



**BLOCK MAXIMA APPROACH TO STUDY EXTREME RAINFALL IN
KIGALI**

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KIGALI**

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DECLARATION

I declare that this Dissertation contains my own work except where specifically acknowledged

Niyotwizera Gerardine

Signed.....

Date: .../.../2022

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ABSTRACT

Extreme rainfall is one of the most devastating natural events. The frequency and intensity of these events has increased. This trend will likely continue as the effects of climate change become more pronounced. As a consequence, it is necessary to evaluate the different statistical methods that assess the occurrence of the extreme rainfalls. This research evaluates some of the most important statistical method that is used for the analysis of the extreme precipitation events. Extreme Value Theory is applied on one station (Kanombe station) data located in Kigali. Furthermore, The Methodology of BMA (Block Maxima Approach) was applied to model monthly rainfall and L-Moment is used as a method of calculation of the extreme distribution parameters. The model Quantile plot and return level plot will be used to test the goodness of the model.

The results showed that the Generalized Extreme Value distribution provides better theoretical justification to evaluate extreme precipitation.

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KEYWORDS

Block Maxima Approach, Generalized Extreme Value Distribution, L-Moment method, Kigali Rainfalls

LIST OF SYMBOLS AND ACRONYMS

BMA: Block Maxima Approach

EVT: Extreme Value Theory

GDP: Gross Domestic Product

GEV: Generalized Extreme Value

ITCZ: Inter Tropical Convergence Zone

MoE: Ministry of Environment

WHO: World Health Organization

NAO: North Atlantic Oscillation

PDO: Pacific Decadal Oscillation

AMO: Atlantic Multidecadal Oscillation

ARs: atmospheric rivers

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CHAPTER ONE

1.0 GENERAL INTRODUCTION

1.1 Background

Extreme High rainfall has been dangerous issue in Rwanda especial in Kigali city, because it leads to floods and landslide (Joseph, 2014).

Kigali city is capital of Rwanda and it is located at Rwanda's geographical heart, Kigali city has become Rwanda's most important business center also the main port of entry. Kigali city has experienced heavy rainfall events caused rapid surges in the flow of rivers and drainage systems leading to flood downstream. Although flooding has been experienced since 1960s in Kigali city, but its frequency has significantly increased since 2000s, and its impact have been great on Human development, properties, infrastructures as well as environment. In 2006, 27 percent of buildings in Kigali city were in flood-prone zones within the Nyabugogo River floodplain where vulnerable populations, infrastructures, and various economic activities were exposed to flash floods. The prevention from this devastation in Kigali city will depends on regularly information on maximum rainfall in Kigali. In this study we will investigate the change of maximum rainfall in order to forecast extreme high rainfall that can occur in Kigali city.

Extreme rainfall is one of the main causes of natural disasters, especially in flood hazards worldwide. Not surprisingly, considerable attention has been paid in recent years to the modeling of extreme rainfall (Garlcia J.A,et al.,2002). A good prediction of extreme rainfall can help for solid infrastructure like roads, ports, education, etc. These structures are designed based on extreme rainfall events that correspond to high return periods. A large amount of the variability of rainfall is related to the occurrence of extreme rainfall events and their intensities (Cooley D, 2009). Rwanda is currently highly vulnerable to climate change as it is strongly reliant on rain-fed agriculture both for rural livelihoods and exports of tea and coffee. The study indicated that rainfall is intensively varying in Rwanda where the average annual rainfall may rise by up to 20% by the 2050s from 1970 (MoE, 2011).The projections have showed that the extreme weather affect the economy, whereby 2030s the negative impacts caused by climate change will cost under 1% of Gross Domestic Product (GDP) each year J.Barret, et.al, (2009).

Extreme rainfall causing significant damage such as flood that can destroy infrastructure, interrupt economic activities and retard development in many part of the world (WHO,2003).

In particularly, Eastern Africa has experienced extreme precipitation changes specifically in Rwanda. Over the past 30 years the frequency, intensity of extreme events such as floods and droughts have increased in Rwanda. Over the past 30-60 years, droughts and heavy rainfall have been experienced more frequently (MINAGRI, 2013). Floods and landslides were increasingly reported in the high altitude western and Northern provinces. Floods occur regularly in Rwanda from heavy rains. Regions in northern Rwanda (provinces of Gisenyi, Ruhengeri, and Byumba), southwestern Rwanda (Gikongoro and Butare), western Rwanda (Kibuye, and Gikongoro), and northern part of Kigali are the most vulnerable regions to floods (MIDIMAR, 2012). In general the problem of heavy rainfall is all around, there are a lot of researchers around the world who have been studying on extreme rainfall. For example in Kenya, Specifically, extreme rainfall is not new and the study of Parry, Echeverria, Dekens, and Maitima (2012) found out that Kenya's exposure to climate risk is high, experiencing major droughts about every 10 years and moderate droughts or floods every 3 to 4 years, and as such regarded as one of the most disaster-prone countries in the world. The study further showed that, floods and droughts in Kenya accounts for a liability of about 2.0 to 2.4 % of the Gross Domestic Product (GDP) each year (Coles S, 2001). Sadly, as global climate change increase it has been shown that there will be a concomitant increase, in climate risks on Kenya's economy. In addition, Huho and Kosonei (2014) found that there is an inverse relationship between economic development and extremes in rainfall that causes disasters. This has been seen in the Long-Rains during March - May of 2018 which was regarded as one of the wettest seasons (Kilavi et al, 2018). In Mawsynram the town in the East Khasi Hills district of Meghalaya state in Northeastern India, It receives the highest rainfall in India with an average annual rainfall of 11,872 millimetres (467.4 in) Luiz Drude de Lacerda (2004) and In October 2020, Vietnam was greatly affected by several heavy rainfall events occurring sequentially during the month. The first event started on 6 October and lasted until 9 October (Jenkinson, 2020). Another studies has indicated in Kenya extreme rainfall is frequency in April Huho J.M & Kosonei R.C (2014). Early detection of Extreme High Rainfall helps to implement strategies and measures, before they occur (Kharrin V.V et al., 2000). Although it is necessary to investigate the future behavior of extreme events especially for regions such as the Kigali, where the flood and drought risks are high because of heavy population, heavy infrastructure and urbanization.

Severally studies have been conducted on the problem of heavy rainfall in different parts of the world using different methods.

The EVT comes to include in those methods. There have been many studies on EVT in the area of hydrology and climatology (Farquharson et al., 1992; Madsen and Rosbjerg, 1997). EVT is based on 2 techniques namely; Block Maxima (BM) and Peack over Threshold (POT) Aproaches (R. L. Smith, 1986). In this study, Block Maxima Aproach will be used. The detail of this method will be given in methodology.

The aim of this work is to use Mathematical techniques to model extreme high rainfall in Rwanda, specifically in Kigali city.

1.2 Statement of the Problem

Extreme high rainfall is dangerous hazard of the nature because extreme high rainfall can lead to flood and landslides, which can threaten human life, disrupt transport. Extreme high rainfall can severely affects both environment and Human lives. Kigali city has experienced extreme high rainfall that hit its several parts, some of the victims were drowned in flood water, and others died after houses collapsed under the heavy rain. This can generally affect economic growth of the country due to the disturbance in agriculture sector that depend on the change in climate. This study aims to use Block Maxima Aproach to study the extreme high rainfall in Kigali city.



1.3 Objectives of the Study

1.3.1 General Objective

The main objective of this research was to study extreme rainfall in Kigali City using Block Maxima Approach.

1.3.2 Specifics Objectives

The specific objectives of this research are:

1. Understanding frequency and Intensity of extreme rainfall in Kigali City
2. To determine the distribution function of extreme rainfall
3. To determine return period corresponding to the return level for extreme rainfall

1.4 Hypothesis of the Study

Improved understanding of the characteristics of extreme rainfall event will lead to improved skill of forecasting rainfall over Rwanda especially in Kigali. This information will enhance the efficiency of policy planning and implementation of early warning systems as well as development and management of agricultural, water resources and other rainfall-dependent sectors of the economy.

1.5 Justification of study

Kigali city has experienced extreme high rainfall, many households; infrastructures have remained vulnerable to extreme rainfall such as flood. Many researches on rain water harvesting, flood in Kigali city have been extensively covered. These have been produced controversial results. Therefore, this study has scope which is different from the previous studies because it attempts to focus in details on the extreme high rainfall model by using Block Maxima Approach (BMA). In addition to this extreme high rainfall model help to know what will happen in future and help to implement measure to prevent extreme high rainfall (flood).

1.6 Area of Study

Rwanda, warmly known as "the land of a thousand hills" is situated in East – central Africa. It lies 121 Km South (75 miles) of the equator in the Tropic of Capricorn, 1416 kilometers (880Miles) west of the Indian Ocean and 1250Km (777Miles) east of the Atlantic Ocean – literally the heart of Africa.

Kigali City is the Capital of Rwanda found in its center and commonly known for being clear, beautiful with well-maintained street as compared to most cities in Africa. Kigali is located in the centre of Rwanda, at 1°57'S 30°4'E. Kigali City has a tropical climate. In winter, there is much less rainfall than in summer.

1.6.1. Precipitation climatology of Kigali

The capital, Kigali, is located at the center of the country, at 1,500 meters (5,000 feet) above sea level, and receives 950 mm (37.5 in) of rain per year, with a maximum in April and a minimum from June to August, when it almost never rains. From December to February, the rains are still quite frequent. This city has a tropical climate. When compared with winter, the summers have much more rainfall. This location is classified as Aw by Köppen and Geiger. In Kigali, the average annual temperature is 20.1 °C | 68.2 °F. Precipitation here is about 1000 mm | 39.4 inch per year.

1.6.2 Precipitation climatology of Rwanda and Regional climatic variability of rainfall

Rwanda rainfall is influenced by Inter Tropical Convergence Zone (ITCZ), regional and local effects, Congo air mass, tropical cyclones, subtropical high-pressure systems and Lake Victoria air mass. The performance of a given rainy season does not only depend in the overall total amount, but also requires an adequate distribution of the rains through the year. This is mainly important especially in the regions where they normally receive small amount of rains fall with a limited period of time. Rainfall information is important to reduce the impacts associated extreme rainfall events.

High frequency of rain days over western part of Rwanda usually occur between the third week of March and the second week of May. The convective activity over this region may be attributed to the strong westerly wind which bring moisture from Congo air mass. The highlands of these regions also enhance convective development through orographic lifting. Abundance of moisture for cloud formation hence rainfall is also promoted by the fact that this region is located near the Nyungwe national park (Joseph, 2014).

On the other hand, most of the active rain days over central part of Rwanda occur within the month of April to the second week of March. The convective activity over this region may be attributed to easterly wind which inject moisture from Lake Victoria and Indian Ocean and the highlands of these region also enhance convective development through orographic lifting (Joseph, 2014).

The rains in Rwanda usually occur in the form of downpours or thunderstorms, but there is not much sun, in fact, the sky is often cloudy, although in the driest months, especially in June and July, the sun comes out more often.

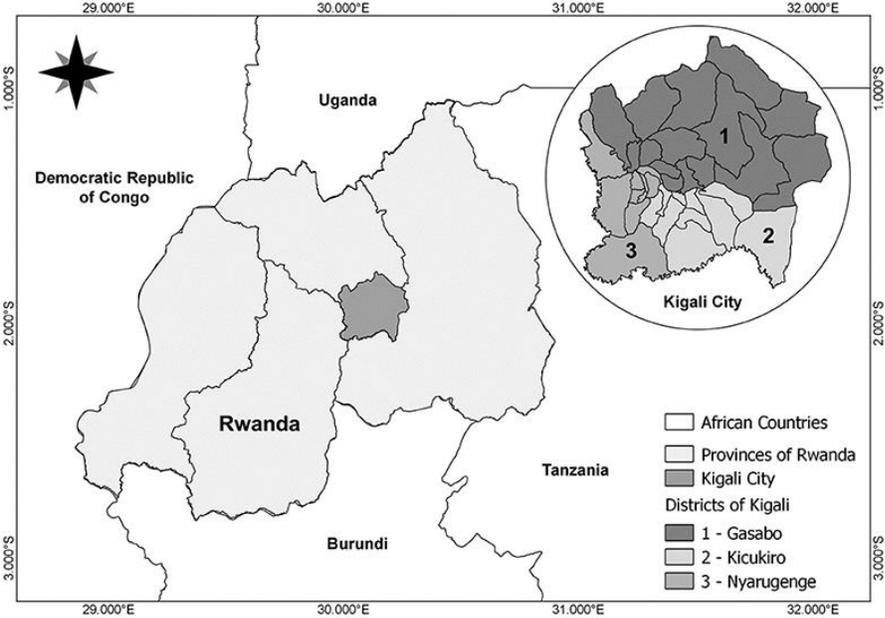


Figure 1: Kigali city

Source: <http://www.kigalicity.gov.rw/IMG/bmp/KCC-Cells.bmp>

CHAPTER TWO

2.0 Literature Review

Rainfall is very important weather and climate parameter that influence economic activities in east Africa as well as Rwanda. It is also climatic parameter over the tropical region with the largest space and time variability. The region of east Africa experiences two main rainfall seasons; March-May (MAM) and October-December (OND) (Omeny and Okoola, 2008). The systems that influence rainfall over East Africa include Intertropical Convergence Zone (ITCZ), monsoon wind, subtropical high-pressure systems, and easterly/westerly waves, tropical cyclones, El-Niño Southern Oscillation (ENSO), jet stream, Quasi Biennial Oscillation (QBO), El Niño Southern Oscillation Index (ENSO), Congo air mass and Indian Ocean Dipole (IOD). These are sometimes linked with excess or deficits of rainfall resulting to floods or droughts (Camberlin and Okoola, 2003).

The study have been done on extreme rainfall in different areas of the world. Particularly in East Africa has suffered both excessive and deficient rainfall in recent years (Webster et al., 1999, Hastenrath et al., 2007). In particular, the frequency of anomalously strong rainfall causing floods has increased. Shongwe, van Oldenborgh and Aalst (2009) report that their analysis of data from the International Disaster Database (EM-DAT) shows that there has been an increase in the number of reported hydrometeorological disasters in the region, from an average of less than 3 events per year in the 1980s to over 7 events per year in the 1990s and 10 events per year from 2000 to 2006, with a particular increase in floods. In the period 2000-2006 these disasters affected on average almost two million people per year.

Flooding caused by extreme rainfall events can have severe social, environmental and economic consequences. Although variations in yearly flood trends have been studied extensively (Robson et al. 1998; Cox et al 2002; Prosdocimi et al 2019), such trends do not help in identifying the processes which lead to the extreme cases. For example, in February 2020 storms Ciara and Dennis passed over the UK, resulting in up to 177 mm of rainfall in a single 24-h period (Met Office 2020), and are estimated to have resulted in insured losses of up to £200 million (Finch 2020). To provide improved risk analysis and support flood management, the processes which cause these extreme rainfall events need to be better understood and differentiated from those of regular rainfall events.

The occurrence of extreme rainfall events in the UK has a strong dependence on the concurrent and prior meteorological conditions across North Western Europe and the North Atlantic. Brown (2018) shows the dependence of extreme daily rainfall in the UK on large-scale meteorological indices: North Atlantic Oscillation (NAO), Pacific Decadal Oscillation (PDO), El Niño – Southern Oscillation (ENSO) and Atlantic Multidecadal Oscillation (AMO). These indices represent the difference in either sea-level pressure (NAO, PDO) or sea-surface temperature (ENSO, AMO) across their specified regions. Brown found the biggest impact was made by the NAO (the difference in sea-level pressure between Iceland and the Azores), a positive NAO increases the likelihood of extra-tropical cyclones developing over the North Atlantic. This relationship is further demonstrated by Richardson *et al.* (2017) and Schillereff *et al.* (2019) which both show a negative NAO correlates strongly with increased high-river flows.

Extra-tropical cyclones are known to be the main contributors to extreme precipitation across the globe (Pfahl & Wernli, 2012) and have also been linked to the development of atmospheric rivers (ARs) (Gimeno *et al.* 2020). ARs are long plumes of highly concentrated water vapour in the atmosphere which originate from the mid to lower latitudes, with those affecting the UK moving upwards towards North Western Europe from the Caribbean. Lavers *et al.* (2011) found ARs occurred during the 10 largest floods in the UK. Following this, Lavers & Villarini (2013) analyzed the frequency and intensity of ARs with several climate change scenarios, concluding that both the intensity and frequency of the strongest ARs are expected to increase in the future. In contrast, Champion *et al.* (2015) found that less than 35% of winter and 15% of summer ARs are associated with an extreme rainfall event. This highlights the need to be able to determine the difference between atmospheric phenomena which cause extreme rainfall events and those which produce more moderate rainfall events of little or no interest in flood management.

Different methods are used to statistically analyze the frequency and magnitude of rainfall. Extreme Value Theory (EVT) is one of the most usual statistical methodologies which is used for the description of rare events (climatic or not) and the one of fundamental approaches which is Generalized Extreme Value distribution (GEV). EVT analyzes the tail of the studied parameter distribution, which describes the extreme values, commonly produced by the Block Maxima technique.

CHAPTER THREE

3.0 DATA AND METHODOLOGY

This chapter is comprised of the data used during the research and Block Maxima Approach is used in order to achieve the objectives.

3.1 Rainfall Data

In our study, data were collected from Rwanda Meteorological Agency (Meteo-Rwanda) headquarters located at Kigali. It consist of daily data (mm/day) at station of Kanombe (Kigali International Airport). Rainfall data cover the period of 38 years, i.e. from 1981 to 2018.

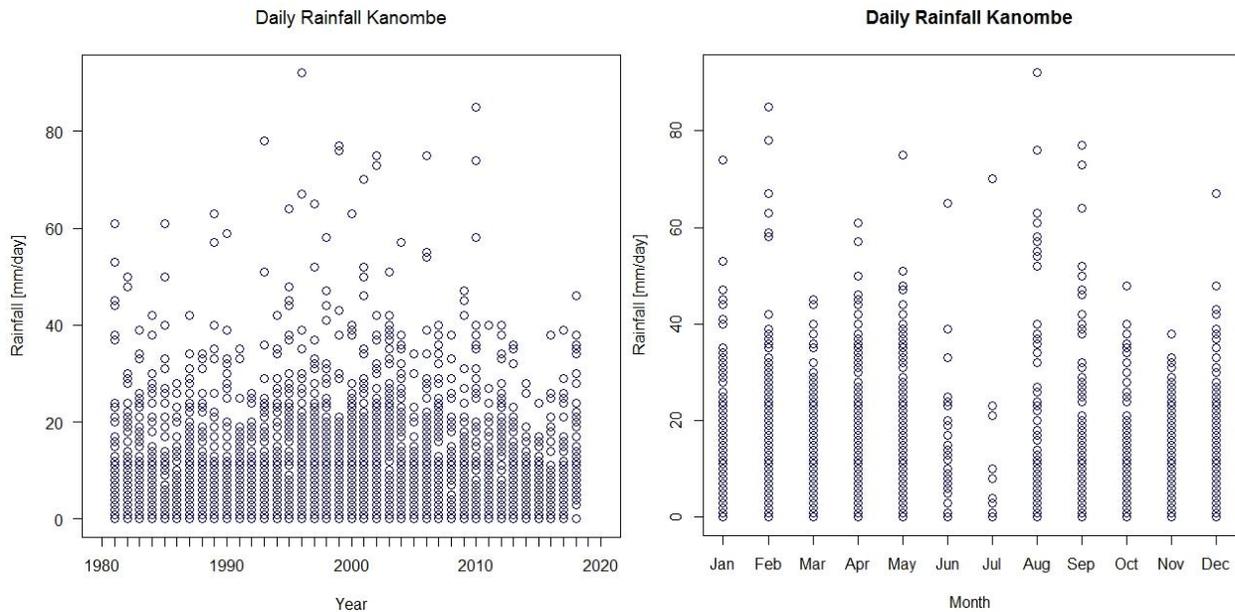


Figure 2: Daily rainfall (mm/day) at Kanombe station for the period of 1981-2018.

3.2. Selection of method

In the present study the Block Maxima Approaches of EVT was applied on the extreme precipitation events of the Kigali City, Firstly the extreme values of precipitation were chosen for Kanombe station using the Block Maxima technique. Due to the fact that the data set is daily values of precipitation, the Block Maxima applied on annual sub-periods. As a result, the maximum precipitation episode of each year was chosen as a block. The data sets Produced by Block Maxima Approach were fitted on the Generalized Extreme Value Distribution (GEV). Finally, the statistical method of L-moments is used for finding the distribution parameters was evaluated.

3.3 Extreme Value Theory (EVT)

EVT provides a tool for modeling the asymptotic distribution of the maximum of a sequence and the sample size based on (Fisher and Tippett, 1928) theorem.

In order to describe the behavior of extreme rainfall at a particular area, it is necessary to identify the distribution (s) which best fit the data. In this study, Generalized Extreme Value Distribution (GEV) is performance. The R Package is used for extremes (Gilleland et al., 2013) that is able to perform parametric inferential analysis of the GEV distribution for Kanombe station.

3.3.1. Generalized Extreme Value Distribution (GEV) and Distribution of Maxima

In this study the Generalized Extreme Value (GEV) distribution is used.

The GEV distribution is a family of continuous probability distributions developed within extreme value theory.

Extreme value theory provides the statistical framework to make inferences about the probability of very rare or extreme events. The GEV distribution unites the Gumbel, Fréchet and Weibull distributions into a single family to allow a continuous range of possible shapes. These three distributions are also known as type I, II and III extreme value distributions. The GEV distribution is parameterized with a shape parameter, location parameter and scale parameter. The GEV is equivalent to the type I, II and III, respectively, when a shape parameter is equal to 0, greater than 0, and lower than 0. Based on the extreme value theorem the GEV distribution is the limit distribution of properly normalized maxima of a sequence of independent and identically distributed random variables. Thus, the GEV distribution is used as an approximation to model the maxima of long (finite) sequences of random variables.

We consider generalized extreme value distribution having the following families, which has all the flexibility of the three extreme value distributions; Gumbel, Frechet and negative Weibull.

This model is appropriate when the maximum observations of each period or block with a predefined and fixed length are assembled from a large number of identically and independently distributed (iid) variables (Coles S, 2001) .The cumulative distribution function of these three distributions can be summarized by the GEV (Jenkinson, A.F., 1955).

Let consider X_1, \dots, X_n as random sample from any distribution

$$\Pr \left\{ \frac{\max \{X_1, \dots, X_n\}}{a_n} \leq z \right\} \rightarrow G(z) \text{ As } n \rightarrow \infty \quad (1)$$

Where $G(z)$ is one of three types of distributions.

$$\text{I. Gumbel) } G(z) = \exp\left\{-\exp\left[-\left(\frac{z-b}{a}\right)\right]\right\}, -\infty < z < \infty \quad (2)$$

$$\text{II. (Frechet) } G(z) = \exp\left\{-\left(\frac{z-b}{a}\right)^{-\alpha}\right\}, z > b \text{ and } 0 \text{ otherwise} \quad (3)$$

$$\text{III. (Weibull) } G(z) = \exp\left\{-\left[\left(\frac{z-b}{a}\right)^\alpha\right]\right\}, z < b \text{ and } 1 \text{ otherwise} \quad (4)$$

(Where $a > 0, \alpha > 0$ and b are parameters)

The above three distributions can be combined into a single family of distributions.

$$G(z) = \exp\left\{-\left[1 + \xi\left(\frac{z-\mu}{\sigma}\right)\right]^{\frac{-1}{\xi}}\right\} \quad (5)$$

G is called the generalized extreme value distribution (GEV).

Three parameters: location (μ), scale (σ) and shape (ξ). Here in our work, use ξ for location, α for scale and κ for shape.

Also, κ is parametrized differently. Specifically, $\kappa = -\xi$ from the above representation.

Where z are the extreme values from the blocks, μ a location parameter; σ a scale parameter; and ξ a shape parameter. For $\xi > 0$ we get the Frechet distribution, $\xi = 0$, the Gumbel distribution and $\xi < 0$ the weibul distribution.

When dealing with extreme events, analysts oftentimes need to be able to determine the distribution of the maximum value or usually be called as maxima. In general, it is important to estimate how likely is it that the greatest value of some sample set will be less than or equal to a particular value. Block maxima model is used in order to estimate the distribution of maxima.

3.3.1.1. Gumbel Distribution

Gumbel distribution or extreme value type I has two forms, one is based on the smallest extreme and other is based on the largest extreme those extreme are called minimum and maximum cases respectively.

It has been used to predict earthquakes, floods and other natural disasters, as well as modelling operational risk in risk management and the life of products that quickly wear out after after certain age. Gumbel distribution has the following characteristics:

- ✓ The shape of the gumbel distribution is skewed to the left. The gumbel pdf has no shape parameter. It means that the gumbel pdf has only one shape, which does not change
- ✓ The gumbel pdf has location parameter μ which is equal to the mode but differs from median and the mean. This is because the Gumbel distribution is not symmetrical about μ
- ✓ As μ decrease, the pdf is shifted to the left. As μ increases, the pdf is shifted to the right.
- ✓ As σ increases, the pdf spreads out and become shallower. As σ decreases, the pdf become taller and narrower. The general form of the pdf of the maximum Gumbel is almost the same (the difference is the change in sign for the first exponent).
- ✓ The change in sign means that the shape of the PDF is identical mirror image of the minimum Gumbel. The following is the formula for probability density function for the Gumbel(minimum).

$$f(x, \mu, \sigma) = \frac{1}{\sigma} \exp\left(-\frac{x-\mu}{\sigma}\right) \exp\left(-\exp\left(-\frac{x-\mu}{\sigma}\right)\right) \quad (6)$$

2.3.1.2. Weibull Distribution

The Weibull distribution is also known as the extreme value type III distribution, it was originally developed to address the problems arising in Material sciences, it is widely used in many other areas thanks to its flexibility.

Weibull model relates to minima (Smallest extreme value) it is characterized by two parameters, one is the shape parameter ε (dimensionless) and the other is the scale parameter σ Probability density function of Weibull Distribution.

Let X be a random variable and let $\varepsilon, \sigma > 0$ then X is said to have a Weibull Distribution with parameters ε and σ if the PDF of X is:

$$f(x, \varepsilon, \sigma) = \left\{ \frac{\sigma}{\varepsilon} \left(\frac{x}{\varepsilon}\right)^{\sigma-1} \exp\left(-\left(\frac{x}{\varepsilon}\right)^\sigma\right) \text{ Where } x \geq 0 \text{ and } 0 \text{ else where} \right. \quad (7)$$

2.3.1.3. Frechet Distribution

Frechet Distribution or extreme value type II, it is slowly converges to 1 and has three parameter: shape parameter ε , scale parameter σ and location parameter μ .

It is defined on interval $\mu \infty$ in other words, it is bounded(restricted) on the lower side.

it has been applied to Extreme events such as Hydrology example for annually maximum one-day rainfalls and river discharges. Flood Analysis, human lifespans. It has the following probability density function.

$$f(x) = \frac{\varepsilon}{\sigma} \left(\frac{\sigma}{x-\mu} \right)^{\varepsilon+1} \exp\left(-\left(\frac{\sigma}{x-\mu}\right)^{\varepsilon}\right) \quad (8)$$

With:

ε : shape parameter

μ : Location parameter

σ : Scalar parameter

3.4. Block maxima Approach

Block maxima Approach is greatly used in analyzing seasonal data, such as climatology and hydrology data. The biggest challenge in block maxima model is determining the size of the blocks. This model is typically used in determining the distribution of maxima.

Block Maxima Approach is one of fundamental approaches used in Extreme Value Theory. It consists of dividing the observation period into non-overlapping periods of equal size and restricts attention to the maximum observation in each period.

Generally, on the BM method studies, it can be seen specific time selection like one month, six months, a year (England, K 2004) or used as arbitrary parameter Santinelli, L., et al, (2014). Selecting a proper and optimum block size for the BM is a crucial issue. Varying across field studies different block sizes were used without any explanation Mudersbach, C. Jensen, J. (2010). Bystorm also indicated that this can be named as optimal block size problem (Byström, 2004). Singh et al gives an example for how to convert the rain fall data set to use the BM. They suggest application by dividing the datasets into yearly, semester, quarterly or monthly blocks without other assumptions Singh, A.K., et al, (2003).

The main intent of Cooley's study is to show weather temperature prediction by using EVT on the area of Central England with the data set starting from 1878 up to 2007 and they used the BM approach by getting daily maximum temperatures Cooley, D. (2009).

The fitting will be inaccurate if block size is too small which may lead to a biased estimation whereas one that is too large may result in a few extracted extreme values, and subsequently, a large variance. Thus, to fully extract the extreme values to constitute the sample loads for fitting a GEV, the block size must be exact. If it is unreasonable, the predictions will be inaccurate.

The few studies have been applied EVT to Kigali monthly rainfall data, the best model was used to forecast, and this was compared to observed data to check if the estimated results were in agreement with reality, Joesph, M.K. (2018). Gumbel distribution was also found to be the best distribution model. Also, Bucchignani.,et al (2015) described the change in extreme rainfall using rainfall indices related to extremes for Dodoma, Tanzania.

In the BM approach a block size is defined and the maximum (or minimum) events are selected for each block. The block size, time interval in this case, is a year and the sample is defined by selecting the maximum precipitation value for each year.

Statistical modeling of extreme events should be performed on samples that satisfy the assumption of randomness, no change points in the mean and no trends in order to assure that the results will be reliable.

In this study, the Block Maxima Approach is used to classify observations that be considered as extreme events and then used in the data analysis process.

3.4.1 Determination of parameters

Determination of parameters cannot be done by analytical methods, only numerical methods are used for their estimation. Different method exist in literature (such as maximum likelihood, L-Moment, Generalized maximum Likelihood and Bayesian methods). In our study, we use the L-Moments (Hosking, 1990).

Let consider, Sample probability weighted moments, computed from data values X_1, X_2, \dots, X_n , arranged in increasing order, are given by

$$b_0 = n^{-1} \sum_{j=1}^n X_j,$$

$$b_r = n^{-1} \sum_{j=r+1}^n \frac{(j-1)(j-2)\dots(j-r)}{(n-1)(n-2)\dots(n-r)} X_j.$$

L -moments are certain linear combinations of probability weighted moments that have simple interpretations as measures of the location, dispersion and shape of the data sample. The first few L -moments are defined by

$$\begin{aligned}\ell_1 &= b_0, \\ \ell_2 &= 2b_1 - b_0, \\ \ell_3 &= 6b_2 - 6b_1 + b_0, \\ \ell_4 &= 20b_3 - 30b_2 + 12b_1 - b_0\end{aligned}$$

the first L -moment is the sample mean, a measure of location. The second L -moment is (a multiple of) Gini's mean difference statistic, which represent the scale parameter. By dividing the higher-order L -moments by the dispersion measure, we obtain the L -moment ratios,

$$t_r = \ell_r / \ell_2.$$

these are dimensionless quantities, independent of the units of measurement of the data. t_3 is a measure of skewness and t_4 is a measure of kurtosis -- these are respectively the L -skewness and L -kurtosis. They take values between -1 and +1 (exception: some even-order L -moment ratios computed from very small samples can be less than -1).

The L -moment analogue of the coefficient of variation (standard deviation divided by the mean), is the L -CV, defined by

$$t = \ell_2 / \ell_1.$$

It takes values between 0 and 1.

2.4.1.1 L -moments for probability distributions

For a probability distribution with cumulative distribution function $F(x)$, probability weighted moments are defined by

$$\beta_r = \int x \{F(x)\}^r dF(x), \quad r = 0, 1, 2, \dots$$

L -moments are defined in terms of probability weighted moments, analogously to the sample L -moments:

$$\lambda_1 = \beta_0,$$

$$\lambda_2 = 2\beta_1 - \beta_0,$$

$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0,$$

$$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0.$$

L -moment ratios are defined by

$$\tau_r = \lambda_r / \lambda_2.$$

CHAPTER FOUR

4.0 RESULT AND DISCUSSION

This section entails the presentation of the results after analysis using EVT and GEV packages in R. Also, discussion about the obtained results is presented.

4.1 Generalized Extreme Value Modelling

This section shows the results and discussion for modelling extreme rainfall in Kigali using GEV

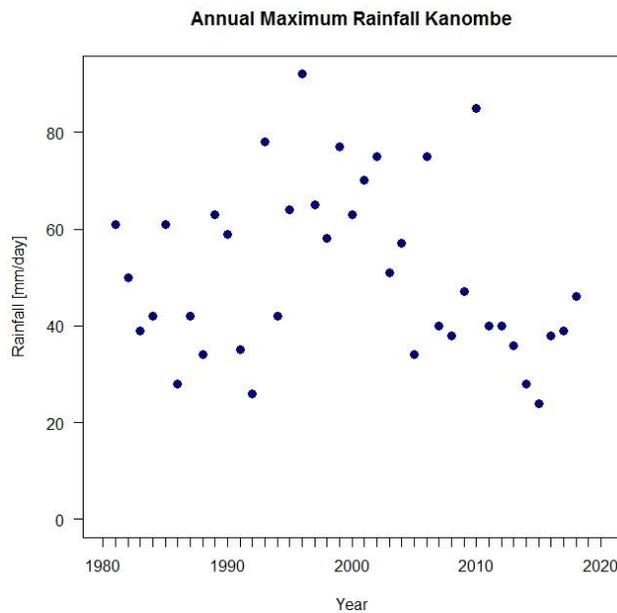


Figure 3: Maximum annual rainfall (mm/day) at Kanombe station for the period of 1981-2018.

It shows extreme rainfall over the region using block maxima method. The outstanding feature is that highest magnitude of 98mm occurred in the year 1996 in Kigali city.

4.2. Parameter estimation for extreme rainfall

L-Moment method is used to estimate the parameters of generalized extreme value distribution with a 95 % confidence interval. The negative shape parameter for GEV distribution (Table 1) implies that the distribution belongs to the Weibull family. In the case when the L-Moment estimate of shape parameter $\xi < 0$ this implies that the support of the GEV distribution is bounden.

4.3. Model diagnostic: QQ-Plots

Quantile-quantile plot (QQ plot) is a useful tool used to check if the assumed distribution fits satisfactorily the studied data set. Generally the QQ plots reveal that the GEV-L-Moment the point are well aligned to the reference line. As it concerns the greatest values of the extreme rainfalls, they are accurately fitted with GEV-L-Moment.

4.4. Return Levels

The estimation of the precipitation return levels, offers a common way to estimate the climatic risk, usually based on historical data. In this study, the return levels of extreme rainfalls were calculated for GEV distribution using L-moments (Table 2).

4.5. Goodness-of-Fit Test

After the application of two goodness of fit tests (Kolmogorov-Smirnov, Anderson Darling), it is proved that GEV distribution can characterize the extreme rainfall behavior with high accuracy. Following that, the parameters of distribution were estimated using L-Moments method. Shape parameter is a helpful tool for checking whether the chosen distribution is appropriate or not. For example the Weibull distribution (negative shape parameter), is inappropriate for precipitation. On the contrary, the Frechet distribution (unbounded above distribution) which has positive shape value, could be appropriate for precipitations.

CHAPTER FIVE

5.1 CONCLUSIONS

The main aim of the present study is to study extreme rainfall in Kigali City by using Block Maxima Approach. For this purpose, daily values of precipitation are used coming from Kanombe station. To analyse the extreme rainfalls, the data was organized into new data sets, the Block Maxima data set which was fitted with the GEV distribution. Finally, their parameters were estimated by L-moments method. The choice of the distribution that can characterize the extreme rainfall's behavior became using goodness of fit tests. In accordance with many researches (Anagnostopoulou, 2012) GEV distribution seem to be one of the most appropriate.

The importance of choosing an appropriate distribution is great, as it can affect the final results (El Adlouni, 2007). The Generalized extreme value distribution has been found as the best distribution model. In this study the extreme maximum rainfall was modeled using Gumbel, Weibull, and Frechet Distribution which are three families of GEV distribution. The model fit suggest that, the Gumbel distribution provides the most appropriate model for the annual maximums of daily rainfall and the Exponential distribution gives the reasonable model for the daily rainfall data over the maximum value of 99 % of Kanombe station. In This research, the return level and predict its coming period was estimated and result show that the floodings might occur once in every 20 to 50 years in Kigali.

Appendix

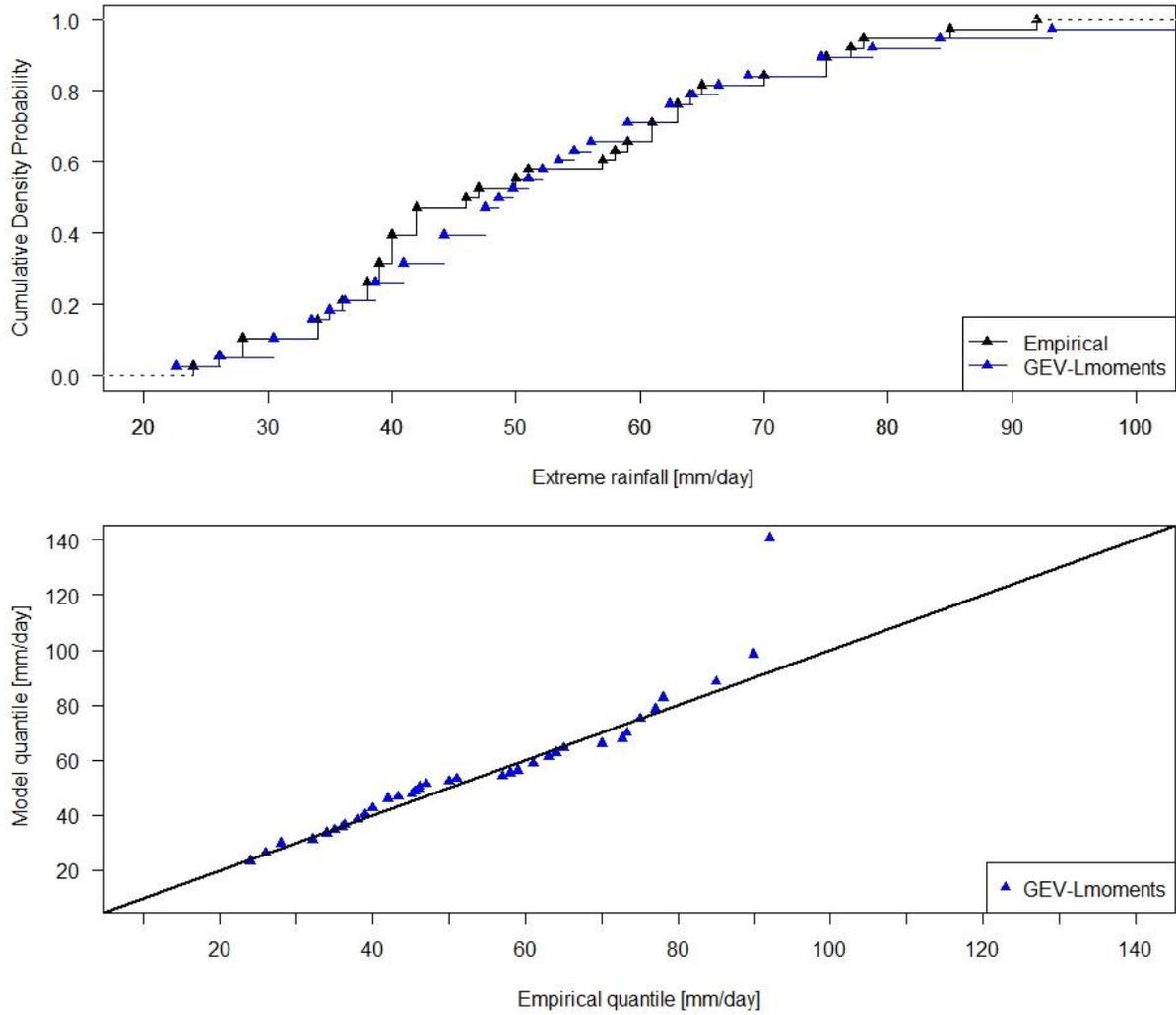


Figure 4: Cumulative distributions and Q-Q plot Kanombe

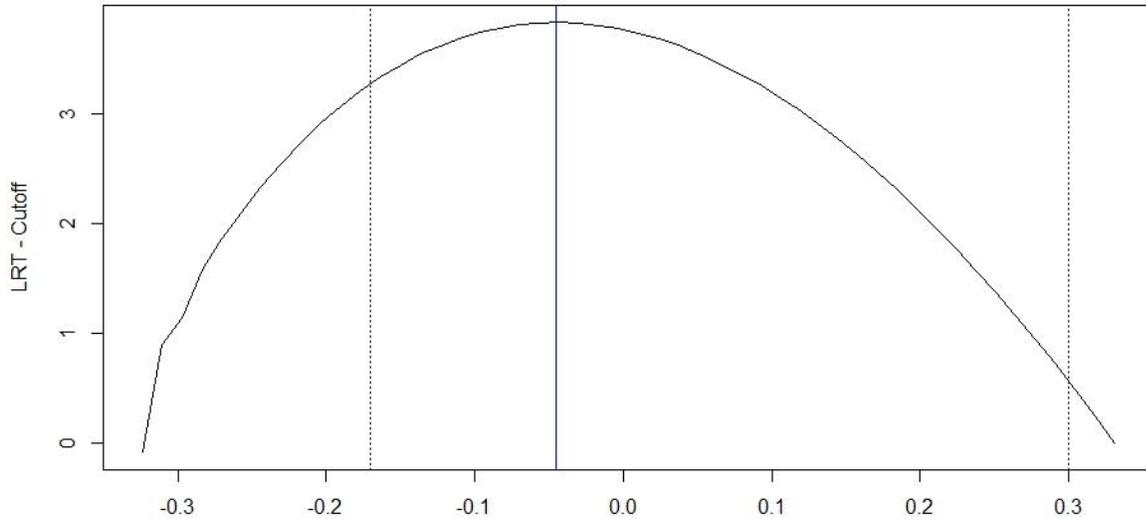


Figure 5: Profile likelihood Kanombe

Table 1: Location, shape and parameters

Parameter	95% CI	Value
Location	Low CI	38.785
	Estimated	43.374
	High CI	48.281
Shape	Low CI	-0.319
	Estimated	-0.056
	High CI	0.167
Scale	Low CI	11.262
	Estimated	15.285
	High CI	19.141

Table 2: Return level plot of extreme rainfall for the station of KANOMBE.

Return period (year)	Return level (mm)
20	85.92
50	99.07
80	105.72
100	108.85
150	114.52
200	118.52

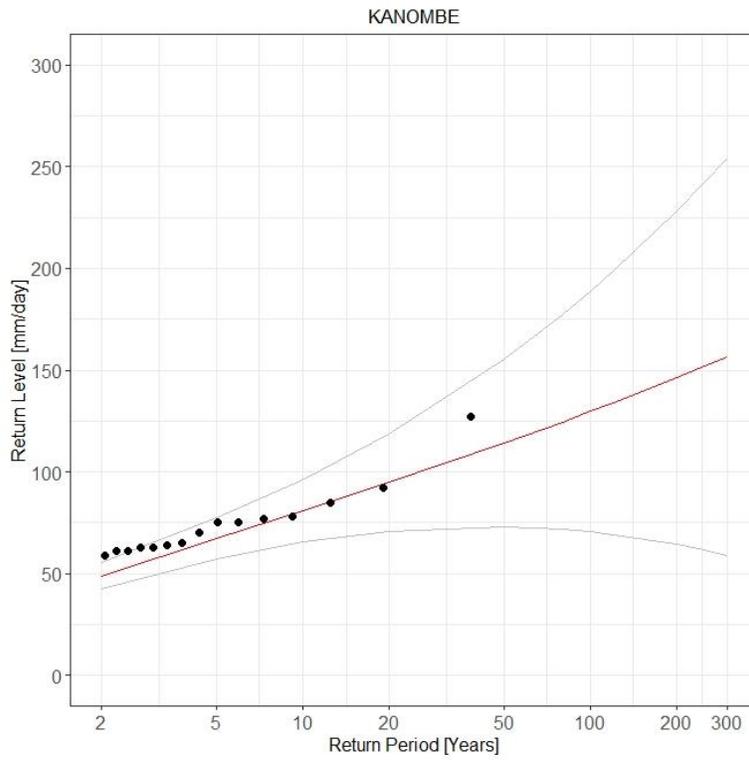


Figure 6: Return level plot of extreme rainfall for the station of KANOMBE.

5.2 RECOMMENDATION

The Generalized extreme value Distribution has proved to be powerful tool to analyze data of extreme event. As we have seen in this project, Gumbel distribution has proved to be a good model for modelling extreme high rainfall. Rwandan Government should take measure of prevention of flood like planting trees, to relocate people's house in high risk zone. The researcher must investigate the use of Generalized pareto Distribution (GPD) to model extreme maximum rainfall in Kigali city. The researcher may also consider to model extreme high rainfall in the whole country. The limitation of the study is that, The Block Maxima method is preferable when the observation are not exactly independent and identically distributed.

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