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A thesis submitted in partial fulfillment of the requirements for the Master of science in High Energy Physics.

TITLE: **Measuring the shape and energy of protohalos in N-body simulations.**

By

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This thesis is submitted in fulfillment of the TURNITIN anti-plagiarism check declaration

DECLARATION

I declare that this thesis is my original work and has not been submitted previously for any degree at the University of Rwanda or any other institution.

All sources used have been acknowledged appropriately.

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ABSTRACT

The universe is initially highly homogeneous but then develops large inhomogeneities through gravitational dynamics. This is due to the classical dynamics of the particles, which tend to fall into the minimum of the potential. Particles that are in underdense regions are attracted to neighboring overdense regions, so that initially overdense regions become denser and denser, and underdense ones become emptier and emptier. This process creates large clumps of matter connected by filaments and separated by large voids. This inhomogeneous distribution of matter in the Universe is known as the cosmic web. The gravitational evolution of the inhomogeneities can be studied by using N-body simulations, a dynamical model of the Universe made up of a large number of particles which can be solved numerically on a computer. Some (many) of these particles end up in high-density regions called halos.

In the initial conditions we refer to protohalos as the regions that will collapse and form halos. This thesis aims to find the initial characteristics of the regions that contain particles that will end in high density regions called halos. The protohalos are the progenitors of halos, in the sense that they are the regions occupied by halo particles in the initial conditions. These regions will collapse and shrink to form halos.

There is a density field in the protohalo region which generates a potential because all the particles in the field attract the other particles and all these particles that attract each other develop a potential energy field and we have to look at the minimum of the potential energy region because this is where the particles will pile up and form a high density region. the physically relevant quantity for the formation of a protohalo is the energy of the protohalo region because dynamically the energy is the quantity that determines the evolution time of a system.

Among the available data in the output of an N-body simulation are the initial positions and velocities of the particles that belong to the protohalos. It is known that the position of the center of mass of the protohalo is well described by minima of the energy field (Marcello Musso and

Ravi K. Sheth, 2021). And the next step will be about determining whether their shape is also described by the energy field. The aim of this thesis is to test numerically whether this hypothesis is correct, constructing the energy of the protohalo regions from the initial positions and velocities of their particles.

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Chapter 1. INTRODUCTION

A system of gravitationally interacting halo particles play an important role in structure formation (Cooray and Sheth, 2002) and N-body simulation imitates the actual process in a dynamical evolution and allows to make the desired measurements. The Universe is everywhere initially homogeneous and isotropic with some tiny fluctuations; these fluctuations grow as time goes on, as a result of gravitational instability. The Friedmann-Lemaître-Robertson-Walker(FLRW) model describes the homogeneous part of the universe through the observations of the distribution of very large scales. the first approximation of the Cosmic Microwave Background radiation motivates to describe the universe to be homogeneous, the space-time is spatially homogeneous and isotropic with its curvature being the same at all points but this can change as time changes, shown by the FLRW metric. These fluctuations will grow with time because of the gravitational evolution. Initially, a perturbative treatment can be used to describing the inhomogeneous part of the universe.

These small perturbations describe the initial growth of density fluctuations. Eventually, when these perturbations become large the perturbative treatment breaks down and one has to move to non-perturbative method like the spherical collapse, which is the simplest model for the formation of non-linear structures.

One of the non-perturbative methods is the spherical collapse which describes the formation of a halo. It predicts the universe to raise some inhomogeneities besides being initially homogeneous (Marcello Musso and Ravi K. sheth, 2021) those behaviors are caused by the gravitational dynamics of the universe. The simplest models of dark matter halo formation rely on the heuristic assumption, motivated by spherical collapse, that virialized halos originate from initial regions that are maxima of the energy field.

In practice, protohalos are not spherical because the inhomogeneities are random and there is no spherical symmetry because the gravitational attraction from one region maybe stronger than the other and particles can be more attracted to one direction than the other. Due to this reason we consider protohalos to be some irregular shape. We want to describe the shape of these regions based on their energy. that is, we choose those shape that are minima of the energy.

1.1 BACKGROUND COSMOLOGY

As motivated form large scale, since the time of Copernicus, it has generally been assumed that we do not occupy an advantaged position in our universe. It means that the basic characteristics of our surroundings appear the same if one were located in a different region of our universe, at the same time it is natural to consider the universe to be isotropic, that means that there are no advantaged directions in space (Daniel Baumann, Cosmology part 3 Mathematical Tripos).

Considering the universe to be homogeneous and isotropic means that there are neither advantaged or privileged position nor direction in our universe, this is more described by the Friedman- Lemaître -Robertson-Walker model of cosmology, it is a model inspired by the large scale homogeneity and isotropy of our universe. It is a space-time in which there is a set of preferred observers called commoving observers who all go through the same experience and every space-time point is passed by the word line of one such observer. This means that their clocks all show the same time t at some maximally symmetric time slices after initially setting their clocks at the same time.

There are three possible choices for the metric of these 3d manifolds that depends on whether their Ricci scalar $R^{(3)}$ is

- Zero
- Positive or Negative

Having the maximal symmetry means that there is no preferred point or direction in space, that simply means that $R^{(3)}$ fully specifies the curvature. As we know our universe is expanding, so for an expanding universe, has its own space-time metric that plays a fundamental role in relativity. And this metric has to be invariant by turning the observer dependent coordinates into the invariant line element. That is x^u= (t, x^i). In our case, here the FLRW model has a metric for an expanding universe, we simply include the Euclidean line element $dl^2 = a^2\gamma_{ij}dx^i dx^j$ into space-time line element and consider the parameter a be an arbitrary function of time t.

$$ds^2 = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j \tag{1}$$

Here we notice that the ten independent components of the space-time metric have been reduced by the symmetries of the universe to an individual function of time, the scale factor $a(t)$ and the curvature parameter k whereas the coordinates $x^i = (x^1, x^2, x^3)$ are referred to as the commoving coordinates. It fits well with our need to use spherical polar coordinates (r, θ, φ) because they make clear the symmetries of the space by using

$$dx^2 = dr^2 + r^2(d\theta^2 + sin^2\theta\, d\varphi^2) \tag{2}$$

so that dl^2 becomes $dl^2 = a^2(\frac{dr^2}{1-Kr^2} + r^2\, d\Omega^2)$ (3)

where $d\Omega^2 = d\theta^2 + sin^2\theta\, d\varphi^2$ (4)

The full metric can be written as:

$$ds^2 = dt^2 - a^2(t)\,(\frac{dr^2}{1-Kr^2} + r^2\, d\Omega^2) \tag{5}$$

Here $a(t)$ is called the scale factor and K is the curvature and we redefine $a(t)$ appropriately and r, k can be taken to be:

- 0 if the spatial slices are 'flat' (3d Euclidean)
- 1 if they are "closed" (spherical/positively curved) or
- −1 if they are "open" (hyperbolic/negatively curved)

The preferred observers that we referred to as the commoving observers can be defined as fixed spatial coordinates moving along the geodesics with a 4 velocity u^u= (1,0,0,0). The stress energy tensor is restricted by the homogeneity and isotropy of the FLRW model as a function of time t.

$$T_0{}^0 = -\rho(t)\,, \quad T_j{}^i = p(t)\delta_j{}^i \qquad (6)$$

And This is how the stress-energy tensor of a perfect fluid with energy density ρ and pressure p looks like, even if the matter content of the universe might not be an actual fluid.

1.1.1 Friedmann equations

A homogeneous and isotropic model of the universe that we are discussing in this chapter is governed by a set of equations in physical cosmology which describes how, based on what is in the universe, its expansion rate will change over time. These equations came from Einstein equations, $G_{uv}= 8\pi G T_{uv}$, once one chooses the metric to be FLRW.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

(8)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho+3p)$$

We should understand ρ and P as the cumulative distributions to the energy density and pressure in the universe. And one can write ρ_m from the contribution by matter, ρ_r for the contribution from radiation with ρ_γ for photons, ρ_v for neutrinos , ρ_c for cold dark matter , ρ_b for baryons and ρ_Λ for vacuum energy contribution.

These set of equations govern the expansion of space in homogeneous and isotropic models of the universe.

1.2 DENSITY PERTURBATIONS

The FLRW metric shows that the space-time is spatially homogeneous and its curvature is the same at all points in space but this can change as time changes. As the universe is gravitationally evolving, this will keep the fluctuations growing with time and the perturbative treatment starts by describing the inhomogeneous part of the universe. The universe is very homogeneous at early time but also very inhomogeneous later on large scale, the way we describe this is by splitting the treatment into a homogeneous background part and inhomogeneous fluctuations. These inhomogeneities are initially small that one can treat them by the perturbation theory. The perturbation theory describes the growth of these inhomogeneities under the effect of gravity. There is the Cold Dark Matter (CDM) particles which has tiny fluctuations, these particles are weakly interacting and their velocities was too low in a galactic scale. The inhomogeneous part of the universe is well described by the perturbation theory of the gravitational evolution of density perturbations. The relevant results of the evolution of large scale structure density perturbations are the Euler and Continuity equations.

Continuity equation

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla \cdot \ (1+\delta)u = 0 \tag{9}$$

- Euler equation

$$\frac{\partial v}{\partial t} + Hu + \frac{1}{a}\left[(u \cdot \nabla)u + 4\pi G\, \bar{\rho}\nabla\varphi\right] = 0 \qquad (10)$$

(Asanta Cooray and Ravi K. Sheth,2002)

We introduced δ as the density contrast

$\delta \equiv \frac{\rho}{\bar{\rho}} - 1$ where $\bar{\rho}$ is the homogeneous background density (11)

And the Poisson equation relates the potential fluctuations due to density perturbations,

$$\nabla^2\Phi = 4\pi G\, \bar{\rho}\, (1+ \delta) \qquad (12)$$

Which is the total Poisson equation.

There is a full Poisson equation that gives $\nabla^2\Phi = 4\pi G\rho$ and therefore the equation for perturbation is simply $\nabla^2\varphi = \delta$.

The continuity equation describes the conservation of mass, basically it tells that the density in a volume changes only when there are particles that are entering or exiting the volume, and the Euler equation tells us the evolution of the velocity of those particles. One can interpret the Euler equation as Newton's law in a different coordinate system.

1.3 Non Linear Methods

The perturbations discussed recently evolve and they become large so that the perturbative treatment breaks down that's because perturbations occur order by order, and one has to move to non-perturbative method which are also the nonlinear methods like the spherical collapse, this is what motivates the halo formation. This is the simplest model for the formation of non-linear structures. There will be the linearity of equations due to the evolved fluctuations and so if things are overdense become more and more overdense and underdense ones become emptier and emptier, matters pile up resulting into filaments and leaving voids emptier. The assumption that non-linear objects formed from a spherical collapse is a simple and useful approximation.

The mean matter over density within a sphere of physical radius R centered at the origin is usually defined as:

$$\delta_R = \int_V \frac{dr}{V}\ \delta(\mathrm{r}) \qquad (13)$$

Where: $\delta(\mathbf{r}) = \rho(\mathbf{r})/\bar{\rho} - 1$ given that $\rho(\mathbf{r})$ is the matter density at r and $\bar{\rho}$ its background value

And $V = 4\pi R^3/3$ (Marcello Musso and Ravi K. Sheth, 2021). The potential energy in the sphere due to matter is:

$$\mathrm{U} = \int_V dr\rho(r)(r - r_{cm}) \cdot g \qquad (14)$$

Where: r_{cm} is the center of mass position, defined as:

$$r_{cm} = \frac{1}{M}\int_V dr\rho(r)r = \frac{\bar{\rho}}{M}\int_V dr\delta(r)r \qquad (15)$$

M is the total mass enclosed in the sphere of radius R. $g \equiv -\nabla\Phi + [\nabla\Phi_{cm}]$ is the acceleration at r induced by matter only, relative to the acceleration of the center of mass.

The acceleration splits into background and peculiar: $\nabla \varphi = 4\pi G \bar{\rho}(r/3 + \nabla \varphi)$, with the potential perturbation normalized so that $\nabla^2 \varphi = \delta$ in physical coordinates. The relative acceleration becomes:

$$g = -4\pi G \bar{\rho}\left[\frac{r-r_{cm}}{3} + \nabla\varphi(r) - \int_V \frac{dr'\rho(r')}{M}\nabla\varphi(r')\right] \tag{16}$$

the last term in the bracket arise to be the peculiar acceleration of the center of mass -$[\nabla\varphi]cm$ the acceleration from the cosmological constant is $(\Lambda/3)\,(r - r_{cm})$ then the potential energy can be written as:

$$U = -4\pi G\bar{\rho}\,\frac{MR_I{}^2}{5}\,(1+\varepsilon_R) \tag{17}$$

From this explains that maximizing epsilon is at the same time minimizing the energy

And finally the potential energy over density(epsilon) associated with the volume of the sphere given by $V = 4\pi R^3/3$ is

$$\varepsilon_R = \frac{5}{MR_I{}^2}\int_V dr\rho(r)(r - r_{cm})\nabla\Phi(r) \tag{18}$$

Where the square of the inertial radius is expressed as

$$R_I{}^2 = \frac{5}{3M}\int_V dr\rho(r)|r - r_{cm}|^2 \equiv \frac{5}{3M}T \tag{19}$$

1.3.1 Evolution equation for the Inertia Radius

A test particle on the surface of an expanding sphere of radius R evolves according to the famous equation of motion $\ddot{R} = -\frac{GM}{R^2}$

From equation (19)

$R_I{}^2 = \frac{5}{3M} \int_V dr\rho(r)|r - r_{cm}|^2 \equiv \frac{5}{3M} T$ and by identification, T= $\int_V dr\rho(r)|r - r_{cm}|^2$ (20)

which is the moment of inertia defined in (19). If one differentiates T,

(S. Chandrasekhar 1969 Ellipsoidal figures of equilibrium)

$$\ddot{T} = \frac{d^2}{dt^2}\left(\int_V dr\rho(r)|r - r_{cm}|^2\right) \equiv \int_V dr\rho(r) \frac{d^2}{dt^2} |r - r_{cm}|^2 = \int_V dr\rho(r) \left(2\,\ddot{r}r + 2\,|\dot{r}^2| \right) \quad (21)$$

Then by considering the potential energy U in (17) in a ΛCDM cosmology.

$$\ddot{T} = 4k + 2U + \frac{2\Lambda T}{3} \quad (22)$$

This is the evolution of the moment of inertia and it can be expressed in term of R_I as,

$$\ddot{R}_I = 2kR_I - \frac{GM_I}{R_I{}^2} + \frac{\Lambda}{3} R_I \quad (23)$$

which is the equation of the evolution of the moment of inertia associated with the collapse of the inertia radius.

The potential energy over density(ε_R,epsilon) has the same impact for the gravitational potential energy U as the mean matter over density impacts the mass $M = V_R \bar{\rho} (1+\delta_R)$. the equation (19) is dropped because of the contribution form the peculiar acceleration of the center of mass which is constant over the sphere. And also due to perturbation, one can replace ρ(r) ≅ 1/$V_R$and in consideration of a spherical volume $R_I \cong R$ and $r_{cm} \cong 0$. Then we can use these notions to redefine the **linearized mean energy over density as**

$$\varepsilon_R = \frac{15}{4\pi R^5} \int_V d\mathbf{r}\,\mathbf{r} . \nabla \varphi(\mathbf{r}) \qquad (24)$$

Here φ is the potential perturbation

We call this the energy overdensity because it quantifies the contribution to the energy due to the perturbation over the homogeneous density of the protohalo.

Chapter 3: METHODOLOGY

We characterize the center of mass of halos as the minima of the potential and the protohalos to be the maxima of the potential because of the classical dynamics of the particles tending to fall into minima potential regions.

In the simulation code, we are computing the energy within the spherical region and considering the initial density field, we can place a sphere centered at some point and compute the gravitational potential energy inside the sphere and get a value and moves the sphere around to get other values because the energy changes as we move the sphere. later, we chose the minima of these values to identify as the center of mass of the protohalo. This approach can only make sense when the protohalo is a sphere.

But now we cannot stick to a sphere, we can randomly deform a sphere by stretching it on one direction and compress it in the other direction, the shape changes as well as the energy because we enclose a different set of particles. This process cannot be done analytically, we used computer simulations to do the measurements.

I did all the intended calculations in a python code, among the selected 10 protohalos, I had to find the center of mass. The protohalo is a collection of particles that the halo finder identifies at the end of the simulation and then trace it back to the initial conditions, and due to gravity the protohalo is in some irregular shape. So the code is computing the energy in the protohalo shape and the energy of the sphere that is centered around the center of mass of the protohalo and we can see how they are related. we choose the configuration of a sphere because we don't know what is the protohalo shape, so we start assuming it to be the sphere since is the easiest thing to do. By moving the sphere around we can find the configuration with the minimum energy and later identify the center of that sphere with the center of the protohalo. All of this is done in the code. Assuming we have the center of mass of the protohalo, the next challenge is to find the shape. The idea is that we deform the sphere at fixed volume in order to find the configuration of minimum energy. It means that we deform a sphere into an ellipsoid and find the ellipsoid of minimum energy and this can be a better description of the protohalo shape, we can validate this by comparing this description with a sphere. the code computes the energy overdensity(epsilon) for a sphere centered around the center of mass of a protohalo itself and also does the same for various configurations or deformations.

3.1 N-body simulation

The calculations are very difficult to be done analytically. There are computer simulations that can help compute the measurements. The simulation has a huge number of particles that are numbered from 1 to a huge number Npart (1024^3) particles, they are initially placed on a square grid making them to be equally spaced, the particles on the grid are not moving because each particle on the grid is attracted by all the neighboring ones because the configuration is symmetric. If there is a gravitational attraction in one direction, there is also an equal gravitational attraction in another direction and the net result is zero and the net acceleration of the particles is also zero. If the particles are on an unperturbed grid, then there is no motion.

However, in a simulation the particles placed on a grid are given small initial velocities that are randomly directed and this keeps them moving. In this case the homogeneity is perturbed and particles are no longer on a symmetric grid, they start feeling an excess acceleration in some direction and then they start moving by picking up a velocity that is proportional to the acceleration, This is why in the theory definition of the energy there is the acceleration or gradient of the potential but in the estimator the velocity appears. And this is what is available from the simulation output. In this way the gravitational evolution begins.

3.2 Estimator

I defined the energy and the energy overdensity(epsilon) considered in a spherical approach but there is a more detailed approach to average all the particles in real space-time positions (Marcello Musso and Ravi K. Sheth, 2021). in practice, we compute averages by explicitly summing over real space positions, rather than using Fourier methods.

This means that we estimate

$$\varepsilon_R = -3\ \frac{\sum_j[(r-r_{cm}).(v-v_{cm})/fDH]_j}{\sum_j[(r-r_{cm}).(r-r_{cm})]_j} \qquad (25)$$

where the sum is over the particles in the protohalo. We used this approach to compute the mean energy over density in the N-body simulation.

3.3 Data

We have a simulation box of 200 Mpc and from this we have a file that contains the Ids of the particles in the box around each protohalo. The box being bigger than the protohalo and constructed around each protohalo because we want to compute the energy in various configurations then need to know the particles around each protohalo. So for each protohalo we have a list of Ids of particles that form a box around a protohalo. Among the available data includes the computed inertia tensor for each protohalo, this is a 3 by 3 tensor having some information about the shape.

In the computation of the measurements, we used data files shared on the quevedo cluster that list the initial positions and velocities of all the particles that are used in the simulation and also another file named the halo catalog, it contains the identities of all the corresponding particles that will end up in a halo. If one reads the halo catalog can identify the identities of the particles that we need in the protohalo which is a long list. Then one can use this notion to identify the position and velocity of each of the particles listed in the catalog resulting in the identification of all the particles' positions and velocities in the protohalo. One can compute the energy of a particle and summing up to the number of particles contained in the protohalo and that gives the energy of the protohalo using the equation (8) but in the simulation we compute the average sum by explicitly summing over real space positions of all the particles in the protohalo by using the estimator in (9).

3.4 Deformations

In the simulation, I first selected 10 protohalos and deformed each 7 times. by measuring the energy of the protohalos in a sphere centered around the protohalo center and deforming each in various 7 directions which make the deformations ellipsoidal. I have initially a sphere,3 spheres of one ellipticity and other 3 spheres of opposite ellipticity which together sums up to 7 deformations.

I measured the inertia tensor of the region of the protohalo and later fit an ellipsoid and deform the axes of the ellipsoid then I computed the energy for each ellipsoidal deformation. The simulation computed the 7 ellipsoidal deformations energies for each selected single protohalo.

All these calculations aim to validate the hypothesis that the regions of the protohalos are minima of the energies. It means that any deformation's energy will give a value that is larger than the energy of the regions of the protohalo, this is conceptually because of replacing the particles that are in the protohalo and enclosing some particles that are not in the protohalo can give a high value of energy

3.5 Quantification of particles in the deformation

As it is mentioned before, protohalo is a collection of particles that will end up in high density regions, so the sphere and the protohalo are obviously not the same configuration but because they have the same volume, the sphere in the simulation contain particles that are not in the protohalo and will also miss some of the protohalo particles. when we start deforming the sphere into an ellipsoid we are being closer to the protohalo, so each deformation contains particles of the protohalo but also miss some particles of the protohalo. In the simulation code, I included a quantitative approach to quantify the number of particles that are in the intersection between the protohalo and the deformed ellipsoidal regions. This quantity should be maximum and later, we should validate by considering how the energy changes as the number of particles in the intersection between the two sets increase or decrease.

Chapter 4: Results

The simulation cube is larger than the sphere because I wanted to deform the sphere into different configurations so that we may not have any configuration which is not enclosed in the simulation box. The simulation code computes the energy in each ellipsoid, then I plotted the energy as function of alpha. Here alpha is a parameter that relates the ellipsoidal deformations with their axes. It creates a simple mapping between the sphere and the ellipsoid as shown in the following formula: $\alpha\, a_1, a_2, \frac{a_3}{\alpha}$ so that the product of the three axes gives the volume of the ellipsoid, which remains constant. Here a_1, a_2 and a_3 are the axes of the ellipsoid that best fits the protohalo which are given by the simulation because first we needed the best fitting ellipsoid and then deforming it later. So choosing α = 1 returns the best-fitting ellipsoid. a smaller value of α returns a more spherical configuration. We explore values of α ranging from np.sqrt($\frac{a_1}{a_3}$) to 2- np.sqrt($\frac{a_1}{a_3}$) where np.sqrt($\frac{a_1}{a_3}$) is the value for which the first and third axes are equal, that is $\alpha\, a_1 = \frac{a_3}{\alpha}$.

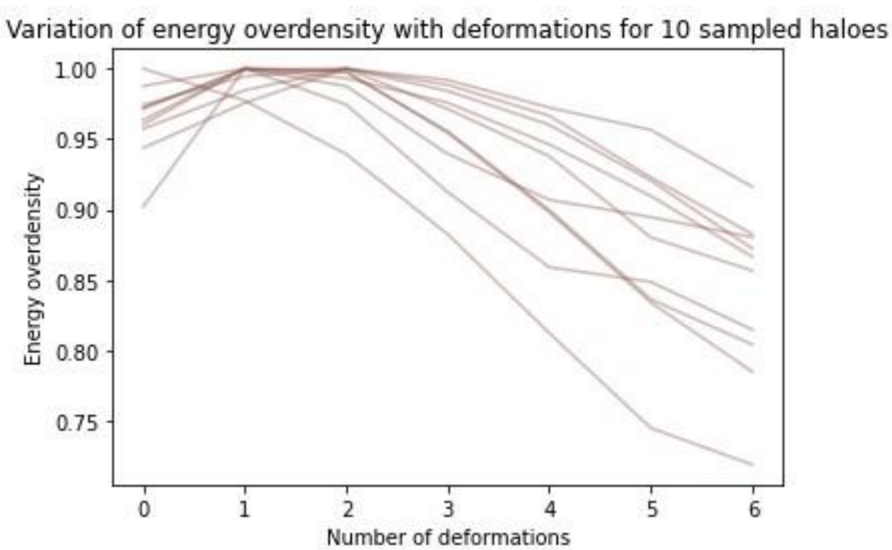

Figure 1: shows the variation of energy over density for 10 randomly selected protohalos with different ellipsoidal deformations and here alpha=1 corresponds to 5

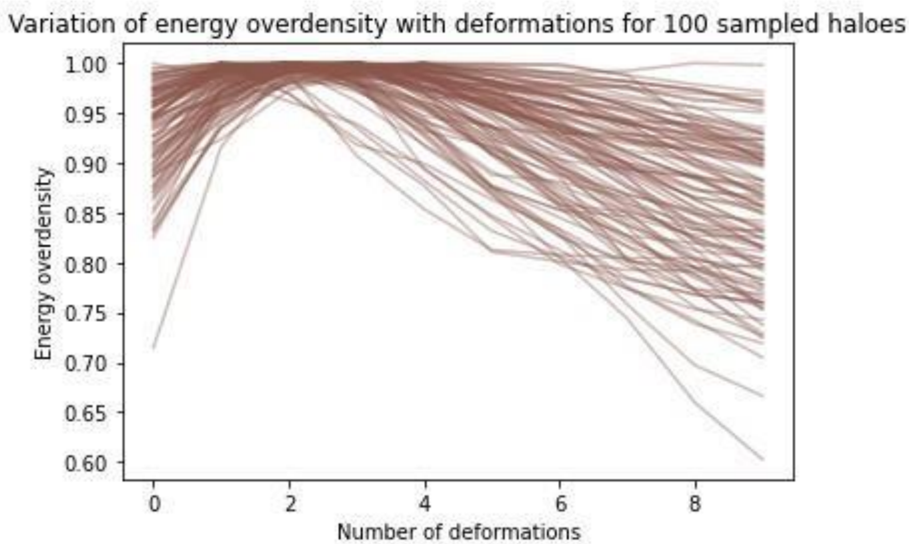

Figure 2: shows the variation of energy over density for 100 randomly selected protohalos.

in the above figures, I first selected 10 protohalos randomly and deforming each in 7 deformations. Later, I increased the sampled protohalos to 100 with 11 deformations for each. we can see that as alpha increases from the minimum value of alpha the energy over density increases, reaches a maximum and then drops. However, the maximum value is attained before alpha=5 which is the best fitting ellipsoid of the inertia tensor. This could be due to two reasons:

1) the best fitting ellipsoid is not the configuration that captures most actual protohalo particles, because of the irregularity of the shape or
2) minimizing the energy is not totally accurate to predict the shape.

A quantification of the intersection of the particles of the protohalo with the ones of each ellipsoid can help to test which one is true. And therefore if plotting the ratio between the number of particles in the intersection and the number of particles in the protohalo against deformations gives a similar plot, from this, we can conclude that the best fitting ellipsoid is not the configuration that captures most actual protohalo particles.

I had to compare the two files containing the IDs of the protohalo particles and the other one with the IDs of the ellipsoid particles. Then later the computation of particles overlap that is the ratio of particles in the intersection between the two sets and the particles in the protohalo can be able to show its peak as a function of alpha and see how it varies with deformations or alpha.

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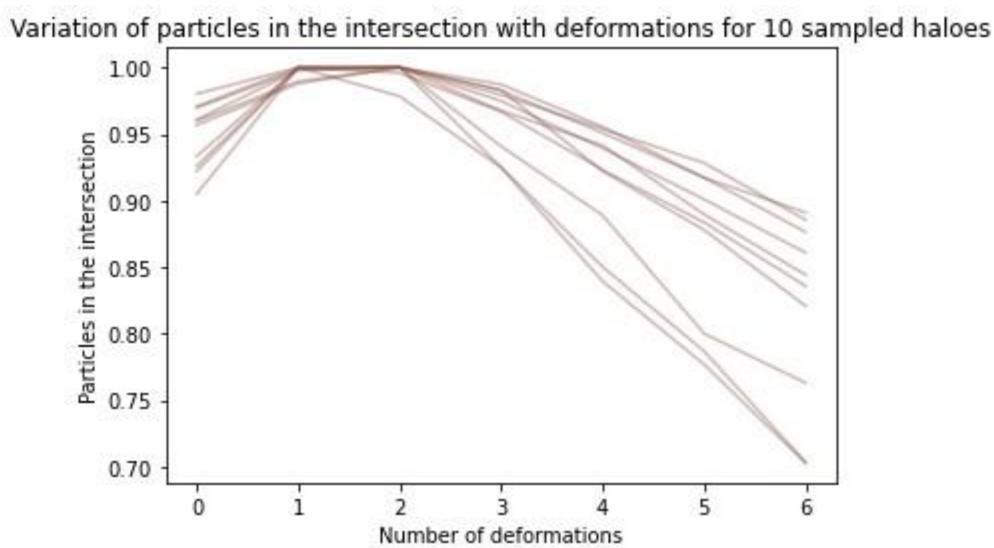

Figure 3: shows how the particles in the intersection between the two sets peak for different ellipsoidal deformations

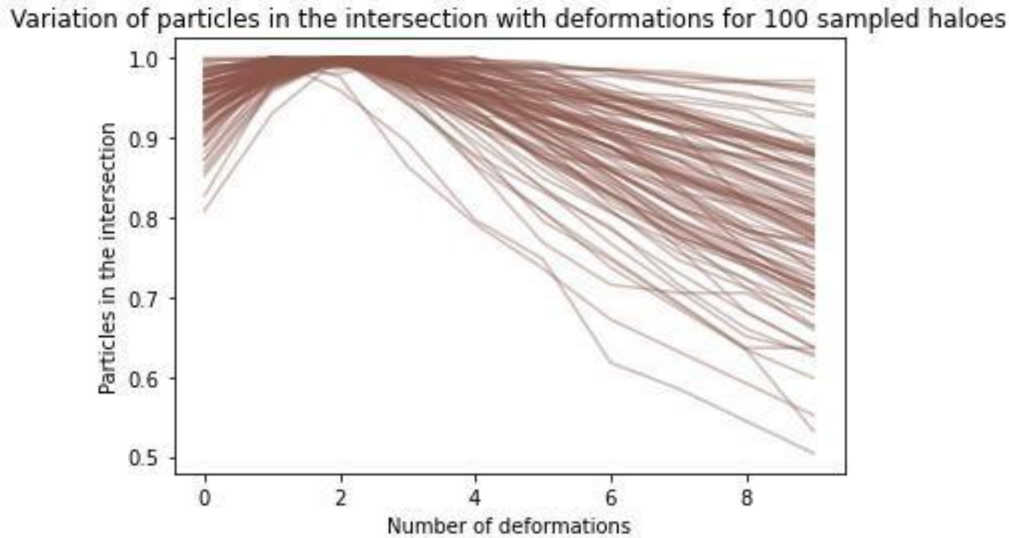

Figure 4: shows how the particles in the intersection between the two sets peak for different ellipsoidal deformations in the 100 sample halos.

We have seen that the alpha of maximum energy overdensity is not alpha=1 (5 in my units) that is the ellipsoid that best fits the inertia tensor, with this new approach of plotting the quantification of particles in the intersection against alpha shows that the energy overdensity peaks for the same alpha that maximizes the intersection with the protohalo. After observing that the energy overdensity peaks for the same alpha that maximizes the intersection with the protohalo assures that the ellipsoid that has the most protohalo particles has also the maximum energy overdensity. Maximizing the energy overdensity is minimizing the energy at the same time, this can be seen mathematically from equation (5). We can conclude that the ellipsoid that includes most of the protohalo particles is also the one that minimizes the energy.

We had already plotted the values of epsilon as a function of the deformation and we saw that the values of epsilon increases to its maximum value and then drops. Now, we are considering a fraction of particle overlap, that is a ratio between the number of particles in the intersection and the number of particles in the protohalo. In the following plot shows how a quantity of particle overlap varies with deformations for 10 and 100 sampled halos. And that is shown from figure 3 and 4, it has a trend similar to the energy overdensity.

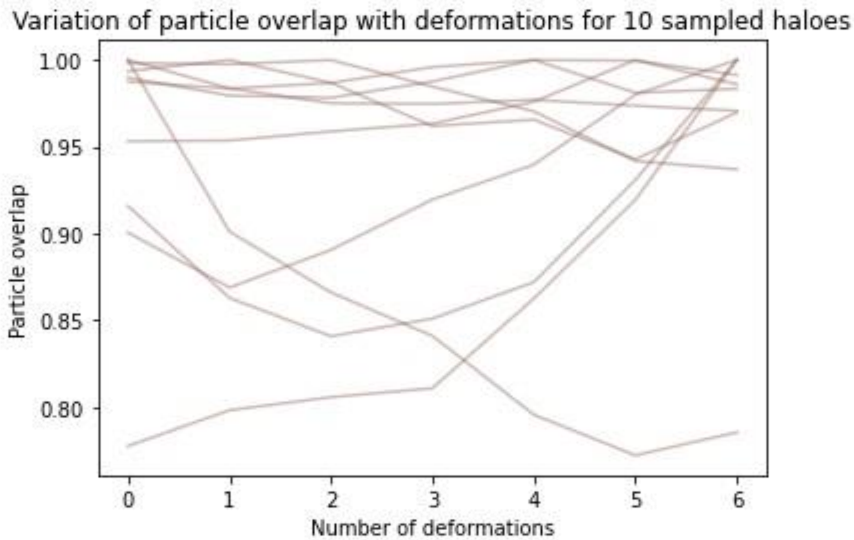

Figure 5: shows the particle overlap peak for different ellipsoidal deformations in the 10 sampled halos

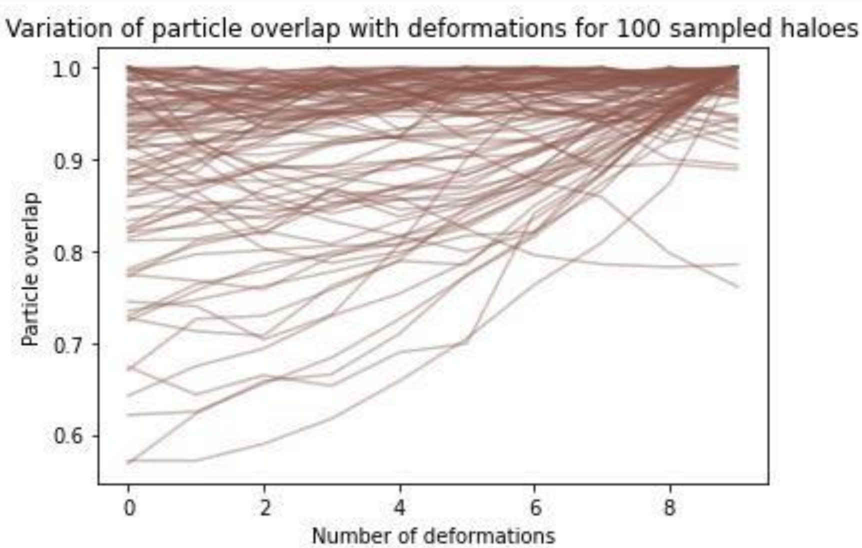

Figure 6: shows the particle overlap peak for different ellipsoidal deformations in the 100 sampled halos

From the figure 5 and 6, show the ratio (normalized to 1) between the energy over density and the particle overlap. This ratio remains roughly constant for most halos, which justify the fact that the value of energy over density traces well the overlap of the ellipsoid with the actual protohalo region. these analyses from the plots showed that the value of alpha that maximizes the intersection is the same alpha that minimizes the energy, then we know that the ellipsoid that include most protohalo particles is also the one that minimizes the energy. Therefore, we know that protohalo particles tend to have a lower energy than the ones that do not belong to the protohalo. That is, to a good approximation to validate the hypothesis that protohalos are configurations of minimal energy.

Chapter 5: Conclusion

This work provides the energy and the shape of protohalos in N-body simulations for a certain elected protohalos containing a collection of particles. A study of the peaks of the energy over density, peaks of the quantification of particles that are in the intersection between the protohalo and the deformations has shown significant results.

According to the analysis on the 10 and 100 selected protohalos we are able to conclude that

- The ellipsoid that includes most of the protohalo particles is also the one that minimizes the energy. This is shown by the energy over density peaking for the same alpha that maximizes the intersection with the protohalo.
- Minimizing the energy is accurate to predict the shape and Therefore, we know that protohalo particles tend to have a lower energy than the ones that do not belong to the protohalo.
- Protohalos are configurations of minimal energy.

List of figures

REFERENCES

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