. Few years ago, these anomalies have evaluated on a new level of importance as possible routes to the **new physics**, a keyword term for phenomena unexplained within SM. Anomalies may be questioned and regarded with doubt, but they often open the playing grounds for theorists. One of the most promising aspect for model builders has been anomalies in physics—interactions involving B mesons. A collection of results from LHCb at CERN, Belle in Japan, and Babar in the US point to potential problems with the standard model predictions for some rare B-meson decays. Anomalies are characterized by their sigma value ( $\sigma$ ).

The observed anomalies in semileptonic B-meson decays represent the most significant deviation from the SM observed to date in particle physics. The largest set of deviations from the Standard Model (SM) observed in the last few years in particle physics experiments are the numerous hints of Lepton Flavour Universality (LFU) violations observed in semi-leptonic B decays, see ref[24].

The two classes of deviations as identified in the ref.[24] are the charged current  $b \rightarrow c\bar{\tau}\bar{\nu}$  transition via the  $R(D)$  and  $R(D^*)$  observables with  $4\sigma$  and in Flavour-Changing Neutral Current (FCNC) transitions  $b \rightarrow sl^+l^-$ . These deviations from the



SM have initiated a series of theoretical speculations about the possible new physics (NP) interpretations. Here the attempt to explain these anomalies is to assume that the NP mainly couples to third-generation fermions of quarks and leptons, with a smaller mixing with light generation. In this case, the proposition is to introduce a  $SU(2)_q \otimes SU(2)_l$  flavour symmetry, where the scale of the mediators responsible for the observed deviations should be close to the 1 TeV scale [24].

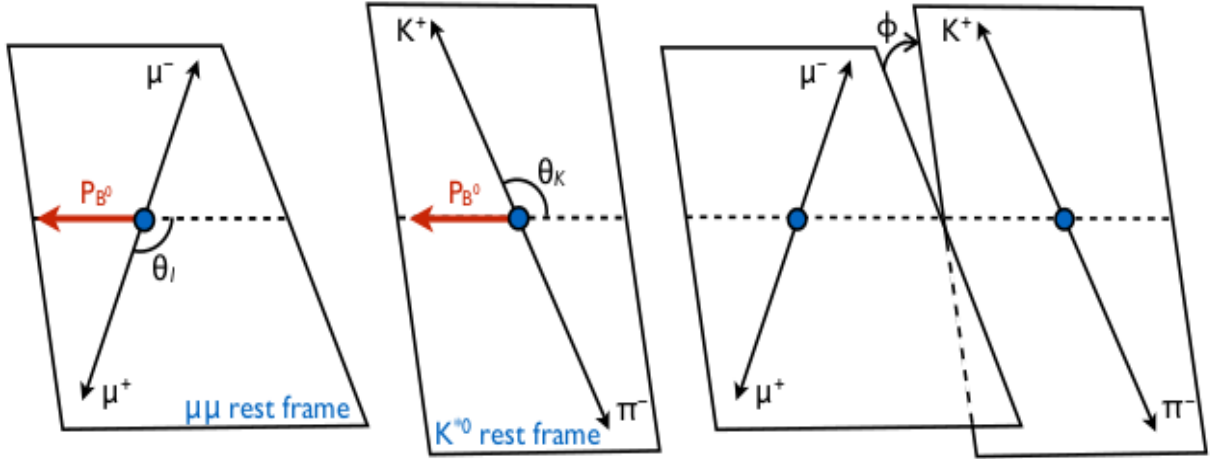
The exact searches for Beyond Standard Model (BSM) particles have been constraining their mass scale (centre of mass energy) to the extent where it is becoming acceptable that such particles are likely to have energy above the one reached by the LHC. Thus, the studies of indirect investigations of BSM physics, with all possible diversity, have been accelerating both in accuracy and in setting up observables where the theoretical errors are remarkable reduced. The observed flavour anomalies in b hadron decays shows an important portion of the program of indirect detection of BSM physics see ref [25].

Recently, numerous deviations from Standard Model (SM) expectations have been building up in B-decay experimental measurements. Now these deviations can be grouped into four categories: [26]

1. **Apparent Suppression of various branching fraction measurements of exclusive decays** ( $B^0 \rightarrow K^{*0} + \mu^+ + \mu^-$ ,  $B^+ \rightarrow K^{*+} + \mu^+ + \mu^-$ ,  $B_s \rightarrow \phi + \mu^+ + \mu^-$ ) or based on Flavour-Changing Neutral Current (FCNC) transition that occurs in the decay:  $b \rightarrow s\mu\mu$ ,
2. **The deviations from standard model expectations in the decays: ( $B^+ \rightarrow K^{*+} + \mu^+ + \mu^-$ ,  $B_s \rightarrow \phi\mu^\pm$ )- angular observables. Also based on the decay:  $b \rightarrow s\mu\mu$  transition)**, the same observation can be identified. The angular distribution and differential branching fraction of the decay:  $B^0 \rightarrow K^{*0}\mu^+\mu^-$ , where  $K^{*0} \rightarrow K^+\pi^-$ , with mass 892 MeV and lifetime  $1.3 \times 10^{-23}s$ , are studied using data sample collected by the LHCb experiment in pp collisions at  $\sqrt{s} = 7$  TeV, corresponding to an integrated luminosity of  $1.0fb^{-1}$ . Several angular observables are measured in bins of the dimuon invariant mass squared,  $q^2$ . The full differential decay distribution for the decay  $B \rightarrow K^*(892)\mu^+\mu^- \rightarrow K\pi\mu^+\mu^-$  is described by four independent kinematic variables:
  - the dimuon invariant mass squared  $q^2 \equiv M_{\mu\mu}^2$  ( $GeV^2$ ).
  - the angle  $\theta_l$  between the  $\mu^+\mu^-$  direction and the direction opposite to the B meson in the dimuon rest frame.

- the angle  $\theta_K$  between the kaon direction and the direction opposite to the B meson in the  $K^*$  rest frame.
- the angle  $\phi$  between the two planes formed by the dimuon and the  $K - \pi$  systems.

The angle  $\phi$  is zero if the two planes are parallel. (See Fig:3.2)



**Figure 3.2:** The plot showing the angular observables

3. **Apparent deviations from the test of Lepton Flavour Universality (LFU) in the decay:  $b \rightarrow sll$ .** This implies the transitions represented in the processes  $B \rightarrow Kll$  and  $B \rightarrow K^*ll$  (through the  $\mu/e$  ratios  $R_K$  and  $R_K^*$  respectively), and
4. **Apparent deviations from the tau-muon leptons ( $\tau - \mu$ ) and the tau-electron leptons ( $\tau - e$ ) universality in  $b \rightarrow cl\nu$  transitions.**

In the SM, the leptonic parts of the three families of fermions have almost the same behaviors, except for the different masses of their constituent particles. In particular, the photon, the W and the Z bosons couple in the same manner to the three lepton generations. This very beautiful aspect of the SM, known as Lepton Universality (LU), was examined and tested in order to verify its validity. Even though in the electroweak sector, the couplings of the W and Z bosons to all lepton species are identical and a large number of experiments has tested this property, there are still some deviations of leptonic universality [27]. These observations came out and accepted after the measurements of LHCb collaboration. For example the

new measurement of  $R_K$ , when combining Run-1 and Run-2 data, showing that the value found was different from what SM predicted. That is:  $R_K = Br(\frac{B \rightarrow K \mu \mu}{B \rightarrow K e e}) = 0.846 \pm 0.060$  (stat)  $\pm 0.014$  (sys) with stat and sys stand for statistical and systematic uncertainties respectively. The experiment was carried out with the dileptonic mass square,  $q^2$  between  $1.1 GeV^2$  and  $6 GeV^2$  ( $1.1 eV^2 < q^2 < 6 GeV^2$ ) [28]. This value is different from that expected by the SM. The lepton flavour universality predicted by SM is unit with very small errors compared to the experimental ( $R_K^{SM} = 1 \pm 0.0004$ ). If counted in terms of standard deviation  $\sigma$ , the deviation is about  $2.5\sigma$  [26].

When performing high precision experiments at intensity frontier experiments both precise results can be found, properties of particles via interactions and magnetic moment can be measured with high accuracy and it can be compared with the SM predictions. If there is deviation between the SM predictions and these precised measurements, this indicates the need for NP. But if the NP involves light particles, then in the lower energy intensity frontier finds no deviations, they can be produced immediately. Like for example in NA62 experiment where  $Z'$ , light particle might be directly produced. But there is another option.

We can see a deviation from the SM predictions but we do not produce new particles because they are heavier than that produced in this lower energy experiment. But indirectly through loop effect, effects can be seen. However, in this case we can not distinguish what kind of high energy physics that leads to this deviation. We can theorise it and then go to search for high energy experiments which lead to this deviation at lower energy.

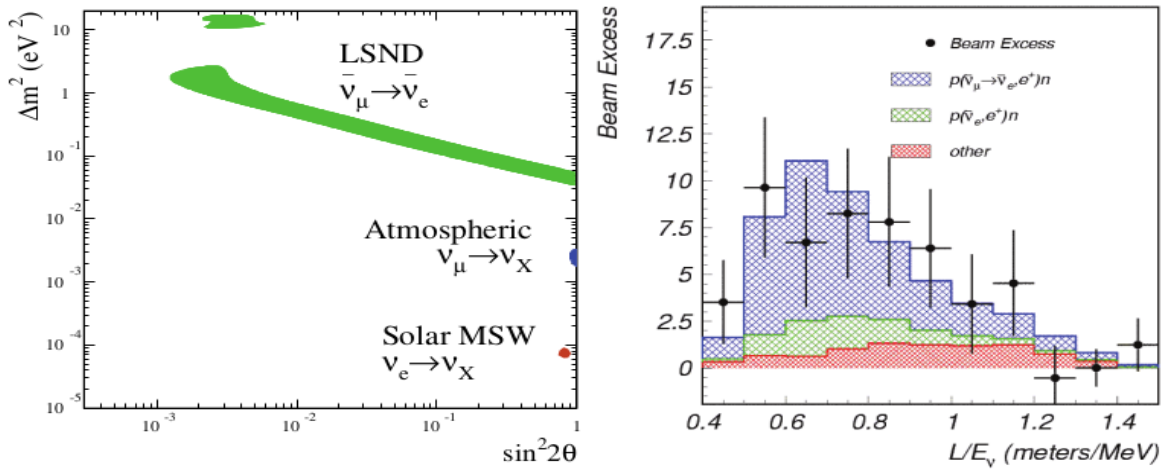
For example, within the SM the electric dipole moment of muon can be predicted. The prediction is consistent. But when measured accurately through the intensity frontier experiment, at this point, deviations occur relatively to the SM predictions. This should come from NP. Here we can have have different hypotheses, then go to search for these effects at high energy intensity frontier. Similarly, if you measure the B-meson decays modes fractions at lower at lower accuracy, you do not find any deviations. Anomalies frequently occur at high accuracy measurements.

### 3.3 Neutrino Anomalies

In the previous section, we discussed the B-anomalies, but we will not skip the neutrino anomalies. In this section, we are going to talk about some discrepancies that are appearing in neutrino physics. The neutrinos have played an important role in the constriction of SM particle physics. On the other hand, the origin of neutrino mass

is among the most crucial problems of the Standard Model (SM) of particle physics ref[8]. The SM expects three mass-less neutrinos ( $\nu_e, \nu_\mu, \nu_\tau$ ). But the experimentally observed solar and atmospheric neutrinos, showed evidence for neutrino oscillations, implying that they posses masses. We review the main concepts of sterile neutrinos, a hypothetical particle, subjected to resolve some anomalies in neutrino data. In the framework of an extension of the SM, a sterile neutrino field is a superposition of fields of massive neutrinos.

The Liquid Scintillator Neutrino Detector (LSND) counter located at Los Alamos National Laboratory, measuring the number of neutrinos that might be produced by a neutrino accelerator source, found an excess of  $\nu_e$  events ( $\nu_\mu \rightarrow \bar{\nu}_e$ ) see ref [29] that would support the neutrino oscillation. LSND observed an excess of  $\nu_e$  in their study of decays  $\mu^+ \rightarrow e^+ + \nu_e + \nu_\mu$  of positively charged muons at rest see ref [30]. This was taken as an anomaly because they expected only muon-neutrino to reach the detector. The flavour change of neutrinos can be explained by the square mass splitting and the mixing angle in the neutrino oscillation mechanism ( $\Delta m^2$  and  $\theta$ ), look at the Fig:3.3



**Figure 3.3:** Two dimensional illustration of neutrino oscillations(left) and final fit result from the LSND experiment(right) This figure was taken from the 34<sup>th</sup> SLAC Summer Institute On Particle Physics (SSI 2006), July 17-28, 2006 "The MiniBooNE Experiment"

The probability for oscillation may be simplified into the following equation: as it is found in [9].

$$P_{osc} = \sin^2(2\theta) \sin^2\left(\frac{1.27 \cdot \Delta m^2 \cdot L}{E}\right) \quad (3.1)$$

where (L, the distance from the neutrino source to the detector, and E, the energy of the neutrino beam), and two terms which are fit for when performing a two-neutrino

oscillation analysis ( $\Delta m^2$  is the difference in squares of the mass states of the neutrinos, and  $\sin^2(2\theta)$ ,  $\theta$  is the mixing angle between the two neutrino states). Note that if the value of  $(\frac{1.27 \cdot \Delta m^2 \cdot L}{E})$  as well as that of  $L$  are very small, then ( $P_{osc}$  tends to zero), and consequently the oscillation does not take place. The first experimental evidence of eV-scale sterile neutrino is from LSND experiment in 2001 by observing  $3.8\sigma$  excess of  $\nu_e$  events from a  $\nu_\mu$  appearance channel at 30 m baseline from 30 MeV  $\nu_\mu$  beam produced by muon decay at rest coming from 800 MeV proton beam on water target more detail see ref [31].

Unfortunately, LSND experimental data are contradictory since they are not consistent with the oscillation features measured by other neutrino experiments. The MiniBooNE experiment at Fermilab was designed to unambiguously verify or refute the LSND controversial result in a controlled environment, increasing  $L$  and  $E$  (in Eq 2.1). After making many experiments, MiniBooNE in 2018, came to report the results that remove the conflicts and agree with that of LSND. The MiniBooNE data are consistent in energy and magnitude with the excess of events reported by LSND, and the significance of the combined LSND and MiniBooNE excesses is  $6.0\sigma$  as was described in ref [9].

### 3.4 Explanations to Neutrino and B-anomalies

As we said before, the LHC did not directly observe any particles beyond the ones we have at present in the SM of particle physics. However, one might have a desire for knowledge about lepton flavour universality violating(LFUV). The  $R_D$  and  $R_D^*$  directly measure LFUV, and in the fit to the  $b \rightarrow s\mu^+\mu^-$  and  $b \rightarrow s\nu^+\nu^-$  data also the LHCb measurement of  $R_K$ , which deviates by  $2.6\sigma$  from the SM, points at LFUV. Therefore, it is highly encouraged to search for a simultaneous explanation of these anomalies, More details see ref[32]. In getting a more careful explanation, new particles were proposed to be added to the SM for explaining  $R_D$  and  $R_D^*$  where they cannot be very heavy and must have sizable couplings. In this case, the following new particles have been proposed: Charged Higgses,  $W'$  gauge bosons and Leptoquarks. Most of the SM predictions agree consistently with the experimental data. However, some theoretical puzzles and experimental results cannot be explained only based on the SM. Therefore, the search for new physics beyond the SM is obvious. The need for new physics beyond the SM is well encountered in the neutrino field where the neutrino oscillation requires at least two neutrinos to have nonzero masses. We know that neutrinos masses are a crucial problem in SM. As the masses and mixing within

the 3-generation neutrino matrix have been attached to solar and long-baseline neutrino experiments, more exotic models are typically used to explain these anomalies, including, for example, 3+N neutrino oscillation models involving three active neutrinos and N additional sterile neutrinos [9]. In attempting to solve such problem new hypothetical neutrinos, called "sterile neutrinos" are proposed for them to mix with the usual active neutrinos. The sterile neutrinos are hypothetical particles that are assumed to interact only via gravity and do not interact via any other of the fundamental forces of the SM (i.e., they do not have weak interactions). By choosing  $\Delta m^2$  of the order of some eV, the problem of LSND and MiniBooNE is now resolved and there are neutrino oscillations.

The other advantage of introducing the sterile neutrinos is the possibility of the existence of right-handed neutrinos. As we know, all other known fermions have been observed with both left and right chirality, and since they could explain in a usual way the small active neutrino masses deduced from neutrino oscillation. Besides, the SM does not provide any dark matter (DM) candidate which could explain the observed DM content of the Universe. Moreover, the neutrino and DM puzzles, a few other experimental results associated with the quarks and charged leptons also pose challenges to the SM. See ref[33]. If they exist, they may be responsible for several unexplained phenomena in cosmology and astrophysics, including dark matter, baryogenesis, or hypothetical dark radiation.

The anomalous magnetic moment of the muon and electron ( $(g - 2)_{\mu,e}$ ) is also one of the considerable deviations of the experimental data from the theoretical predictions of the SM. As described in ref [8] and [6], a class of models have been proposed that inter-correlates and offers a simultaneous explanation of neutrino mass, dark matter, the long-standing puzzle of the muon anomalous magnetic moment, and the recently observed tension in the electron anomalous magnetic moment

# Chapter 4

## New light gauge boson

In the previous chapter, (section 2.2), we talked about the gauge bosons ( $W^\pm$  and  $Z^0$ ) with masses about 80.4 GeV and 91.2 GeV respectively. The ( $W^\pm$  and  $Z^0$ ) are gauge bosons of the standard model particles already observed and are heavier than the mesons. The CMS and ATLAS Collaborations at LHC have searched for various BSM theories like dark matter, neutrino mass and neutrino oscillation, grand unification (supersymmetry), and connection between standard model and gravity, but they have not succeeded in finding any new particle except Higgs boson. In this section, we discuss a new light neutral gauge boson which appears in the beyond standard model (BSM) theory and how it couples with neutrinos. This gauge boson is a hypothetical (BSM) particle that can interact with neutrinos and it is much lighter than the ( $W^\pm$  and  $Z^0$ ). We denoted such new gauge boson by ( $Z'$ ) and its gauge coupling by  $g_{new}$ . Its mass is small, less than 100 MeV ( here we assume it to be less than 50 MeV). Neutrinos are not easily accessible particles even though they contribute so much in governing numerous physical phenomena in SM. Their advantages are well noticed while describing their weak interactions. Nevertheless, neutrino detection remains technically a hard task and new interactions that affect neutrinos might be undiscovered. These unseen neutrino interactions have been called upon for solving a variety of problems relating to cosmological structure formation, neutrino oscillation anomalies, and dark matter. As shown in ref [5], the big motivation in searching for this new vector gauge boson comes from the fact it can be useful in solving such theoretical problems and provides advantages when searching for the corrections to some anomalies identified in SM. In this regard, many scenarios have taken place for possible explanations of various anomalies within particle physics and cosmology. Taking into account of how  $Z'$  couples to neutrinos, the problem of missing satellite that is considered a drawback for the canonical cold dark matter scenarios could be

solved. Also if the  $Z'$  couples to matter fields, its effect can be probed by scattering experiments or by its impact on neutrino oscillation in the matter. However, if the only standard particles that couple to  $Z'$  are neutrinos, it will not have any impact on neutrino oscillation in the matter or on the elastic scattering of neutrinos off nuclei. It is reasonable to speculate about the possibility of interaction of neutrinos with new light particles. This can be done by introducing a new gauge interaction which involves neutrinos without coupling to charged leptons. The same thing happens to electromagnetic interactions that involve charged leptons but not the neutrinos. In this case, a new gauge symmetry  $U(1)'$  can be considered rather than  $U(1)_{EM}$ . This is shown in the decays of charged mesons ( $\pi^+, K^+$ ) as we are going to see in the next section. see ref [4].

## 4.1 Coupling of light gauge boson to neutrinos

In this section we investigate how much the leptonic decay of charged mesons is sensible to the interaction of neutrinos with a new light neutral gauge boson.

From the neutrino physics framework and various of kaon and pion decay experiments, it is possible to think about the possibility of interaction of neutrinos with new light particles.

The decays of charged mesons revealed the secret of neutrinos to interacting with light new particles. The decay,  $K \rightarrow \mu +$  missing energy, has been used to constrain the gauge coupling of muon to a light gauge boson, which is a motivation as a solution to the (g-2) muon anomaly.

The missing energy is since  $Z'$  can decay only into neutrinos and hence appears as missing energy in experiments. The tree-level decay of the charged meson is generalized as

$$M \rightarrow l\nu Z' \tag{4.1}$$

where  $l = e^+, \mu^+$  and  $M$  charged meson ( $\pi^+, K^+$ ). The decay of  $Z'$  to the pair of neutrino and anti-neutrino is given below

$$Z' \rightarrow \nu\bar{\nu} \tag{4.2}$$

The  $Z'$  has longitudinal component and because of this, the rate of three-body decay of  $M$ , in equation (4.1), receives a factor of  $(\frac{m_M^2}{m_{Z'}^2})$  while that of two-body decay ( $M \rightarrow l\nu$ ) is suppressed by a factor of  $(\frac{m_l^2}{m_M^2})$ . Therefore, if the new gauge coupling is



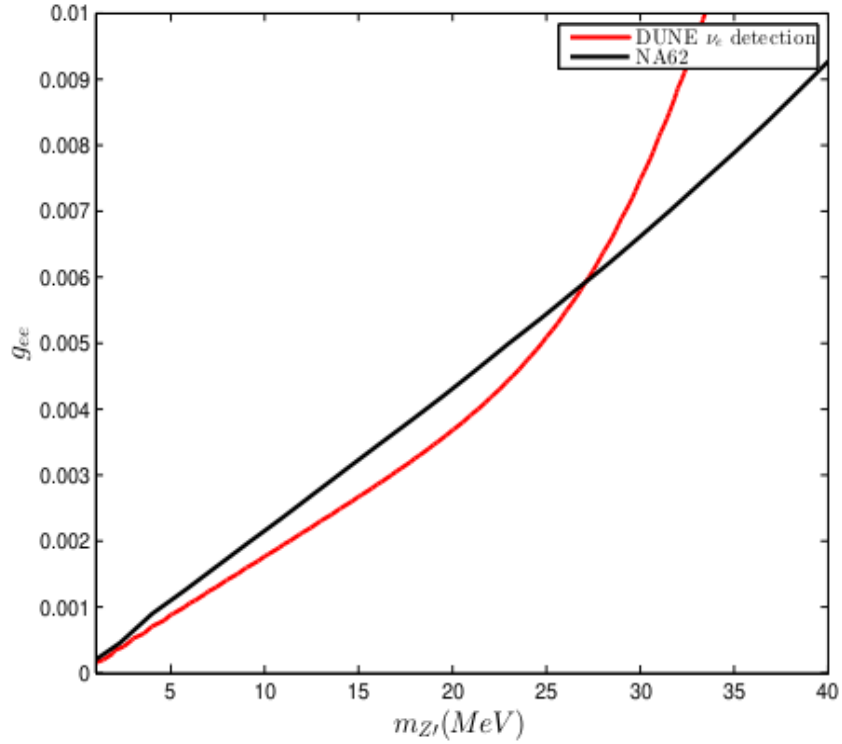
moderate (not too small), one expects a significant contribution to ( $\pi^+$  or  $K^+ \rightarrow e^+$  + missing energy). Otherwise, the huge enhancement of order  $O(\frac{m_K^4}{m_e^2 m_Z^2}) \sim 10^8 - 10^{10}$  invokes about an opportunity of probing very small gauge couplings of  $\nu_e$ . The interaction amplitude has the form:  $g_{new} k_\alpha k_\beta^* \nu_\alpha \gamma^\mu \bar{\nu}_\beta Z'_\mu$  with  $g_{new}$ , new coupling constant and  $k_\alpha$ , mixing coupling.

In this regard, many experiments have been performed concerning the determination of the decay fractions,  $R_\pi$  and  $R_K$ , of charged mesons ( $\pi^+, K^+$ ) to leptons. Here we discuss: (i) the measurement of  $K^+ \rightarrow e + \nu\nu\nu$  by TRIUMF, (ii) the measurement of  $R_\pi \equiv \frac{\Gamma(\pi^+ \rightarrow e^+ \nu)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu)}$  by PIENU experiment, and (iii) the measurement of  $R_K \equiv \frac{\Gamma(K^+ \rightarrow e^+ \nu)}{\Gamma(K^+ \rightarrow \mu^+ \nu)}$  by KLOE and NA62. From all these measurements, NA62 gives the strongest bound (accurate value) on the new gauge interaction of  $\nu_e$  and  $\nu_\mu$ . Comparing the experimental value found by NA62: [4]  $R_K^{exp} = (2.488 \pm 0.001) \times 10^{-5}$  and the SM predicted value:  $R_K^{SM} = (2.477 \pm 0.001) \times 10^{-5}$ , we realise a small difference (about  $1\sigma$  excess relative to the SM prediction). Similarly for pion,  $R_\pi^{SM} = (1.2352 \pm 0.0002) \times 10^{-4}$  and  $R_\pi^{exp} = (1.2344 \pm 0.0023) \times 10^{-4}$ . Considering the results from various experiments, the average value taken by Physics Data Group (PDG) is  $R_\pi^{PDG} = (1.2327 \pm 0.0023) \times 10^{-4}$ , which agrees with the strong bound as measured by NA62 experiment.

## 4.2 Detection of light gauge boson at DUNE

It was shown that the neutrinos and  $Z'$  can be inserted within electroweak symmetric models. The possibility of neutrinos and  $Z'$  interaction was studied at DUNE ND (Deep Underground Neutrino Experiment Near Detector), based at Sanford Underground Research Facility (SURF). It is a long-baseline set up with two detectors: Far detector (FD) and Near detector (ND). The particles recognition and energy-momentum measurement receive much flux of neutrinos from the source and makes the ND a formal site to search for new light particles to couple with leptons. The new gauge boson  $Z'$  is produced via  $\pi^+(K^+) \rightarrow l\nu Z'$  before reaching ND. The produced neutrinos are detected in ND and provide us information on intermediate  $Z'$ . Taking into account the different values of the coupling,  $g_{e\alpha} = \sqrt{\sum_\alpha g_{e\alpha}^2}$  and different values  $m_{Z'}$ , both at DUNE ND and at NA62, The graph see the Fig:4.1, of  $g_{ee}$  versus  $m_{Z'}$  at 90% C.L at high bound, can be produced.

We remember that the contribution from  $\pi$  to the value of  $m_{Z'}$  is important at energy less than 25 MeV. And that of K when the energy is greater than 30 MeV. Here the NA62 experiment indicates stronger bound. The other advantage of the DUNE ND is its sensitivity to the detection of tau neutrinos ( $\nu_\tau$ ). In this case for



**Figure 4.1:** the plot  $g_{ee}$  versus  $m_{Z'}$  at 90% C.L.(Confidential Level) [3]

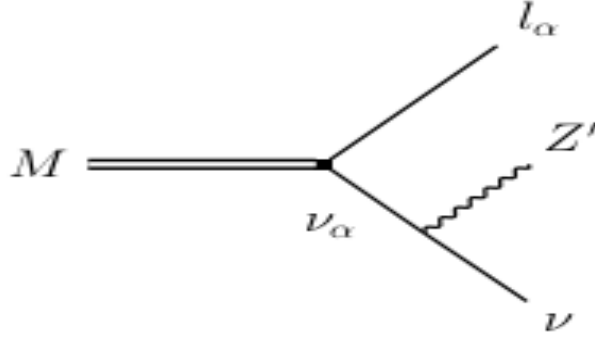
$g_{e\tau} \neq 0$ ,  $\nu_\tau, \bar{\nu}_\tau$  are produced via  $\pi^+ \rightarrow e^+ \nu_\tau Z'$  and  $Z' \rightarrow \nu_\tau \bar{\nu}_\tau$  [3].

### 4.2.1 Light gauge boson decay rate

The amplitude of interaction between neutrinos and the new gauge boson is of the form :  $\mathcal{M} = g_{\alpha i} \bar{\nu}_i \gamma^\mu \nu_\alpha Z'$ , where  $\nu_i$  is any neutrino mass eigenstate much lighter than ( $\pi$ )'s mass (approximately 100 MeV),  $g_{\alpha i}$  new coupling between  $\nu_i$  and  $Z'$  and generally given by  $g_{\alpha i} = \sum k_\alpha k_\beta^* U_{\beta i}^*$ ,  $U_{\beta i}^*$  is CKM mixing matrix element. In this case, the decay rate is :

$$\Gamma(M \rightarrow l_\alpha \nu Z') = \frac{1}{64\pi^3 m_M} \int_{E_l^{min}}^{E_l^{max}} \int_{E_\nu^{min}}^{E_\nu^{max}} dE_l dE_\nu \sum_{spins} |\mathcal{M}|^2. \text{ Neglecting the leptonic and neutrino masses } (m_{l_\alpha}, m_\nu \approx 0), \text{ the square of the amplitude can be calculated in a given integration boundaries : } E_l^{min} = m_l, E_l^{max} = \frac{m_M^2 - m_Z^2}{2m_M}, E_\nu^{min} = \frac{m_M^2 - m_Z^2 - 2m_M E_l}{2m_M}, \text{ and } E_\nu^{max} = \frac{m_M^2 - m_Z^2 - 2m_M E_l}{2m_M - 2E_l}.$$

As result, the decay rate of  $Z' \rightarrow \nu_\alpha \bar{\nu}_\beta$  is :  $\Gamma(Z' \rightarrow \nu_\alpha \bar{\nu}_\beta) = \frac{g_{\alpha\beta}^2 m_{Z'}}{24\pi}$ . The Feynman diagram corresponding to the decay  $M \rightarrow l_\alpha \nu Z'$  is shown in Fig:4.2 The decays of usual gauge bosons ( $W^\pm$  and  $Z^0$ ) can also contribute to the emission of the light vector gauge boson  $Z'$ . A light vector boson  $Z'$  that couples to neutrinos may be

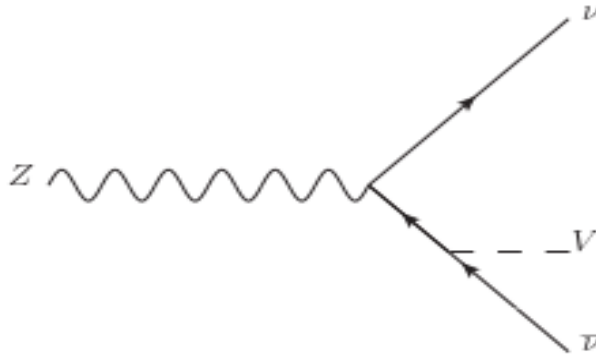


**Figure 4.2:** Feynman diagram of charged meson to lepton, neutrino, and light gauge boson [4]

constrained by the invisible decay width of the boson  $Z^0$ . The so-call invisible decay of  $Z^0$  given as

$$Z^0 \rightarrow \nu\bar{\nu} \quad (4.3)$$

The decay fraction ratio of this decaying process is approximately 20% with the mass of  $Z'$  much smaller than that of  $Z^0$ . By replacing  $Z'$  by  $V$ , similar Feynman diagrams as Fig: 4.2 might be reproduced ensuring that the vector boson's mass is smaller than that of  $Z^0$ . The decay width of  $Z^0$ , as measured in the rest frame, is  $(2.4949 \pm 0.0071)\text{GeV}$  and is in good agreement with the theoretical value calculated  $(2.4952 \pm 0.0023)$ . Note that this constraint must apply only to the standard model neutrinos considered in question. If  $Z'$  is replaced by  $V$  (light vector boson) as it was used in [5], we have a Fig: 4.3 below A similar approach can work for  $W$ -boson.



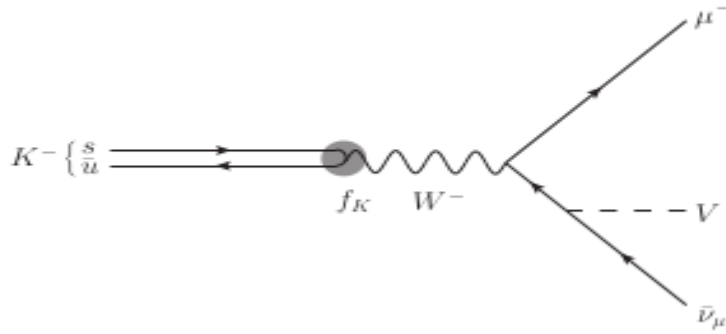
**Figure 4.3:** Feynman diagram showing  $Z$ -boson decay to neutrinos where a  $V$  is radiated from the final state antineutrino [5].

Because  $W$ -boson is charged, our restriction on the light vector boson coupling to

neutrinos can become stronger if the final state in the decay contains charged leptons as well. Here we focus on leptonic decay like

$$W^\pm \rightarrow l^\pm \bar{\nu}_l \quad (4.4)$$

, which is related to the  $Z^0$  decay and with an averaged branching ratio of around ten per cent ( $\sim 10\%$ ). The difference from what we know is that a V-boson can be radiated from the charged lepton, instead of that from neutrinos. The experimental measurements of the total decay width of the W was found to be  $2.085 \pm 0.042$  GeV, which is almost the same as the one theoretical calculated value,  $2.091 \pm 0.002$  GeV. The corresponding Feynmann diagram of the W boson is shown in the Fig:4.4 below



**Figure 4.4:** Feynman diagram showing  $K^- (\bar{u}s)$  decay to a muon where a V is radiated from the final state antineutrino. [5].

Recently, many observations of a tree- and loop-level B- decays reveal possibilities of LU violation. As consequence, a violation of LU would be an unambiguous sign that probes physics beyond the SM. The most hints of new physics, like neutrino masses and dark matter, point towards the existence of a hidden sector weakly coupled to the Standard Model particles. Instead, it is traditionally believed to be mediated by particles at a heavier mass scale. Now, it seems that the new physics is also at low energies and weakly coupled. Light neutral vector bosons can accomplish such a pattern, and we believe that our constraints on their interactions to neutrinos and charged leptons will serve as a useful guide to phenomenology.

# Chapter 5

## Discussion and Conclusion

In this work, we did a review of SM particles (mesons) where we discussed fundamental interactions (forces) and other main characteristics (mass, spin, quark content, electric charge, beauty, charm, strangeness,...). Within SM framework, we studied the main decay modes of different categories of mesons ( $\pi, \rho, \eta, K, D$ , and  $B$ ) providing their lifetimes and branching ratio.

We reviewed the CP violation that appears in the weak interaction of neutral kaon and bottom mesons where among the four CKM parameters (three real and one complex), the complex one bears the responsibility of CP asymmetry.

We discussed the B-anomalies presented in the B-meson decays ( $B^0 \rightarrow K^{*0} + \mu^+ + \mu^-$ ,  $B^+ \rightarrow K^{*+} + \mu^+ + \mu^-$ ,  $B_s \rightarrow \phi + \mu^+ + \mu^-$ ,  $B^+ \rightarrow K^{*+} + \mu^+ + \mu^-$ ,  $B_s \rightarrow \phi \mu^\pm$ ). Concerning the B-anomalies, the LHCb collaboration has confirmed evidence of odd behaviour in the way B mesons decay into a  $K^*$  and pair of muons ( $B^0 \rightarrow K^{*0} + \mu^+ + \mu^-$ ), bringing fresh intrigue to the pattern of flavour anomalies that has emerged during the past few years. At a seminar at CERN on 10 March 2019, it was presented an updated analysis of the angular distributions of  $B^0 \rightarrow K^{*0} + \mu^+ + \mu^-$  decays based on around twice as many events than were used for the collaboration's previous measurement reported in 2015. The latest LHCb result includes additional Run 2 data collected during 2016, corresponding to a total integrated luminosity of  $4.7 fb^{-1}$  showed the discrepancy that could be explained within in BSM framework. The deviations in the test of Lepton Flavour Universality (LFU) make LHCb continuing measurements of LFU. In recent years, LHCb has also found that the ratio of the rates of muonic and electronic B decays departs from the SM prediction, suggesting a violation of the key SM principle of LFU.

Also, the anomalous magnetic moment of the muon and electron as deviations of the experimental data from the theoretical predictions of the SM were discussed. We

showed the need for the extension of the SM towards the New physics beyond the standard model theory in attempting to find the explanations to these anomalies.

In this regard, new theories were searched for and new particles beyond SM were proposed. This is because even though the SM of particle physics is a very successful, and mathematically consistent theory of the elementary particles, some theoretical deviations and experimental results that cannot be explained only based on the SM are still there.

The search for new physics beyond the SM is well established in both: neutrino sector (neutrino masses), dark matter (DM), origin of the anomalous magnetic moment of the muon and electron and in the origin of CP violation of the electroweak field.

Finally, we discussed a new hypothetical light gauge boson ( $Z'$ ) generated from the charged meson ( $\pi^+$  or  $K^+$ ) which can couple to neutrinos. The great motivation of many scenarios in searching for this new vector gauge boson comes from the fact it can serve in solving some theoretical problems and moderate some anomalies within particle physics and cosmology.

## 5.1 Beyond Standard Model (BSM) theories

In this section, we briefly discuss a few BSM theories. As it can be understood, BSM theories are theories that resulted from the extensions of the SM theory to provide the explanations with the deficiencies and shortcomings presented in the SM. Such BSM theories, one can list the following.

- **Two-Higgs-doublet model (2HDM).** It is one of the simple extended SM theory. The 2HDM models are one of the natural choices for beyond-SM models containing two Higgs doublets instead of just one described in the Higgs mechanism where it was a spin zero scalar. Through Higgs mechanism particles got their masses, but not all of them (like for example, neutrinos did not). There are also models with more than two Higgs doublets, for example, three Higgs doublet models etc. Such a model can be described in terms of six physical parameters: four Higgs masses ( $m_h, m_H, m_A, m_{H^\pm}$ , two neutral and two charged Higgs), the ratio of the two vacuum expectation values,  $\tan(\beta)$  and the mixing angle ( $\alpha$ ) which diagonalizes the mass matrix of the neutral CP even Higgses while the SM uses only 2 parameters: the mass of the Higgs and its vacuum

expectation value, see ref [34].

- **Supersymmetry (SUSY)**. It is a superposition that ensure the relationship between two basic classes of elementary particles: bosons, which have an integer-valued spin, and fermions, which have a half-integer spin. This hypothesis could resolve the major hierarchy problems within gauge theory, by guaranteeing that quadratic divergences of all orders to cancel out in perturbation theory. In supersymmetry theory, each particle (boson or fermion) from one group would have an associated particle in the other, known as its **superpartner**, the spin of which differs by a half-integer. These superpartners might be thought of as new and undiscovered particles. For example, there would be a particle called a "selectron" (superpartner electron), a bosonic partner of the electron , See ref [35]. As the main purpose, the supersymmetry model, if successful, contributes to finding the solutions to the dark matter candidate and neutrino masses.
- **Grand Unified Theory (GUT) [36]**. It is a model in particle physics, where at high energies, the three gauge interactions of the Standard Model comprising the three fundamental forces ( electromagnetic, weak, and strong forces) are combined into a single force. Although this unified force has not yet been directly observed, several GUT models theorize/predict its existence. If the unification of these three interactions is possible and realizable, it raises the possibility that there was a grand unification epoch in the very early universe in which these three fundamental interactions were not yet distinct. The three Physicists, Weinberg, Glashow, and Salam, confirmed that at high energy the electromagnetic interaction and weak interaction unify into a single electroweak interaction. GUT models predict that at even higher energy, the strong interaction and the electroweak interaction will unify into a single electronuclear interaction. This interaction is characterized by one larger gauge symmetry and thus several force carriers, but one unified coupling constant. In addition, if the electronuclear interaction is unified with gravity, gives the Theory of Everything (TOE). For more details look at ref [37],[38].

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# References

- [1] M. Segond, L. Szymanowski, and S. Wallon, “Diffractive production of two  $\rho^0$  mesons in  $e^+e^-$ -collisions,” *The European Physical Journal C*, vol. 52, no. 1, pp. 93–112, 2007.
- [2] A. Oyanguren, “B decay anomalies at lhcB,” in *EPJ Web of Conferences*, vol. 175, p. 01004, EDP Sciences, 2018.
- [3] P. Bakhti, Y. Farzan, and M. Rajaei, “Secret interactions of neutrinos with light gauge boson at the dUNE near detector,” *Physical Review D*, vol. 99, no. 5, p. 055019, 2019.
- [4] P. Bakhti and Y. Farzan, “Constraining secret gauge interactions of neutrinos by meson decays,” *Physical Review D*, vol. 95, no. 9, p. 095008, 2017.
- [5] R. Laha, B. Dasgupta, and J. F. Beacom, “Constraints on new neutrino interactions via light abelian vector bosons,” *Physical Review D*, vol. 89, no. 9, p. 093025, 2014.
- [6] C. Arbeláez, R. Cepedello, R. M. Fonseca, and M. Hirsch, “ $(g-2)$  anomalies and neutrino mass,” *Physical Review D*, vol. 102, no. 7, p. 075005, 2020.
- [7] D. Buttazzo, A. Greljo, G. Isidori, and D. Marzocca, “B-physics anomalies: a guide to combined explanations,” *Journal of High Energy Physics*, vol. 2017, no. 11, p. 44, 2017.
- [8] S. Jana, W. Rodejohann, S. Saad, *et al.*, “Dark matter assisted lepton anomalous magnetic moments and neutrino masses,” *Physical Review D*, vol. 102, no. 7, p. 075003, 2020.
- [9] A. Aguilar-Arevalo, B. Brown, L. Bugel, G. Cheng, J. Conrad, R. Cooper, R. Dharmapalan, A. Diaz, Z. Djuricic, D. Finley, *et al.*, “Significant excess of

- electronlike events in the miniboone short-baseline neutrino experiment,” *Physical review letters*, vol. 121, no. 22, p. 221801, 2018.
- [10] M. H. M. Hilo *et al.*, “Using of the generalized special relativity (gsr) in estimating the neutrino masses to explain the conversion of electron neutrinos,” *Natural Science*, vol. 3, no. 4, pp. 334–338, 2011.
- [11] M. Thomson, *Modern particle physics*. Cambridge University Press, 2013.
- [12] S. Godfrey and J. Napolitano, “Light-meson spectroscopy,” *Reviews of Modern Physics*, vol. 71, no. 5, p. 1411, 1999.
- [13] L. A. Anchordoqui and T. Montaruli, “In search of extraterrestrial high-energy neutrinos,” *Annual Review of Nuclear and Particle Science*, vol. 60, pp. 129–162, 2010.
- [14] A. De Rújula, H. Georgi, and S. Glashow, “Hadron masses in a gauge theory,” *Physical Review D*, vol. 12, no. 1, p. 147, 1975.
- [15] B. Nefkens and J. Price, “The neutral decay modes of the eta-meson,” *Physica Scripta*, vol. 2002, no. T99, p. 114, 2002.
- [16] A. Kupsc, “What is interesting in eta and eta’ meson decays?,” *arXiv preprint arXiv:0709.0603*, 2007.
- [17] M. Artuso, E. Barberio, and S. Stone, “B meson decays,” *PMC Physics A*, vol. 3, no. 1, p. 3, 2009.
- [18] G. D’AMBROSIO and G. Isidori, “Cp violation in kaon decays,” *International Journal of Modern Physics A*, vol. 13, no. 01, pp. 1–93, 1998.
- [19] M. A. Baak, “Measurement of ckm-angle  $\gamma$  with charmed b0 meson decays,” tech. rep., SLAC National Accelerator Lab., Menlo Park, CA (United States), 2007.
- [20] N. Tuning, “Lectures notes on cp violation,” 2020.
- [21] M. Kobayashi, “Cp violation and flavor mixing (nobel lecture),” *ChemPhysChem*, vol. 10, no. 11, pp. 1706–1713, 2009.
- [22] M. E. Peskin, *Concepts of elementary particle physics*, vol. 26. Oxford Master Series in Physics, 2019.

- [23] T. E. Browder, K. Honscheid, and S. Playfer, “A review of hadronic and rare  $b$  decays,” in *B Decays*, pp. 158–230, World Scientific, 1994.
- [24] D. Marzocca, “Combined interpretation of  $b$ -physics anomalies and model building,” *Acta Physica Polonica B*, vol. 49, no. 6, 2018.
- [25] A. Hicheur, “Flavour anomalies in  $b$  decays at lhcb,” *arXiv preprint arXiv:1910.13121*, 2019.
- [26] J. Aebischer, W. Altmannshofer, D. Guadagnoli, M. Reboud, P. Stangl, and D. M. Straub, “ $B$ -decay discrepancies after moriond 2019,” *The European Physical Journal C*, vol. 80, no. 3, pp. 1–27, 2020.
- [27] S. Bifani, S. Descotes-Genon, A. R. Vidal, and M.-H. Schune, “Review of lepton universality tests in  $b$  decays,” *Journal of Physics G: Nuclear and Particle Physics*, vol. 46, no. 2, p. 023001, 2018.
- [28] R. Aaij, C. A. Beteta, B. Adeva, M. Adinolfi, A. Affolder, Z. Ajaltouni, S. Akar, J. Albrecht, F. Alessio, M. Alexander, *et al.*, “Angular analysis of the  $b \rightarrow k^* 0 \mu^+ \mu^-$  decay using 3 fb<sup>-1</sup> of integrated luminosity,” *Journal of High Energy Physics*, vol. 2016, no. 2, p. 104, 2016.
- [29] Y. Farzan, T. Schwetz, and A. Y. Smirnov, “Reconciling results of lsnd, mini-boone and other experiments with soft decoherence,” *Journal of High Energy Physics*, vol. 2008, no. 07, p. 067, 2008.
- [30] D. V. Naumov, “The sterile neutrino: a short introduction,” in *EPJ Web of Conferences*, vol. 207, p. 04004, EDP Sciences, 2019.
- [31] S.-H. Seo, “Review of sterile neutrino experiments,” *arXiv preprint arXiv:2001.03349*, 2020.
- [32] A. Crivellin, D. Müller, and T. Ota, “Simultaneous explanation of  $r(d)$  and  $b \rightarrow s \mu^+ \mu^-$ : the last scalar leptoquarks standing,” *Journal of High Energy Physics*, vol. 2017, no. 9, p. 40, 2017.
- [33] P. Ballett, M. Hostert, and S. Pascoli, “Dark neutrinos and a three-portal connection to the standard model,” *Physical Review D*, vol. 101, no. 11, p. 115025, 2020.

- [34] D. A. Camargo, M. D. Campos, T. B. de Melo, and F. S. Queiroz, “A two higgs doublet model for dark matter and neutrino masses,” *Physics Letters B*, vol. 795, pp. 319–326, 2019.
- [35] S. P. Martin, “A supersymmetry primer,” in *Perspectives on supersymmetry II*, pp. 1–153, World Scientific, 2010.
- [36] D. Croon, T. E. Gonzalo, L. Graf, N. Košnik, and G. White, “Gut physics in the era of the lhc,” *Frontiers in Physics*, vol. 7, p. 76, 2019.
- [37] P. Langacker, “Grand unified theories and proton decay,” *Physics Reports*, vol. 72, no. 4, pp. 185–385, 1981.
- [38] J. Baez and J. Huerta, “The algebra of grand unified theories,” *Bulletin of the American Mathematical Society*, vol. 47, no. 3, pp. 483–552, 2010.