华北电力大学

硕士学位论文

水火电机组联合发电优化调度方法

Generation Scheduling Approach for Hydrothermal Power Systems

SINGIRANKABO YOUSSOUF （辛格）

2016 年 6 月

国内图书分类号：TM721.3 学校代码：10079
国际图书分类号：621.3 密级：公开

硕士学位论文

Generation Scheduling Approach for Hydrothermal Power System

硕 士 研 究 生：辛格
导 师：周明 教授
申 请 学 位：工学硕士
学 科：电气工程
专 业：电力系统及其自动化
所 在 学 院：电气与电子工程学院
答 辩 日 期：2016 年 6 月
授予学位单位：华北电力大学

Classified Index: TM721.3

U.D.C: 621.3

Thesis for the Master Degree

Generation Scheduling Approach for Hydrothermal Power System

Candidate：	SINGIRANKABO Youssouf
Supervisor：	Prof. Zhou Ming
Academic Degree:	Master of Engineering
Major:	Power System and Its Automation
School：	School of Electrical & Electronic Engineering
Date of Defense：	June, 2016
Degree-Conferring-Institution：	North China Electric Power University

华北电力大学硕士学位论文原创性声明

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摘要

在电力系统运行方面，由于水电厂在一定时间内可用水量的一系列限制，同时包含水电和火电系统的调度比全火电系统更加复杂。与火电厂相比，水电厂的运行成本可以忽略不计；因此，水火联合调度优化的目标就是在给定时期和给定的水量约束下将火电厂的运行成本最小化。

很多传统和人工智能优化方法已经成功运用到水火联合调度问题当中。但是，这些优化方法中仍有许多缺点，如：计算量随维数急剧增长、算法复杂和收敛性差。

本文提出的基于混合整数规划的“水火发电系统的发电调度方法”（GSAHTPS）为短期水火联合调度问题提供了一种解决方法。它采用 IBM 公司开发的 ILOG Cplex 优化软件进行线性化编程与求解；使用 Cplex 中的优化编程语言（OPL）作为组合优化的建模语言可大幅简化优化问题，这种 OPL 语言在为模型线性化、整数规划提供支撑的同时还为最顶尖线性规划算法创造了捷径。

水电厂与火电厂的发电功率特性都是非线性的，在 ILOG Cplex 中采用分段线性化的方法近似求解。本文根据水电厂与火电厂发电功率特性的不同特点提出了一维、三角形和矩形三种方法来进行分段线性化。在研究中我们主要着眼于经济调度与机组组合问题。

为了评估该方法的高效和强大性能，验证算例采用了包含 46 个火电厂以及额外 4 个级联水力发电厂的 IEEE118 节点测试系统。

计算结果表明，本文提出的方法计算高效、简单，可为短期水火联合调度问题的决策提供支持。

关键词：混合整数线性规划（MILP）；经济调度（ED）；机组组合（UC）；短期水火调度

ABSTRACT

In power systems operation, a system consisting of both hydro and thermal power plants is more complex than the scheduling operation of an all thermal generation due to a number of constraints of water available in a given period for hydro generation. The hydro power plants having a negligible operating cost compared to the thermal power plants; the objective of generation scheduling approach of hydrothermal power plant is to minimize the thermal operating cost under the constraints of water available to operate the hydro plant during a given period of operation.

A variety of optimization techniques; conventional and artificial intelligence techniques has been applied to solve the hydrothermal scheduling problems, however these techniques consist of a certain drawbacks like drastic growth of computational and dimensional requirement, complex algorithm and convergence characteristics.

The work done in this thesis named "Generation Scheduling Approach of Hydrothermal Power Systems" (GSAHTPS) develops a solution of short term hydrothermal scheduling problems; the solution approach is based on mixed integer linear programming (MILP). The IBM ILOG Cplex Optimization Studio is used to build linear programming codes; the Optimization Programming Language (OPL) in Cplex taken as modeling language for combinatorial optimization that may simplify the optimization problems substantially is used, this OPL provides support for modeling linear and integer programs and also provide access to state-of-the-art linear programming algorithm.

The generating power characteristics of both hydro and thermal power plants being non-linear, the piecewise linear approximations are used to solve this scheduling problems using IBM ILOG Cplex Optimization Studio. Three methods for piecewise linear approximation that are one dimension, triangle and rectangle method are developed in this thesis due to the non-linear output power characteristics of hydro and thermal power plants. In this research we mainly focus on the economic dispatch and unit commitment problems.

To assess the efficiency and the powerful performance of the proposed method; a typical case study of forty six thermal power plants from the standard of IEEE118 bus system is investigated with four additional hydropower plants connected in cascade.

The numerical results show that the solution techniques are computationally efficiency, simple and suitable for decision support of short term hydrothermal scheduling problems.

Keywords: Mixed Integer Linear Programming (MILP); Economic Dispatch (ED); Unit Commitment (UC) and short term hydrothermal scheduling.

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CHAPTER 1. INTRODUCTION

1.1 Background and meaning of the topic

The Generation Scheduling Approach of Hydrothermal Power Systems (GSAHTPS), it is the method used for solving the thermal unit commitment and generation dispatch by considering hydro schedules. The main objective is to minimize the thermal production cost, to meet the forecast demand and the operating constraints like power balance, water balance, reservoir volume limits, and the flow of water of the turbine as well as the operation limit of hydro and thermal unit. The generation scheduling plays an important role in power system operation and planning. The hydropower plants scheduling have many challenges than that of an all thermal power plants due to the fact that the hydropower plant can be coupled electrically and hydraulically[1].

The water releases during the operation of hydropower plants should be scheduled which in turn give rise to the long range hydro scheduling that goes from one week to one year or several years and the short range hydro scheduling which take one day to one week [2]. The long range involves meteorological and statistical analysis for a system with a capacity impounding water over several seasons; this scheduling is a very complex practical problem whose optimization contrasts the general economic objective with conflicting operative constraints, water management, energy market and regulatory matters. The stochastic dynamic program models are the most theoretically appealing solution for this problem and it has been widely used in long term scheduling[1], [3], [4].

In optimal economic operation of electrical power system through a given time period; the short term scheduling play an important role, typically the amount of water per hour needed for an hydro unit to generate the power is specified, in a given total demand; the hydro unit supply up to its maximum limit and the rest of energy that cannot be supplied by this plant is met by the thermal unit. Moreover the use of electrical energy by the human being is not constant but varies with time, in addition of this the time delay of water discharge from hydro unit have made the short term scheduling problem to become a complex issue and no linear [5].

In the recent decades, different suggestions have been considered for solving scheduling problem of hydrothermal generating units; various method have been suggested and developed such as gradient based search, lambda-gamma iteration, classical Linear Programming (LP), Lagrangian Relaxation (LR) and Dynamic Programming (DP), but those techniques has been applied for small and simplified problems [1]. DP can deal with the non- linear, non-convex and stochastic problems fortunately its main drawback is dimensionality [6]. To reduce the computational and memory pattern burden for dimensionality problem, multi-pass DP [7], differential DP [8], and Constructive DP (CDP) have been developed [9 – 10]. Benders decomposition [11] and its extension, stochastic dual DP has been applied for large scale hydrothermal scheduling[12]. Furthermore LR was the most popular technique used for large scale and complex problem of hydrothermal scheduling; however its efficiency was depending to the size and duality gap which made the implementation of this technique to become more complicated[13]. Apart the dynamic programming, other conventional methods usually require a certain assumption and simplifications to make the original model easy to be solved in this case the suboptimal solutions can contain great loss of revenue.

In the recent years, the drawbacks of the conventional system has been solved by several artificial intelligence techniques like gravitational search algorithm, particle swam optimization**,** neural network, simulated annealing, genetic algorithm[14 − 17]. Some artificial intelligence techniques, evolutionary computation has received more interest because of its ability to find the global optimum, flexibility in dealing with complex systems and simple in implementation. In [16 − 17] the simulated evolution is performed on the primal variable such as water releases and thermal generator outputs. Other proposed evolutionary techniques evolved the system marginal cost and the future benefit [18]as problem solutions, showing that there are other ways of coding the problem solutions.

The use of renewable energy such as hydro, wind, solar, and biomass; in nowadays, has become the main goal of different country in the world due to theirs advantages over the application of fossil fuels generation which are mostly limited by the increasing fuel cost and environment challenges such as the emission of C_{O2} which are the main component of the greenhouse effect.

1.2 Research contents and objectives

- The aims of this research will be focusing on ED, UC and scheduling approach of hydrothermal plants; analysis and mathematical models are built by using IBM ILOG CPLEX optimization Studio.
- To develop and study the performance of MILP in solving hydrothermal scheduling problem.
- Based on the development of the researcher’s country, Rwanda; the strategic plan of Economic Development and Poverty Reduction Strategy second phase (EDPRS II) in energy sector to increase the generation capacity from 117MW to more than 1000MW by 2017; using sources of energy like hydro, geothermal, methane gas peat and solar[19], this topic will contribute in coordination of generating units connected to the national power grid in Rwanda.

1.3 Significance of the study

The operating cost of hydroelectric plant is low but its capital cost is high while the operating cost of the thermal power plant is very high though their capital cost is low. To overcome problems by compensating the drawback of one system using another system in economic manner, it is convenient to have both thermal and hydro power plants units in the same power grid. The hydroelectric power plants can take up fluctuating loads due to its high reliability and greater speed of response; these plants can also be started quickly. To minimize the cost of thermal units and reduce the emission of Co_2 which are the main component of greenhouse effect, in this research we develop the coordination method of the hydro and thermal power plants by using modern and advanced software programming tools that are computationally efficient, fast and require less programming effort. This research will contribute to the government of Rwanda's plan decisions to increase the generation capacity and keep sustainable development and remarkable innovative solutions that can rely on solving challenges, protect environment and the use of renewable energy.

1.4 Organization of the Thesis

This thesis consists of six chapters which are organized as follow:

- The first chapter describes the general introduction; it summarizes the background and meaning of the topic, research contents and objectives and the organization of the thesis.
- The second chapter is based on the literature review of the topic; it highlights on the hydrothermal coordination, systematic coordination and operation of hydrothermal scheduling, need of hydrothermal coordination, classification of hydro plants and different scheduling techniques that has been used to solve the hydrothermal scheduling problems.
- The chapter three focuses on mathematical models formulation of MILP; it introduces the overview of the piecewise linear approximation of a nonlinear function, various methods to solve the piecewise linear approximation of a function of two variables are considered such as one dimensional, triangle, rectangle methods.
- The Chapter four highlights the problem formulation of the case study and the generation scheduling approach of hydrothermal power systems.
- The Chapter five represents the results and discussion from test of the system.
- The Chapter six represents the conclusion drawn and also presents the scope and future work.

CHAPTER 2. HYDROTHEMAL COORDINATION

2.1 Introduction on literature review

Different papers and report had been published in hydrothermal scheduling in the past years (mostly in 1980). After 1990, most of papers had been reviewed by many researchers. The review has been mainly focusing on the artificial intelligence methods developed with purpose to overcome the drawbacks provided by the conventional method and the consideration of various variables.

The literature review can be subdivided into two main group based on the technique used:

- The conventional techniques
- Artificial Intelligence techniques

However, based on the period considered during the hydrothermal scheduling techniques, the literature review can be roughly categorized into two classes:

- Short term hydro scheduling.
- Long term hydro scheduling.

Different research in this field of hydrothermal scheduling has been reviewed on the conceptual level based on the results, developments, and conclusions from the conventional research. Furthermore different techniques that have been used tend to analyze, evaluate and compare techniques that have been reported in the literature using one or several new sets of data.

2.2 Systematic coordination and operation of hydrothermal scheduling

Scheduling the hydro and thermal energy in the most economic manner has become an important task in modern power generation because of increasing competition in power market. The scheduling approach of a system of hydroelectric generation power plant is usually more complex than the scheduling of an all thermal generation system. This is due to the fact that the hydroelectric plants may very well be coupled electrically and hydraulically [1].

The Hydrothermal Coordination (HTC) problem can be taken as the operation planning of hydrothermal systems. The main aims of HTC problem is based on solving the thermal unit ED and UC as well as the hydro schedules; the target of this coordination system is to minimize the thermal production cost subjected to the varying load demand at a particular time and other operating constraints. The HTC can be solved by classifying the problem according to the period taken during the scheduling process; this will lead to the short term and long term scheduling approach.

The hydro system consists of two important basic aspects:

- The available water quantity is stochastic in nature.
- The decision for the energy allocated to the hydro units is deterministic.

2.3 Classification of hydro plant

The hydroelectric systems are different due to the natural differences in the watersheds, the differences in the manmade storage and release elements used to control the water flows; they can be grouped into the following categories , [20].

2.3.1 Classification on basis of manmade storage

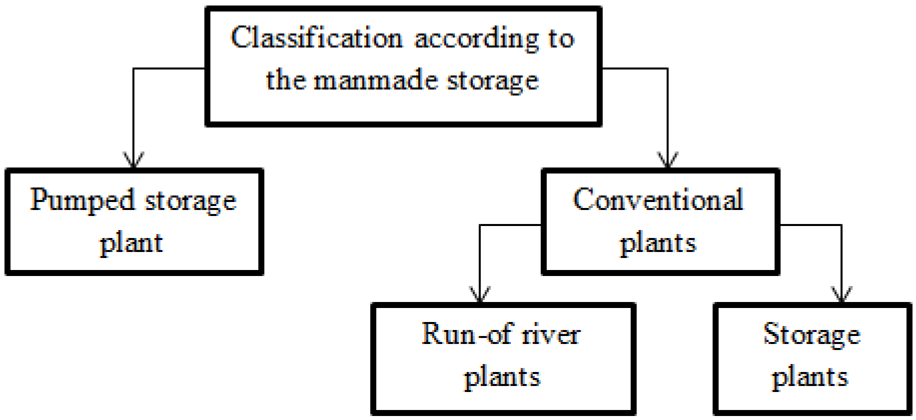

Fig. 2-1 Classification of hydro-plant based on manmade storage

The Fig. 2-1 shows the classification according to the manmade storage; the conventional storage consists of storage plants which are associated with the reservoirs which have significant storage capacity, during periods of lower power requirements, water can be stored and then released when the demand is high; the run-of river (ROR) have a small storage capacity and use water as it becomes available furthermore the water not utilized is spilled. The output of the ROR therefore depends heavily on river flow conditions and thus might not be optimally dispatched.

Pumped storage (P/S) plant which consists of an upper and lower reservoir is designed to save fuel costs by generating during peak load with water in the upper reservoir which would be pumped up during light load hours. The pumped storage plant is operated until the added pumping cost exceeds the saving in the thermal costs due to the peak sharing operation.

2.3.2 Classification on basis of natural differences in watersheds

The Fig. 2-2 shows the classification according to natural differences in watersheds.

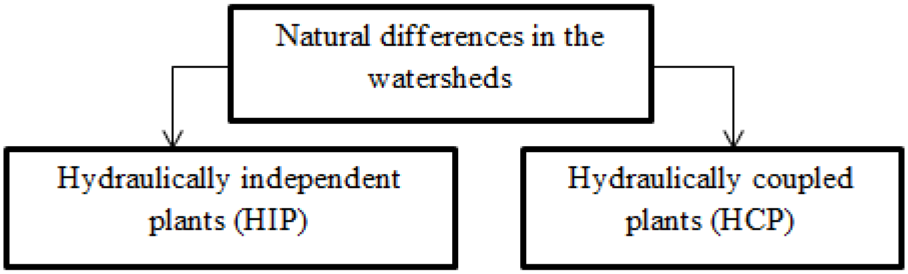

Fig. 2-2 Classification on basis of natural differences in watersheds

2.3.2.1 Hydraulically independent plants

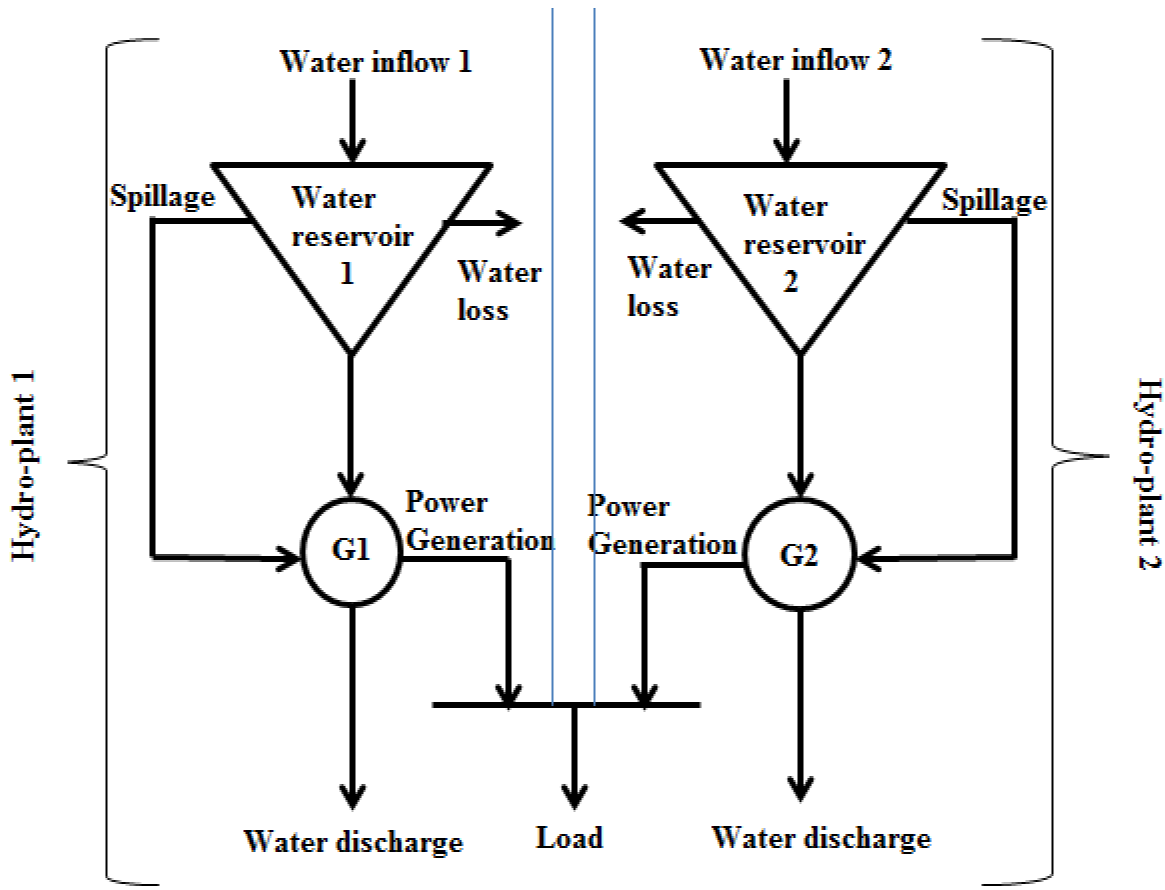

Fig. 2-3 Hydraulically independent plants

Two hydraulic plants are isolated from each other as it is shown in the schematic arrangement Fig. 2-3, but these plants are connected through the same electric network. G1 and G2 represent hydro turbine generators; in most cases these plants use to have same storage capacity. These two pants are said to be on different streams.

2.3.2.2 Hydraulically coupled plants

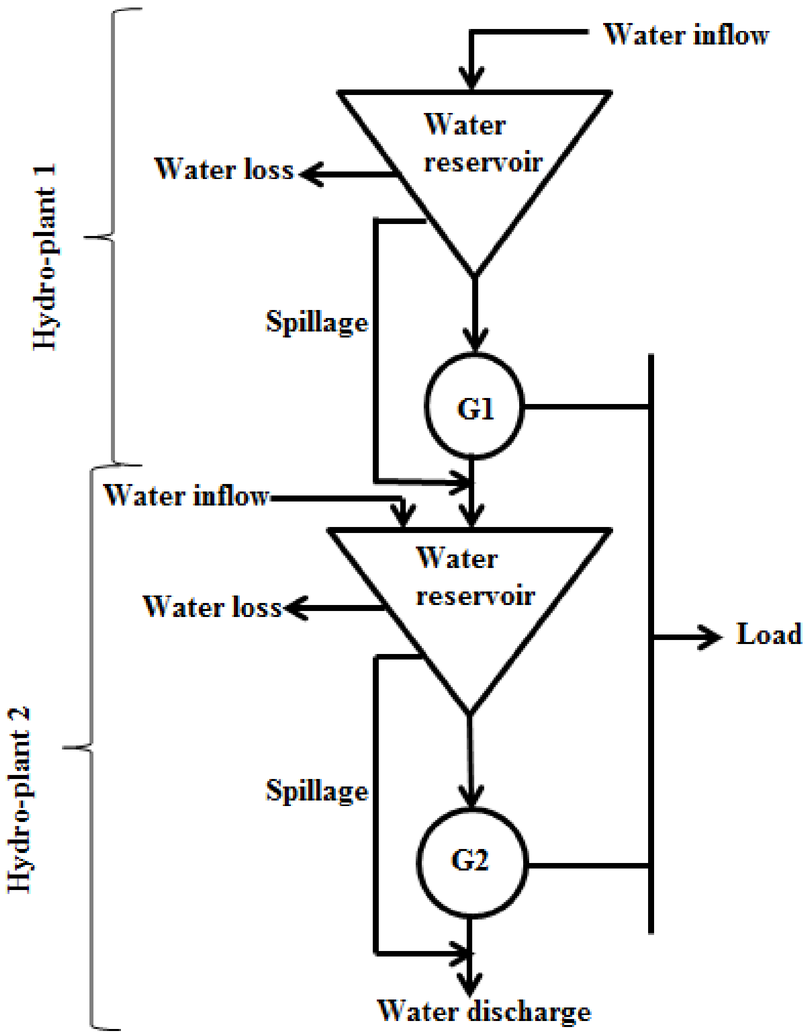

Fig. 2-4 hydraulically coupled plants

In hydraulically coupled plants (HCP) schematic; the water outflow from one plant may be very significant portion of the inflow to one or more other plants which are located downstream. Fig. 2-4 shows HCP schematic arrangement with G1 and G2 representing the hydro turbine-generators. The HIP and HCP are considered as two basic schematic arrangements of hydro plant unit， furthermore; we should mention that the scheduling process of these plants located on the same stream is more complex since both units are connected electrically and hydraulically.

2.3.2.3 Multi chain hydro plants configuration

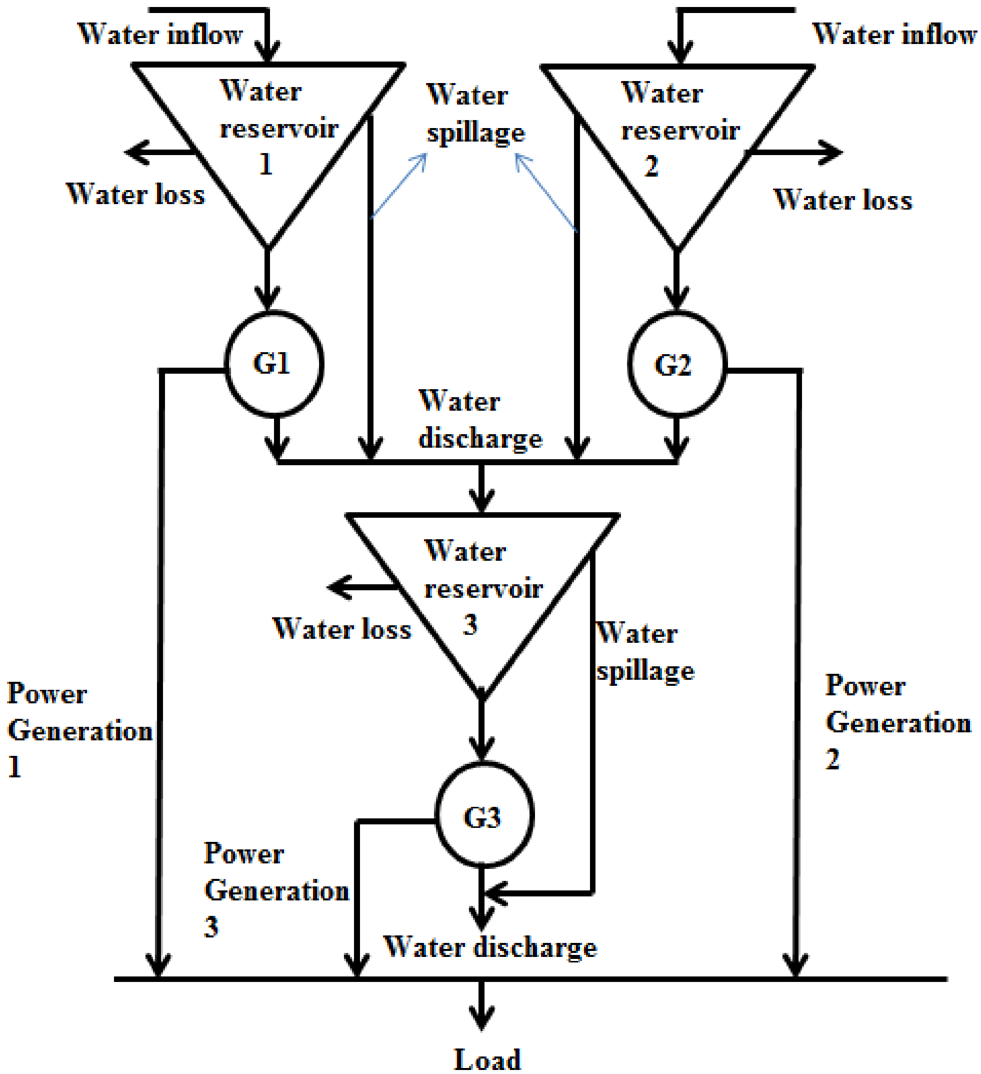

Fig. 2-5 Multi-chain hydro plants

Several combinations consisting of both HIP and HCP can be made; these combinations are taken as multi-chain hydro-plants, as it is depicted on Fig. 2-5.

2.4 Classification of hydrothermal scheduling problem

The hydrothermal scheduling problems can be classified into two main group based on the period taken during the scheduling process:

- Short term hydrothermal scheduling
- Long term hydrothermal scheduling

These scheduling problems in turn; consist of:

- Problem characteristics.
- Problem formulation
- Solution approach

2.4.1 Short term hydrothermal scheduling

The short term scheduling for hydrothermal unit is one of the most significant issue in economic operation of electrical power systems; this scheduling involves the hour by hour from one day to one week scheduling of all generation on a system in other to achieve minimum production cost for a given time period[1]. The amount of water per hour needed for generating power by hydro units should be specified. The main objective during the scheduling is to provide minimum production cost, however; this objective will be subjected to different constraints[5]. The most significant constraints subjected to the hydro and thermal units are:

- Power balance.
- Water balance.
- Water reservoir volume limit.
- Flow of water of the turbine.
- Characteristics operation limit of hydro and thermal units.

The short term hydrothermal scheduling problem is classified into two groups:

- Fixed head hydro thermal scheduling.
- Variable head hydrothermal scheduling.

In case of fixed head hydro thermal scheduling, the head of reservoirs can be assumed fixed if the hydro plants have reservoirs of large capacity while in variable head reservoir, the head of reservoir is variable if the hydro plants have reservoirs of small capacity.

In such scheduling problem; the load, hydraulic inflows, and unit availabilities are assumed to be known. A set of starting conditions such as reservoir levels are given, and the optimal hourly schedule that minimizes a desired objective, while meeting hydraulic steam, and electric system constraints, is sought. Part of the hydraulic contsraints may involve meeting “end-point” conditions at the end of the scheduling interval in order to conform to a long range, water release schedule previously established [39].

2.4.2 Long term hydrothermal scheduling

The long term scheduling uses a period of one week to one year or several years. For hydro schemes with a capacity of impounding water over several seasons, the long term problem involves meteorological and statistical analyses. It involves optimizing a policy in the context of unknowns such as load, hydraulic inflows, and unit availabilities; these unknown are treated statically and this will involves optimization of statistical variables[1] , [21].

In the long term planning level, an operating strategy must be calculated to define the proportion of hydro and thermal that will be used to meet the load in each month of the planning period. This is a complex problem since it is a large scale model, with stochastic variables and nonlinear equations. Dynamic programming (DP) is one of the most popular and successful technique mainly because it can handle well the stochastic and nonlinear issues of such problems. Still the applicable extension of DP techniques is limited on the real systems due to the fact that it requires strong computational efforts as this requirements increase exponentially with the number of state variables. Apart the DP, other useful techniques used to solve the long range hydro scheduling problems are:

- Composite hydraulic simulation models, which can represent several reservoirs.
- Statistical production cost models.

2.5 Hydrothermal scheduling techniques

The water discharge from the hydro turbine and the use of electrical energy by the human being is not constant but varies with time; this variation is not linear which have made the short term and long term scheduling problems to become more complex issue. Different techniques have been suggested for solving the scheduling problem of hydrothermal units; some of those techniques are enumerated in Table 2-1.

Table 2-1: Hydrothermal scheduling techniques

No	Conventional method	Artificial Intelligence techniques
1	Gradient based search[1]	Gravitational search algorithm[27][28]
2	Lambda-gamma iteration [1]	Particle swam optimization [29], [30], [31]
3	Classical linear programming [1]	Neural network[32]
4	Decomposition technique [22]	Simulated annealing[16]
5	Network flow [23], [24]	Genetic algorithm[33], [17], [20], [34]
6	Dynamic programming [1], [3], [4]	Honey Bee Optimization Algorithm[5], [35]
7	Lagrangian relaxation [1], [25], [26]	Fuzzy inference system approach [36]

The researcher emphasize to the useful techniques that have been mostly used by different authors to solve the hydrothermal scheduling problems.

- **Dynamic programming:**

The dynamic programming has been developed by Dr. Richard Bellman and his associates in 1950. This technique is useful in solving a variety of problems and can greatly reduce the computational effort in finding optimal trajectories or control policies[1] [3]. In the scheduling of power generation systems, DP techniques have been used in the following area:

- The economic dispatch of thermal system.
- The solution of hydrothermal economic scheduling.
- Practical solution of unit commitment.

In DP, the choice of the route is made in sequence; there are various stages traversed. The optimum sequence is called the optimal policy and the subsequence is called sub-policy, from this; it may be seen that the minimum cost route contains only sub-policies which give raise the theorem of optimality “an optimal policy must contain only optimal sub-policies” [1].

A policy is optimal if at a stated stage, whatever preceding decision may have been taken, the decision still to be taken constitutes an optimal policy when the result of the previous decision is included[1].

Lagrangian relaxation technique:

We can solve an optimization problem by using a technique that solves for Lagrange variables directly and then solves for the problem variables themselves. This formulation is known as a dual solution and in it the Lagrange multiplier are called dual variable[1]. Different researchers have used this technique to solve the thermal scheduling unit by using Lagrangian relaxation.

Lagrangian relaxation method offers a new approach for solving hydrothermal scheduling problem for large scale power systems. The basic idea of Lagrangian relaxation methods is to relax demand and reserve requirement using Lagrangian multipliers. The new problem called the dual problem can then be decomposed into the scheduling of individual thermal and hydro units[26]. The multipliers are then adjusted iteratively to maximize the dual function. The relaxation of the constraints causes a gap between the solution of primal and dual problems called dual gap. Therefore dual optimal solution rarely satisfies the power balance and reserve constraints. Hence it is necessary to search for a suboptimal feasible solution near the dual optimal point[1][26]. The method of search is an iterative process that solves relaxed sub-problems and updates Lagrangian multipliers according to the extent of violation of the power balance and spinning reserve constraints.

A comparative study of Lagrangian relaxation and dynamic programming methods for solving large scale hydrothermal coordination problems has been done[37]. In the Lagrangian relaxation approach, commitment states of the thermal units are obtained by solving only thermal sub-problems. In the dynamic programming approach, truncated dynamic programming is used to get the commitment states of the thermal units. An efficient hydrothermal scheduling algorithm is used to solve for output level of hydro units in both methods. The demand and spinning reserve constraints, the capacity limits, the minimum up and down time constraints, ramp rate and hydro constraints are considered in the problem formulation of both methods. Non-linear cost function and dispatches have been used and the transmission losses have been incorporated; those two methods were compared with speed of execution and operating cost by testing them on a practical utility system.

- **Genetic Algorithm:**

In optimal operation hydrothermal system in short term, the author tried to find the scheduling solution by using genetic algorithm. The hydrothermal coordination is solved by using an equivalent cost function for available thermal units, while the unit commitment and the economic dispatch are solved considering the more capable individuals that are obtained from the hydrothermal coordination stage [34].

The hydrothermal generation scheduling has been decomposed into sub- problems which are the hydrothermal coordination problem (HCP), unit commitment problem (UCP) and economic dispatch problem (EDP). The economic dispatch and unit commitment was solved in other to minimize the costs of the thermal generation subjected to the operative restrictions. Mathematically, the HGSP is a high dimensionality nonlinear optimization; it consists of continuous and discrete variables, including equality and inequality constraint. Due to the discrete representation of thermal units states, ON and OFF, the formulation results in a non- convex optimization problem, hindering the employment of conventional techniques of solution. To approach this inconvenience several numerical optimization tools have emerged; among these, can stand out the dynamic programming Lagrange techniques, the semi definite programming, and genetic algorithm.

- **Particle Swarm Optimization:**

Among the entire stochastic search algorithm, the Particle Swarm Optimization (PSO) gives the reasonable solution in a short CPU time[38]. The PSO was first introduced by Kennedy and Eberhart in 1995 for nonlinear continuous optimization problems. The application of PSO technique into the linear discontinuous constrained power system such as Economic Dispatch (ED), Unit Commitment (UC), state estimation (SE), load forecasting (LF), Optimal Power Flow (OPF) etc., have been reported by several authors.

PSO is initialized with a group of random particle and then searches for optima by updating generation. Every iteration particle is updated by two best values:

- Based on its own best exploration called pbest
- Based on best swarm overall experience called gbest

After finding two best values, the particle velocity and position are updated by using equation.

- **Scheduling by Gravitational Search algorithm:**

The Gravitational search algorithm (GSA) is one of the heuristic optimization algorithms based on law of gravity and mass interaction; it can be also applied to solve the problem of short term hydrothermal scheduling system. It has been developed by Rashedi et Al. in 2009[28]. GSA is followed by the physical law of gravity and law of motion. In the proposed algorithm[27], agents are considered as objects and their performance is measured by their masses. All these objects attract each other by their gravitational force, and this force causes a global movement of all objects towards the object with heaviest masses. Hence, each mass will be informed through gravitational force and updated by exchanging the information.

- **Neuro Fuzzy Approaches:**

In Fuzzy inference systems approach for long term hydrothermal Scheduling; the author proposed an alternative technique to deal with the long term hydro scheduling problem. The proposed approach is based on an adaptive Neuro Fuzzy Interference System (ANFIS) technique[36]. The problem is first solved through a determinist optimization model with a project inflow forecast and the data of monthly optimal operation is used to train the Neuro Fuzzy System. After the training process, the network produces a Fuzzy rule based interference system that will reproduce the optimal behavior through the amount of water discharge at each stage.

The Neuro Fuzzy approaches have been successfully used in nonlinear system modeling and parameter estimation process control and time series forecasting. Fuzzy logic has been used in optimal operation rules (OOR) for coupled operation of hydroelectric power plants. In the hydrothermal scheduling problem this approach allows the representation of hydro plant individually including its operation constraints and nonlinear production characteristics, since it is a production of the optimal deterministic model, moreover, it permits a better interpretation of the results and the physical relation of the variables as its model consists of Fuzzy rules of the type IF <antecedent> then<consequent>[38].

CHAPTER 3. MATHEMATICAL MODELS FORMULATION OF MIXED INTEGER LINEAR PROGRAMMING

3.1 Introduction

In the recent years, the increased efficiency of MILP software tools has encouraged their use also in the solution of non-linear problems, bringing to the need for efficient techniques to linearize non-linear functions of one or more variables. The standard methodologies consist in the piecewise linear approximation of such functions.

For a function of single variable, let us assume a function $f(x)$, as it is depicted in the Fig. 3-1; the piecewise linear approximation are obtained by introducing the breakpoints on x axis, we have the coordinates $x_1, \ldots, x_n$. The function is then approximated by the linear segments $\left[\left(x_i, f\left(x_i\right)\right), \left(x_{i+1}, f\left(x_{i+1}\right)\right)\right]$ for all $(i = 1, \ldots, n-1)$. For any given x value, say $\overline{x}$ if $x_i \leq \overline{x} \leq x_{i+1}$; the function value is approximated by convex combination of $f\left(x_i\right)$ and $f\left(x_{i+1}\right)$; let λ be the value in the interval $\left[0,1\right]$ such that:

$$\overline{x} = \lambda x_i + (1-\lambda) x_{i+1} \quad \text{(3-1)}$$

The approximated value is:

$$f^a(\overline{x}) = \lambda f(x_i) + (1-\lambda) f(x_{i+1}) \quad \text{(3-2)}$$

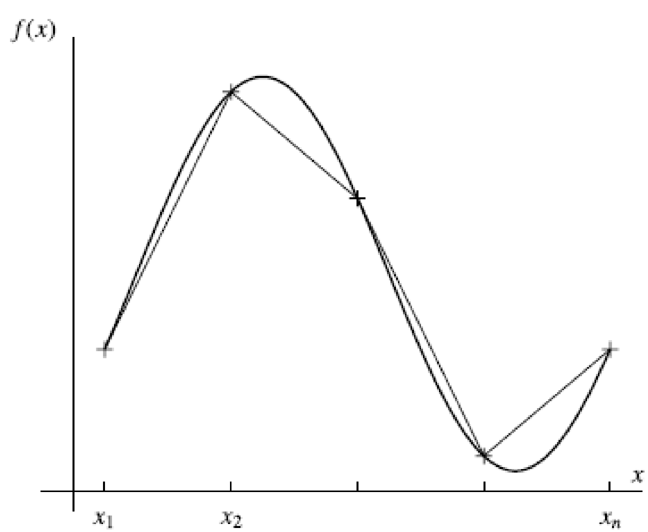

Fig. 3-1: Piecewise linear approximation of non-linear function

This methodology can be alternatively be described through the slope:

$$\frac{f(x_{i+1})-f(x_i)}{x_{i+1}-x_i}$$

The interpolating function will be:

$$f^a(\bar{x}) = f(x_i) + (\bar{x}-x_i)\frac{f(x_{i+1})-f(x_i)}{x_{i+1}-x_i} \quad \text{(3-3)}$$

From the (3-3), the equation (3-1) has:

$$\lambda = \frac{x_{i+1}-\bar{x}}{x_{i+1}-x_i} \quad \text{(3-4)}$$

In order to use the above technique in MILP solver, it is necessary to include in the model variables and constraints that force any x value to be associated with the proper pair of consecutive breakpoints or with single one, in case $x \in \{x_1,\ldots,x_n\}$. Let us introduce a continuous variables $\alpha_i \in [0,1]$ with $(i=1,\ldots,n)$. Let h_i be a binary variable associated with the i[th] interval $[x_i, x_{i+1}]$ with $(i=1,\ldots,n-1)$, with dummy values $h_0 = h_n = 0$ at the extremes. The approximated value f^a can then be obtained by imposing the following constraints:

$$\sum_{i=1}^{n-1} h_i = 1 \quad \text{(3-5)}$$

$$\alpha_i \le h_{i-1} + h_i \text{ With } (i=1,...,n) \quad \text{(3-6)}$$

$$\sum_{i=1}^{n} \alpha_i = 1 \quad \text{(3-7)}$$

$$x = \sum_{i=1}^{n} \alpha_i x_i \quad \text{(3-8)}$$

$$f^a = \sum_{i=1}^{n} \alpha_i f(x_i) \quad \text{(3-9)}$$

The constraint (3-5) impose that only one h_i , say $h_{\bar{i}}$, takes the value 1. Hence, constraints (3-6) impose that the only α_i values different from 0 can be α_i and α_{i+1} . It follows from (3-7) and (3-8) that $\alpha_i = \lambda$ and $\alpha_{i+1} = 1-\lambda$ (see (1)). Constraint (3-9) ensures then the correct computation of the approximate value according to (3-2).

In the contexts of this type, the MILP constraints can be simplified by the so called special order sets (SOS), introduced by Beale and Tomlin [40], and extensively studied in [41- 43].

3.2 Piecewise linear approximation of a functions of two variables

We describe three techniques for the piecewise linear approximation of a function of two variables, these are:

- One dimensional method.
- Triangle method.
- Rectangle method.

3.2.1 One dimensional method

Let us introduce a number m of coordinates on the y axis, $y_1,\ldots,y_m$. For j[th] interval $\left[y_j, y_{j+1}\right)$, let $\hat{y}_j$ be the associated sampling coordinate, leading to $m-1$ variable functions $f(x,\hat{y}_j)$ with $(j=1,..,m-1)$. For any given value, say $\bar{y}\in[y_j, y_{j+1})$, the approximated function values $f^a(x,\bar{y})$ are then given by the piecewise linear approximation of $f(x,\hat{y}_j)$ with breakpoints $x_1,\ldots,x_n$ as it is depicted in Fig. 3-2. In this case we assume that the sampling coordinate is the left extreme of interval, which means $\hat{y}_j = y_j$; in this way the approximating function agrees with the given function at the breakpoints. In practical applications; it can often be preferable to use the central point of the interval as the sampling coordinate, thus loosing such property.

Let $\beta_1,\ldots,\beta_{m-1}$ be the binary variables, defined as SOS1, with β_j taking the value 1 if and only the given value $\bar{y}$ belongs to $[y_j, y_{j+1})$. The approximated value f^a is then obtained through (3-7) and (3-8) in the following way:

$$y \le \sum_{j=1}^{m-1} \beta_j y_{j+1} \quad \text{(3-10)}$$

$$y \ge \sum_{j=1}^{m-1} \beta_j y_j \quad \text{(3-11)}$$

$$\sum_{j=1}^{m-1} \beta_j = 1 \quad \text{(3-12)}$$

$$f^a \le \sum_{i=1}^{n} \alpha_i f(x_i, y_j) + M(1-\beta_J) \text{ With } (j=1,\ldots,m-1) \quad \text{(3-13)}$$

$f^a \geq \sum_{i=1}^{n} \alpha_i f(x_i y_j) - M(1-\beta_J)$ With $(j = 1,\ldots,m-1)$ ....(3-14)

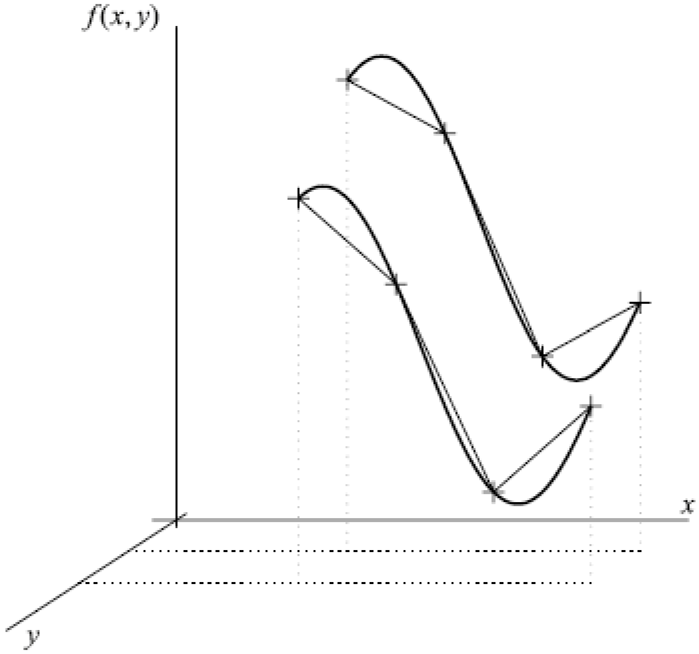

Fig. 3-2 One dimension method

Where α the SOS2 is introduced in previous section and M is a very large value. Constraint (3-10)-(3-12) impose $\beta_j = 1$ and $\beta_j = 0$, for $j \neq \bar{j}$, $\bar{j}$ being the interval containing y. Constraints (13) and (14) are inactive if $\beta_j = 0$ hence providing $f^a = \sum_{i=1}^{n} \alpha_i (f(x_i, \hat{y}_j)$ for the correct interval $\bar{j}$.

The model (3-10) to (3-14), for a given x value, f^a can take two value for $y = y_{\bar{j}}$, as either β_{j-1} or β_j can equivalently take the value. Although this can be inessential in practice, such theoretical drawback can be corrected by replacing (3-10) with $y \leq \sum_{i=1}^{m-1} \beta_j y_{j+1} - 2\theta$, where θ is the feasible tolerance of linear constraints in the specific MILP solver.

3.2.2 Triangle Method

A more complex method can be obtained by extending the one variable technique to the two variable cases. Consider again n sampling coordinates $x_1,\ldots,x_n$ on the x axis and m sampling coordinates $y_1,\ldots,y_m$ on the y axis, the function $f(x,y)$ is evaluated for each breakpoint (x_i, y_j)

$(i=1,\ldots,n)$ and $(j=1,\ldots,m)$. For any given (x,y) point, say $(\bar{x},\bar{y})$, with $x_i \le \bar{x} \le x_{i+1}$ and $y_j \le \bar{y} \le y_{j+1}$, let us consider the rectangle shown in Fig.3-3.a of vertices (x_i, y_j), (x_{i+1}, y_j), (x_{i+1}, y_{j+1}), (x_i, y_{j+1}) and two triangles produced by its diagonal $\left[(x_i,y_j)(x_{i+1},y_{j+1})\right]$.The function value is approximated by convex combination of function values evaluated at the vertices of the triangle containing $(\bar{x},\bar{y})$. As it is shown on Fig.3-3.b.

$$f^a(\bar{x},\bar{y}) = \lambda f(x_i,y_j) + \mu f(x_{i+1},y_{j+1}) + (1-\lambda-\mu)\bar{f} \quad \text{(3-15)}$$

With

$$\bar{f} = f(x_{i+1},y_j) \text{ if } \bar{y} \le y_j + (\bar{x}-x_i)\frac{y_{j+1}-y_j}{x_{i+1}-x_i} \quad \text{(3-16)}$$

Or $\bar{f} = f(x_i, y_{j+1})$ otherwise

Where $\lambda \in [0,1]$, $\mu \in [0,1]$ and $(1-\lambda-\mu) \in [0,1]$ are the weight of the convex combination of the vertices of the appropriate triangle which contains $(\bar{x},\bar{y})$.

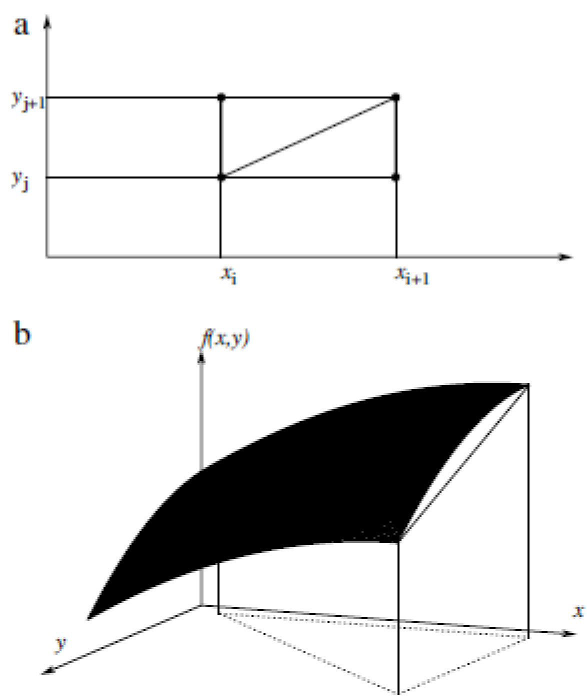

Fig. 3-3 Geometric representation of the triangle method

In MILP model, (3-14) and (3-15) are implemented by introducing $n\ m$ continuous variable $\alpha_{ij} \in [0,1]$ (one per breakpoint), and computing the convex combination by extending (3-7)-(3-9) to three -dimensional space as follows:

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\alpha_{ij} = 1 \qquad (3\text{-}17)$$

$$x = \sum_{i=1}^{n}\sum_{j=1}^{m}\alpha_{ij} x_i \qquad (3\text{-}18)$$

$$y = \sum_{i=1}^{n}\sum_{j=1}^{m}\alpha_{ij} y_j \qquad (3\text{-}19)$$

$$f^a = \sum_{i=1}^{n}\sum_{j=1}^{m}\alpha_{ij} f(x_i, y_j) \qquad (3\text{-}20)$$

Variables α_{ij} should be defined as specific SOS3. However, differently from what happens for SOS1 and SO2, current MILP solver do not have an automatic syntax to impose an SOS3, so far the sake of completeness, we give here the analogue of constraints (3-5) and (3-6), to be added to the

Consider the rectangle corresponding to the intervals $[x_i, x_{i+1})$ and $[y_j, y_{j+1})$, we associate binary variables h_{ij}^u and h_{ij}^l, respectively, to the upper and lower triangle in rectangle shown in Fig. 3-3.a, The additional constraints are then:

$$\sum_{i=1}^{n-1}\sum_{j=1}^{m-1}(h_{ij}^u + h_{ij}^l) = 1 \qquad (3\text{-}21)$$

$$\alpha_{ij} \le h_{ij}^u + h_{ij}^l + h_{i,j-1}^u + h_{i-1,j-1}^l + h_{i-1,j-1}^u + h_{i-1,j}^l \qquad (3\text{-}22)$$

With $(i = 1,\ldots,n;\ j = 1,\ldots,m)$

The constraint (3-21) imposes that, among all triangles, only one is used for the convex combination. Then, constraints (3-22) impose that the only α_{ij} values different from 0 can be those associated with the three vertices of such triangle.

Formulation (3-17)-(3-22), known as lambda 9 (or convex) combination [41], is probably the most popular piecewise linear approximation technique for a functions of two variables. Different formulations of the triangle method have been introduced in [44, 46-48].

3.3.3 Rectangle method

Let us introduce n coordinates $x_1, \ldots, x_n$ on the x axis and m sampling coordinates $y_1, \ldots, y_m$ on the y axis. For any given y value, say, $\bar{y} \in [y_j, y_{j+1})$, instead of associating a prefixed $\hat{y}_j$ to interval $[y_j, y_{j+1})$ as in the one dimensional method, we use the piecewise linear approximation of $f(x, y_j)$ with a linear correction depending on $\bar{y}$. More precisely, for a given point $(\bar{x}, \bar{y})$, with $\bar{x} \in [x_i, x_{i+1})$, the approximate value $f^a(\bar{x}, \bar{y})$ is given by:

$$f^a(\bar{x}, \bar{y}) = \lambda f(x_i, y_j) + (1-\lambda) f(x_{i+1}, y_j) + \delta \min\{\Delta(i, j), \Delta(i+1, j)\} \quad \text{....(3-23)}$$

With

$\Delta(l, j) = f(x_l, y_{j+1}) - f(x_l, y_j)$ And $\delta = (\bar{y} - y_j)/(y_{j+1} - y_j)$. The function is evaluated using the rectangle a, b, c, r that lies on the plane passing through vertices $f(x_i, y_j)$, $f(x_{i+1}, y_j)$ and $f(x_i, y_{j+1})$ as shown in Fig. 3-4.

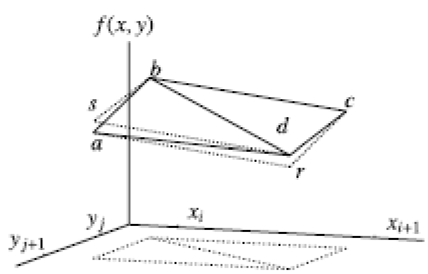

Fig. 3-4 Geometric representation of the rectangle method

If instead of the minimum in (3-23) occurs for $\Delta(i+1, j)$, rectangle *sbcd* is used for approximation.

We finally observe (3-23) produces an underestimate with respect to the triangle approach, due to the "min" operator in the third term. If according to the specific application, overestimate is desired, it is enough to substitute it with "max' operator.

Let us now consider how the method can be modeled within a MILP. As in (3-10)-(3-14), let $\beta_1, \dots, \beta_{m-1}$ be a SOS1 with β_j taking the value 1 if $\bar{y}$ belongs to $[y_j, y_{j+1})$, and the value 0 otherwise. In addition let $\gamma_1, \dots, \gamma_{m-1}$ be continuous variables, taking values in the interval [0, 1]. If $\bar{y} \in [y_j, y_{j+1})$ then $(\bar{y} - y_j)/(y_{j+1} - y_j)$, and $\gamma_k = 0$ for all $k \neq j$. In other words, when y lies on the j[th] interval, γ_j represents the relative position of y within the j[th] interval. The approximate value f^a is then given by:

(3-5),(3-6),(3-7),(3-8) and (3-9)

$$y = \sum_{j=1}^{m-1} (\beta_j y_j + \gamma_j (y_{j+1} - y_j)) \quad \text{(3-24)}$$

$$\gamma_j \le \beta_j \text{ for all } j = 1, \dots, m-1 \quad \text{(3-25)}$$

$$f^a \le \sum_{k=1}^{n} \alpha_k f(x_k, y_j) + \gamma_j K_{ij} + M(2 - \beta_j - h_i) \quad \text{(3-26)}$$

For all $(j = 1, \dots m-1; i = 1, \dots, n-1)$

$$f^a \ge \sum_{k=1}^{n} \alpha_k f(x_k, y_j) + \gamma_j k_{ij} - M(2 - \beta_j - h_i) \quad \text{(3-27)}$$

For all $(j = 1, \dots m-1; i = 1, \dots, n-1)$

Where $K_{ij} = \min\{\Delta(i, j), \Delta(i+1, j)\}$ as it is shown in (3-23). Due to the above definition of the β_j and γ_j variables, equations (3-24) and (3-25) impose that y is given by the unique non zero term of the summation. Equations (3-26) and (3-27) are inactive when $\beta_j = 0$ or $h_i = 0$, hence providing $f^a \ge \sum_{k=1}^{n} \alpha_k f\left(x_k, y_j\right) + \gamma_j K_{ij}$ for the correct interval. In this case too, for a given x value f^a can take two value for $y = y_{\bar{j}}$, if such that behavior is undesired, one can replace (3-24) with $y = \sum_{j=1}^{m-1} (\beta_j y_j + \gamma_j \left(y_{j+1} - y_j - 2\theta\right))$ where θ is a feasible tolerance.

Note that in order to have a constraint matrix with few non zero elements, one can efficiently reformulate constraints (3-26) and (3-27) by replacing the first term of the right hand side with a corresponding variable, say, φ_j, at the price of the addition of $m-1$ constraints as shown in (3-28). The final formulation is:

(3-5),(3-6),(3-7),(3-8),(3-12),(3-24),(3-25)

$$\varphi_j = \sum_{k=1}^{n} \alpha_k f(x_k, y_j) \quad \text{......(3-28)}$$

For all $(j = 1, \ldots, m-1)$

$$f^a \le \varphi_j + \gamma_j K_{ij} + M(2 - \beta_j - h_i) \quad \text{......(3-29)}$$

For all $(j = 1, \ldots, m-1; i = 1, \ldots n-1)$

$$f^a \ge \varphi_j + \gamma_j K_{ij} - M(2 - \beta_j - h_i) \quad \text{......(3-30)}$$

For all $(j = 1, \ldots, m-1; i = 1, \ldots n-1)$

3.3 Technique adopted for building a wide project in IBM Cplex Ilog Optimization Studio

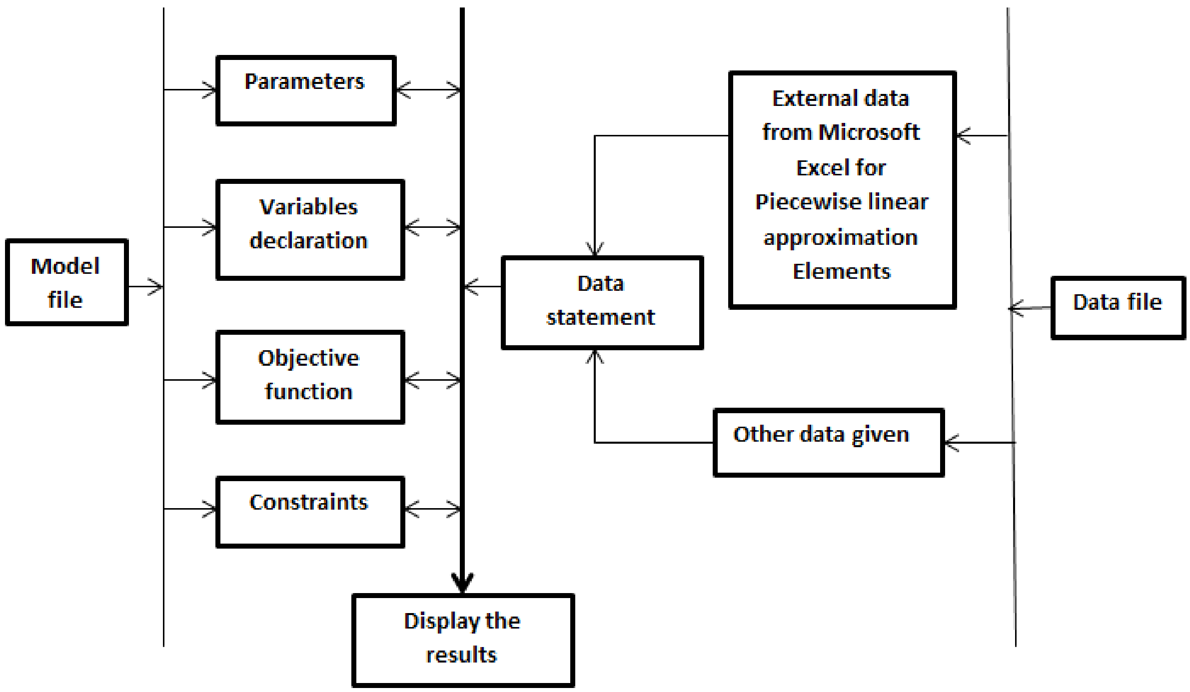

Fig. 3-5 Schematic arrangement for building a model in IBM Ilog Cplex Optimization studio using the Optimization Programming Language (OPL)

The Fig. 3-5 depicts the schematic arrangement technique used to build wide projects such as the scheduling of hydrothermal power plants using IBM Ilog Cplex Optimization Studio. Two files are created, one for model statement and another for data. External data from Microsoft excel are connected to the optimization software by the use of the appropriate codes. The

Microsoft Excel being a good tool that can be easily used to write equation and formula; this leads to execute less programming effort.

CHAPTER 4. PROBLEM FORMULATION OF THE CASE STUDY AND GENERATION SCHEDULING APPROACH OF HYDROTHERMAL POWER SYSTEM

4.1 Problem formulation

In different power systems, the use of hydroelectric and thermal power plants are widely used but the economic scheduling of this system is a serious problem due to the scheduling of water releases to satisfy all the hydraulic constraints and meet the demand for electrical energy.

The hydrothermal systems where the hydroelectric system is taken as the largest generation unit may be scheduled by economically scheduling the system to produce the minimum cost for the thermal system. These are basically the problem in scheduling energy.

4.2 Objective function

$$MinF_T = \sum_{t=1}^{T}\sum_{j=1}^{N_S} F_j(P_{Sj}(t)) \qquad (4\text{-}1)$$

F_T : Total thermal production cost function

$P_{Sj}(t)$: Power generation by the thermal unit j at time t

N_s : Number of thermal units

T : Number of time intervals

The thermal production cost function is expressed as a quadratic function as follow:

$$F_j(P_{Sj}) = a_j + b_j P_{Sj} + c_j P_{Sj}^2 \qquad (4\text{-}2)$$

a_j, b_j and c_j : cost coefficient of thermal unit j

4.3 Constraints function

In other to operate the thermal unit at a minimum operating cost, the objective function represented by (4-1) will be subjected to several operating constraints such as:

- **Load balance:**

$$\sum_{j=1}^{N_H} P_{Hj}(t) + \sum_{j=1}^{N_S} P_{Sj}(t) - (P_D(t) + P_L(t)) = 0 \qquad (4\text{-}3)$$

N_H : Number of hydro units

$P_{Hj}(t)$: Output power of hydro unit j at time interval t

$P_D(t)$: Load demand at time interval t

$P_L(t)$: Total losses at time interval t

The output power of hydro unit will be expressed as a function of water discharge rate and the reservoir storage:

$$P_{Hj}(t) = f(q_j(t), V_j(t)) \quad (4\text{-}4)$$

$V_j(t)$: Storage volume of the reservoir j at time interval t

q_j : Water discharge for hydro unit j during the time interval t

- **Power generation limit for hydro and thermal units**

$$P_{Hj}^{\min} \le P_{Hj}(t) \le P_{Hj}^{\max} \quad (4\text{-}5)$$

P_{Hj}^{min} and P_{Hj}^{max} : minimum and maximum output power of hydro unit j.

$$P_{Sj}^{\min} \le P_{Sj}(t) \le P_{Sj}^{\max} \quad (4\text{-}6)$$

P_{Sj}^{min} and P_{Sj}^{max} : Minimum and maximum output power of thermal unit j.

- **Water balance:**

$$Q_{Totj} = \sum_{t=1}^{T} q_j(t) \quad (4\text{-}7)$$

Q_{Totj} : Total water discharge

- **Hydraulic continuity:**

$$V_j(t-1) + n_t\left[r_j(t) - q_j(t) - s_j(t)\right] = V_j(t) \quad (4\text{-}8)$$

$r_j(t)$: Water inflow into the reservoir j during the time interval t.

$s_j(t)$: Water spillage of the reservoir j during the time interval t.

n_t : The length of the time interval t.

➢ **Water storage limit**

The volume limit should be specified such that:

$$V_j^{\min} \le V_j(t) \le V_j^{\max} \qquad (4\text{-}9)$$

V_j^{min} and V_j^{max} : Minimum and maximum volume of the reservoir j.

The starting volume and ending volume should also be specified in the following way:

$$V_j(t)\big|_{t=0} = V_s \qquad (4\text{-}10)$$

$$V_j(t)\big|_{t=T} = V_E \qquad (4\text{-}11)$$

V_s : Starting volume

V_E : Ending volume

➢ **Water discharge limit**

The water flow should be kept in the following range:

$$q_j^{\min} \le q_j(t) \le q_j^{\max} \qquad (4\text{-}12)$$

q_j^{min} and q_j^{max} : Minimum and maximum discharge respectively.

The water discharge should be also fixed at particular time t such that:

$$q_j(t) = Q_j(t) \qquad (4\text{-}13)$$

4.4 Case study for generation scheduling approach of hydrothermal power system

The particular case study will deals with the GSAHTPS with a hydraulically coupled plant (HCP). The hydrothermal generation system consists of forty six thermal generating units and four hydro power plants. The scheduling method considered will be the short thermal scheduling approach which leads to the scheduling horizon of twenty four hours with one hour time interval. The thermal unit characteristics are shown on the Table 4-1; load demands and the hydropower plants characteristics are also provided in Table 4-2 and Table 4-3 respectively; furthermore thermal and hydro units output power value all are given in MW.

Table 4-1: Thermal unit data from the IEEE 118 BUS system

T. Unit	Bus No.	$\mathbf{a}/(\$/h)$	$\mathbf{b}/(\$/MWh)$	$\mathbf{c}/(\$/MW^2h)$	$\mathbf{P}_{\mathbf{Sj}}^{\mathbf{min}}$	$\mathbf{P}_{\mathbf{Sj}}^{\mathbf{max}}$	Ramp Up	Ramp Down
1	4	31.67	26.2438	0.069663	5	30	15	15
2	6	31.67	26.2438	0.069663	5	30	15	15
3	8	31.67	26.2438	0.069663	5	30	15	15
4	10	6.78	12.8875	0.010875	150	300	150	150
5	12	6.78	12.8875	0.010875	100	300	150	150
6	15	31.67	26.2438	0.069663	10	30	15	15
7	18	10.15	17.82	0.0128	25	100	50	50
8	19	31.67	26.2438	0.069663	5	30	15	15
9	24	31.67	26.2438	0.069663	5	30	15	15
10	25	6.78	12.8875	0.010875	100	300	150	150
11	26	32.96	10.76	0.003	100	350	175	175
12	27	31.67	26.2438	0.069663	8	30	15	15
13	31	31.67	26.2438	0.069663	8	30	15	15
14	32	10.15	17.82	0.0128	25	100	50	50
15	34	31.67	26.2438	0.069663	8	30	15	15
16	36	10.15	17.82	0.0128	25	100	50	50
17	40	31.67	26.2438	0.069663	8	30	15	15
18	42	31.67	26.2438	0.069663	8	30	15	15
19	46	10.15	17.82	0.0128	25	100	50	50
20	49	28	12.3299	0.002401	50	250	125	125
21	54	28	12.3299	0.002401	50	250	125	125
22	55	10.15	17.82	0.0128	25	100	50	50
23	56	10.15	17.82	0.0128	25	100	50	50
24	59	39	13.29	0.0044	50	200	100	100
25	61	39	13.29	0.0044	50	200	100	100
26	62	10.15	17.82	0.0128	25	100	50	50
27	65	64.16	8.3391	0.01059	100	420	210	210
28	66	64.16	8.3391	0.01059	100	420	210	210
29	69	6.78	12.8875	0.010875	80	300	150	150
30	70	74.33	15.4708	0.045923	30	80	40	40
31	72	31.67	26.2438	0.069663	10	30	15	15
32	73	31.67	26.2438	0.069663	5	30	15	15
33	74	17.95	37.6968	0.028302	5	20	10	10
34	76	10.15	17.82	0.0128	25	100	50	50
35	77	10.15	17.82	0.0128	25	100	50	50
36	80	6.78	12.8875	0.010875	150	300	150	150
37	82	10.15	17.82	0.0128	25	100	50	50
38	85	31.67	26.2438	0.069663	10	30	15	15
39	87	32.96	10.76	0.003	100	300	150	150

40	89	6.78	12.8875	0.010875	50	200	100	100
41	90	17.95	37.6968	0.028302	8	20	10	10
42	91	58.81	22.9423	0.009774	20	50	25	25
43	92	6.78	12.8875	0.010875	100	300	150	150
44	99	6.78	12.8875	0.010875	100	300	150	150
45	100	6.78	12.8875	0.010875	100	300	150	150
46	104	10.15	17.82	0.0128	25	100	50	50

Table 4-2: Hourly load demand

Hours	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Load	4620	4356	3828	2640	3300	3960	4620	5148	5412	5808	5874	5544
Hours	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Load	5280	5016	5808	5940	5610	5874	6204	6468	6600	5940	5742	5412

Table 4-3: Hydro power plants data

Unit	q_j^{min}	q_j^{max}	V_s	V_j^{Low}	V_j^{Up}	V_j^{min}	V_j^{max}	$r_j(t)$	**Start Up Cost**
1	2	62	100	152	200	6	225	0.051	110
2	5	163	80	100	150	6	162	0.058	150
3	14	464	790	500	1000	6	1200	0.603	200
4	19	662	33	50	60	6	66	0.051	250

Four hydro power plants are connected in cascade, the time delay between two successive plants are ignored in the scheduling problem for simplicity of the problem. These plants characteristics have been given in the reference [49].

V_j^{Low}: Lower level of the content of reservoir associated with a given plant j in the discretization of the performance curve.

V_j^{Up}: Upper level of the content of reservoir associated with a given plant j in the discretization of the performance curve.

Table 4-4: Piecewise linear approximation of the performance curve of hydropower plants

Unit	S_{1j}	S_{2j}	S_{3j}	S_{4j}	q_l^{max}
1	0.80	0.30	0.20	0.10	15
2	0.40	0.30	0.50	0.10	39.50
3	0.20	0.10	0.30	0.20	112.50
4	0.10	0.10	0.05	0.05	160.75

where S_{1j}, S_{2j}, S_{3j} and S_{4j} are slopes respectively for the lower level content of the performance content. In this case study a piecewise linear approximation with four blocks has been implemented. The performance curve for the intermediate and high level contents are obtained by adding 0.05 and 0.1 respectively to each slope in Table 4-4.

With P_{Hj}^{1}, P_{Hj}^{2} and P_{Hj}^{3} minimum power of lower, intermediate and upper level respectively of the performance curve of a given plant j as shown in Table 4-5.

Table 4-5: Power output limits of hydropower plants

Unit	P_{Hj}^{1}	P_{Hj}^{2}	P_{Hj}^{3}	P_{Hj}^{max}
1	1.440	1.530	1.62	28.62
2	1.896	2.133	2.37	69.52
3	2.7	3.375	4.05	60.05
4	1.929	2.894	3.858	40.38

4.5 Important consideration in the analysis of the case study

In the problem formulation; it is clear that the objective function is a function of output power of the thermal unit; in addition of this the output power of hydro unit is a function of water discharge rate and reservoir volume storage, both functions are non-linear problem (NLP) which are more complex to be solved. Furthermore; the author will use the IBM ILOG Cplex optimization Studio to solve GSAHTPS while this optimization software is used to solve the optimization of linear problems (LP); in this case study it is convenient to convert the NLP function into piecewise linear function.

4.6 Formulation of Mixed Integer Linear Programming Model

4.6.1 Piecewise linear programming

The piecewise linear programs are more useful in simplifying the model for a variety of application. Piecewise linear program are in fact syntactic sugar for linear, integer or MILP. In other words, a piecewise linear program can always be transformed into MILP and, sometimes into LP. The piecewise linear function are often specified by giving a set of slopes, a set of breakpoints at which the slopes change and the value of the function at a given point.

4.6.2 Piecewise formulation of Thermal generation units

The production cost function of thermal unit is expressed as a quadratic function; furthermore the problem of the load balance constraint and different characteristic of the thermal and hydro unit are nonlinear optimization problem that seem to be difficult to be solved by the standard nonlinear programming methods. Consider for instance; the formulation of optimization for the minimum cost of thermal unit into optimization of piecewise linear expression, MILP; that can be solved by IBM ILOG Cplex optimization Studio.

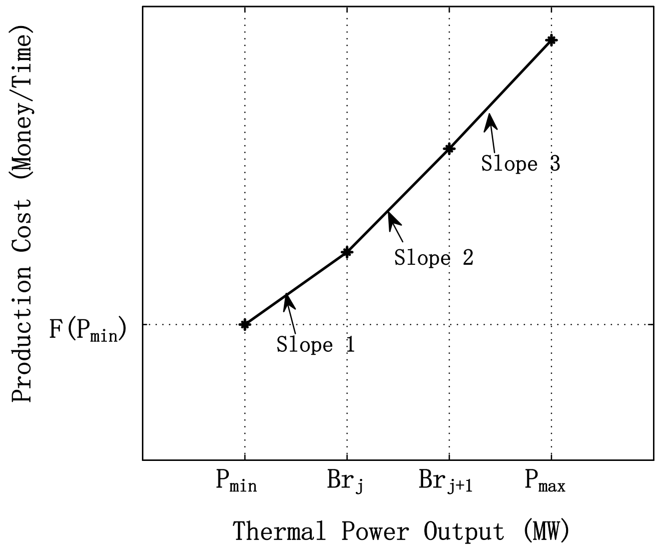

Fig. 4-1 Piecewise function of thermal production cost

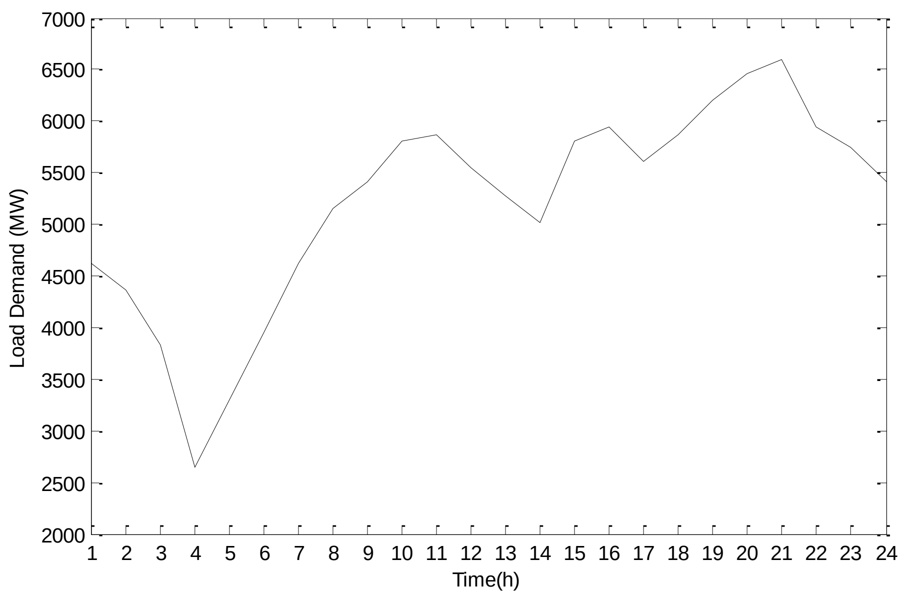

Fig. 4-2 The load curve demand of the case study

With thermal production cost expressed as:

$$F(P_S) = a + bP_S + cP_S^2$$

And with a given thermal power plant operating limit

$$P_S^{\min} \le P_S \le P_S^{\max}$$

To formulate the piecewise linear function we should specify:

- A set of slopes
- Set of breakpoints at which the slopes change.

The quadratic production cost function converted into the piecewise linear function will be:

$$F_j(P_{Sj}) = F(P_S^{\min}) + \sum_{j=1}^{N_{slopes}} s_j Br_j \quad \text{(4-14)}$$

Where

$$F(P_S^{\min}) = c_j = a + bP_S^{\min} + cP_S^{\min} \quad \text{(4-15)}$$

$N_{breakpoints}$: Number of breakpoints

s_j : Slope value at the number j

Br_j : Breakpoints value at value number j

C_j : Production cost of thermal unit at the minimum power

In Cplex Integrated Development Environment - Optimization Programming Language (Cplex IDE OPL), the programming language code notation of (45) will be:

$$piecewise\left(j\, in 1..N_{breakpoints}\right)\left\{s_j -> Br_j; s_{j+1}\right\}\left(P_s^{min}, c_j\right)P_s;$$

The corresponding graph of this piecewise linear function is shown on Fig.4-1, two breakpoints have been assumed. The accuracy of the results increase with the increasing of breakpoint considered.

4.6.2.1 Thermal unit commitment

It is quite very expensive to run too many generating units; a great deal of money can be saved by de-committing some units when they are not needed. The mixed integer linear programming for the unit commitment problem of thermal units is considered in this scheduling process and the formulation proposed requires fewer binary variables. The dynamic ramping up and dynamic ramping down limit functions provided in reference [50] are used in this scheduling problem. The generation limits of each period are set as follows:

$$P_{Sj}^{\min}U_j(t) \le P_{Sj}(t); \forall j \in J, \forall t \in T \qquad (4\text{-}16)$$

$$P_{Sj}(t) \le P_{Sj}^{\max}U_j(t); \forall j \in J, \forall t \in T \qquad (4\text{-}17)$$

The power generated at a period t are bounded by the minimum power output in (4-16), this power generated will be also bound by the unit maximum capacity as it is shown by the constraint (4-17). The binary variable $U_j(t)$ is introduced in the generation limits constraints. If the unit j is offline in the period t then $U_j(t)=0$ and $U_j(t)=1$ otherwise. The variable $P_{Sj}(t)$ will be always a non-negative variable.

4.6.3 Linearizing the hydropower production function

The mathematical model for linearizing the hydro plant generation in this research thesis consist of the linearization by ignoring the net head variation and the linearization by considering the net head variation.

4.6.3.1 Linearization by ignoring the net head variation

The performance of the hydropower plant depends on the rate of water discharge and on the net hydraulic head. The value of the net head depends on the water level in the reservoir, the tail-race level and the penstock losses between the reservoir and the turbine all these are a function of water flow. The water flow and the reservoir characteristics determine the power generated from a hydro unit; this power output of the hydro unit shown in (4-4) can be expressed as a nonlinear function of the water flow and the water volume in the reservoir. In reference [17] it has been shown that:

$$V_j(t) = f(H_j(t)) \qquad (4\text{-}18)$$

And $H_j(t)$ is the net head; this can be ignored for relatively large reservoir, in which lead the hydropower generation to be solely depend on the water discharge.

$$P_{Hj}(t) = f(q_j(t)) \qquad (4\text{-}19)$$

The approximation of the hydropower generation of (4-19) will be achieved by segments linear curve, with breakpoints at the full gate position, best efficient positions and the point representing the minimum flow as it is shown in Fig. 4-4 where $qb1(t)$ and $qb2(t)$ represent the breakpoints at the water discharge axis for the best efficient points with S1, S2 and S3 representing slopes respectively.

The mathematical model in the Fig. 4.4 gives the net electrical output as follow:

$$P_{Hj}(t) = P_{Hj}^{\min} + S1qb1(t) + S2qb2(t) + S3q\max(t) \quad \text{(4-20)}$$

The control schemes of hydrothermal power plant using IBM ILOG Cplex Studio software is depicted in Fig.4-3.

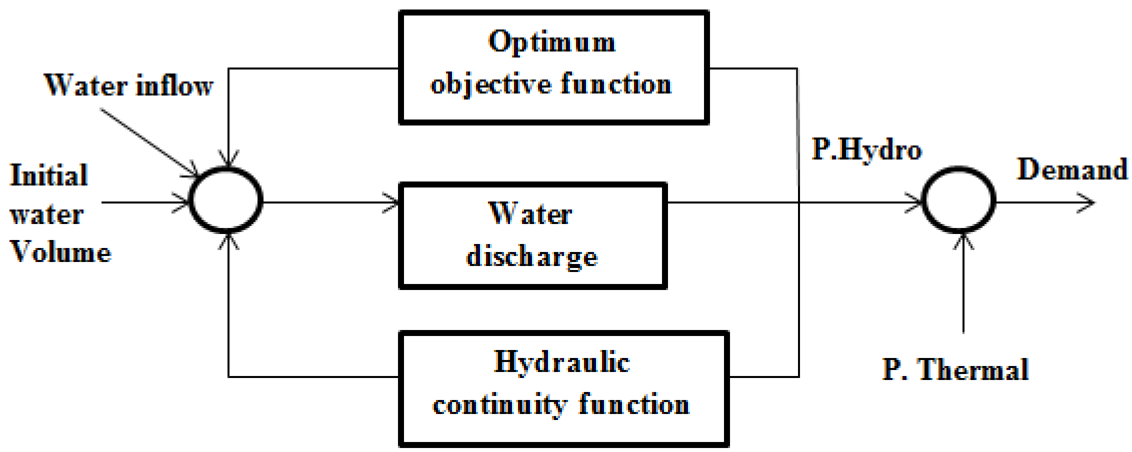

Fig. 4-3 Control diagram of hydrothermal power plant using IBM Cplex optimization software

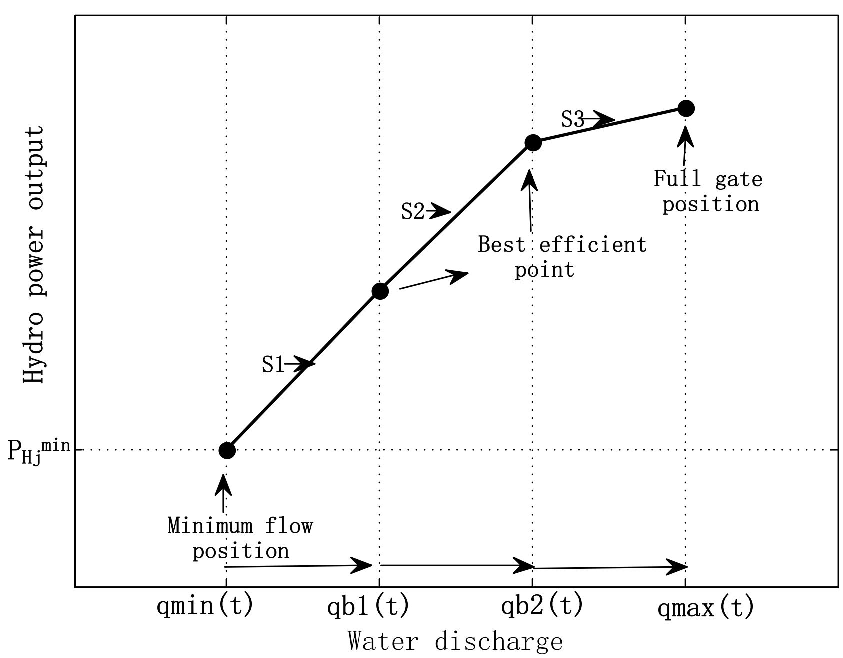

Fig. 4-4 Linear approximation of the unit performance curve

4.6.3.2 Linearization by considering the net head variation

The generated power can also be written as a function of water volume and water discharge as follow:

$$P_{Hj}(t) = f(q_j(t), V_j(t)) \quad \text{(4-21)}$$

The linearization of this model is developed in this research thesis in the following way:

Let us introduce n coordinates on q axis, $q_1,...,q_n$ and m coordinates on V axis $v_1,...,v_m$. The x axis coordinates are considered as the breakpoints that will help us to create the piecewise functions while V during a given interval of time will be kept constant and by the use of binary variables; the choice of the right curve due to water level in the reservoir will be possible.

Assume that a is a given interval between two successive water volumes, such that

$A = \{1, \ldots, a\}$,

Let $\bar{v}_j \in \left[V_j^{min}, V_j^{max}\right]$, suppose that $a = 2$ and locating in the middle of the two extreme water volumes such that $\bar{v}_j = \left(V^{min} + V^{max}\right)/2$; this leads to determine three starting point of the hydro power output at three different given volumes of water with a minimum water discharge q_{min}.

$$P_{Hj}^1 = (q_j^{\min}, V_j^{\min}) \quad \text{(4-22)}$$

$$P_{Hj}^2 = (q_j^{\min}, \bar{v}_j) \quad \text{(4-23)}$$

$$P_{Hj}^3 = (q_j^{\min}, V_j^{\max}) \quad \text{(4-24)}$$

With P_{Hj}^1, P_{Hj}^2 and P_{Hj}^3 are hydro power outputs when the water volumes are $V_j^{min}, \bar{v}_j$ and V_j^{max} respectively with minimum water discharge q_j^{min}.

To determine the right choice of the curve as shown on Fig. 4-5; the binary variables will be used; the MILP formulation with discretization will be formulated in the following way:

- Suppose that $b_{1j}(t)$ and $b_{2j}(t)$ are the binary variable such that:

$$b_{1j}(t), b_{2j}(t) \in \{0,1\} \quad \forall t \in T \text{ And } \forall j \in J$$

The output power approximation of the non-concave unit performance curve with $p-q$ relationship will be formulated as:

$$P_{Hj}(t) \le P_{Hj}^{1} + \sum_{i=1}^{n} S_{1j}(i) X_i(t) + P_{Hj}^{\max}\left[b_{1j}(t) + b_{2j}(t)\right] \quad \forall j \in J \text{ And } \forall t \in T \quad \ldots\ldots\ldots\ldots\text{(4-25)}$$

$$P_{Hj}(t) \ge P_{Hj}^{1} + \sum_{i=1}^{n} S_{1j}(i) X_i(t) - P_{Hj}^{\max}\left[b_{1j}(t) + b_{2j}(t)\right] \quad \forall j \in J \text{ And } \forall t \in T \quad \ldots\ldots\ldots\text{(4-26)}$$

$$P_{Hj}(t) \le P_{Hj}^{2} + \sum_{i=1}^{n} S_{2j}(i) X_i(t) + P_{Hj}^{\max}\left[1 - b_{1j}(t) + b_{2j}(t)\right] \quad \forall j \in J \text{ And } \forall t \in T \quad \ldots\ldots\text{(4-27)}$$

$$P_{Hj}(t) \ge P_{Hj}^{2} + \sum_{i=1}^{n} S_{2j}(i) X_i(t) - P_{Hj}^{\max}\left[1 - b_{1j}(t) + b_{2j}(t)\right] \quad \forall j \in J \text{ And } \forall t \in T \quad \ldots\ldots\ldots\text{(4-28)}$$

$$P_{Hj}(t) \le P_{Hj}^{3} + \sum_{i=1}^{n} S_{3j}(i) X_i(t) + P_{Hj}^{\max}\left[2 - b_{1j}(t) + b_{2j}(t)\right] \quad \forall j \in J \text{ And } \forall t \in T \quad \ldots\ldots\ldots\text{(4-29)}$$

$$P_{Hj}(t) \ge P_{Hj}^{3} + \sum_{i=1}^{n} S_{3j}(i) X_i(t) - P_{Hj}^{\max}\left[2 - b_{1j}(t) + b_{2j}(t)\right] \quad \forall j \in J \text{ And } \forall t \in T \quad \ldots\ldots\ldots\ldots\text{(4-30)}$$

Where S_{1j}, S_{2j} and S_{3j} are slopes respectively corresponding to different level of water volumes;

$$x_i(t) = Breakpoint_{i+1} - Breakpoint_i$$

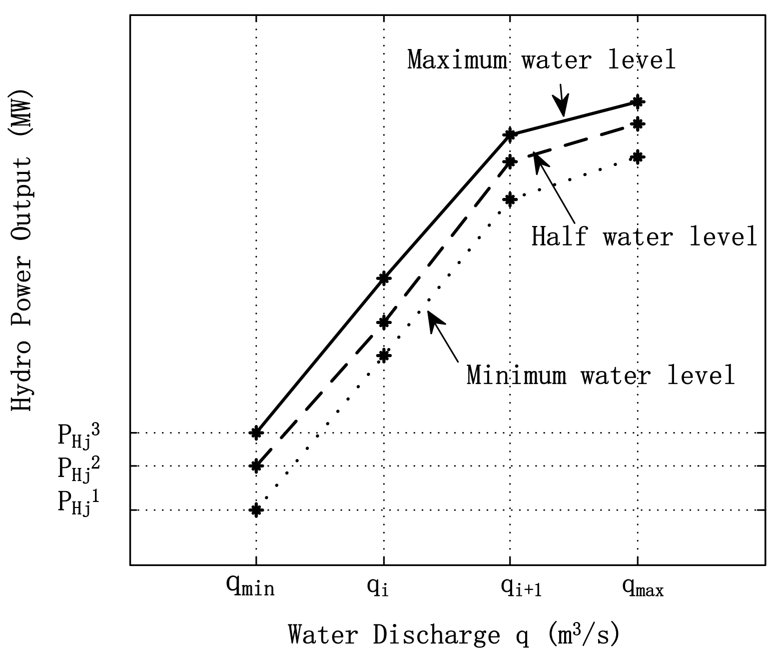

Fig. 4-5 Simplification of three dimensional hydro unit performance characteristics

- When both $b_{1j}(t)$ and $b_{2j}(t)$ are zero; (4-25) and (4-26) which represent the hydro power output for lower level will be used.
- When $b_{1j}(t)=1$ and $b_{2j}(t)=0$; the middle curve at the medium water volume level will be used and the approximated hydro power output will be determined by (4-28) and (4-29).
- When both $b_{1j}(t)$ and $b_{2j}(t)$ are one; the curve will be forced to move to the up level corresponding to the maximum water volume; the approximated hydro power output will be determined by (4-29) and (4-30).

The choice of the right curve to approximate the hydro power output should also be imposed by the water level as follows:

$$V_j(t) \geq V_j^{\min}\left[b_{1j}(t)-b_{2j}(t)\right]+\bar{v}_j b_{2j}(t) \quad \forall t \in T \text{ ....(4-31)}$$

$$V_j(t) \leq V_j^{\max} b_{2j}(t)+V_j^{\min}\left[1-b_{1j}(t)\right]+\bar{v}_j\left[b_{1j}(t)-b_{2j}(t)\right] \quad \forall t \in T \text{ ....(4-32)}$$

This leads to following constraints:

$$\left.\begin{matrix} b_{1j}(t)=1 \\ b_{2j}(t)=0 \end{matrix}\right\} \quad \left.\begin{matrix} b_{1j}(t)=0 \\ b_{2j}(t)=0 \end{matrix}\right\} \quad \begin{matrix} V_j(t)\geq 0 \\ V_j(t)\leq V_j^{min} \end{matrix} \quad \textit{conditions leading to minimum water level}$$

$$\left.\begin{matrix} b_{1j}(t)=1 \\ b_{2j}(t)=0 \end{matrix}\right\} \quad \begin{matrix} V_j(t)\geq V_j^{min} \\ V_j(t)\leq \bar{v}_j \end{matrix} \quad \textit{conditions leading to half water level}$$

$$\left.\begin{matrix} b_{1j}(t)=1 \\ b_{2j}(t)=1 \end{matrix}\right\} \quad \begin{matrix} V_j(t)\geq \bar{v}_j \\ V_j(t)\leq V_j^{max} \end{matrix} \quad \textit{conditions leading to maximum water level}$$

CHAPTER 5. NUMERICAL RESULTS OF CASE STUDY AND DISCUSSION

To demonstrate the effectiveness of hydropower plants taken as one of renewable source of energy; the scheduling results obtained are divided into two sections:

- The scheduling of thermal units without hydropower plants.
- The scheduling combination of both thermal units and hydropower plants.

5.1 Scheduling of thermal generating units without hydropower plant

In this scheduling process of thermal units shown in Table 5-1; the statistic results given by the IBM ILOG Cplex optimization studio shows that the objective solution is 2208945.352 and 2278 constraints are executed in the mathematical model. Total variables are 2301 with 5566 non- zero coefficients.

The tried aggregator is 1 time. The MIP pre-solving eliminated 2306 rows and 3121 column with zero binaries. The MIP search was dynamic search method.

Most of expensive units are supplying the hourly load demands at their minimum power capacity, while the expensive unit supply to their maximum capacity in other to satisfy the hurly load demand with minimum operating costs.

The accuracy of our proposed results are also shown in Table 5-2; the thermal unit commitment are executed with purpose of turning on or off some generators that are not needed to supply the hourly load demands. In this process of unit commitment; expensive units were first turned off in other to minimize operating costs and at the same time satisfied hourly load demands.

Table 5-1: Hourly power generated by forty six thermal power plants without hydropower plants

Unit	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5	Hour 6	Hour 7	Hour 8	Hour 9	Hour 10	Hour 11	Hour 12
1	5	5	5	5	5	5	5	5	5	5	5	5
2	5	5	5	5	5	5	5	5	5	5	5	5
3	5	5	5	5	5	5	5	5	5	5	5	5
4	300	300	150	150	150	150	300	300	300	300	300	300
5	300	100	100	100	100	100	100	300	300	300	300	300
6	10	10	10	10	10	10	10	10	10	10	10	10
7	25	25	25	25	25	25	25	25	25	30	25	25
8	5	5	5	5	5	5	5	5	5	5	5	5
9	5	5	5	5	5	5	5	5	5	5	5	5
10	112	100	100	100	100	100	300	300	300	300	300	300
11	350	350	350	350	350	350	350	350	350	350	350	350
12	8	8	8	8	8	8	8	8	8	8	8	8

13	8	8	8	8	8	8	8	8	8	8	8	8
14	25	25	25	25	25	25	25	25	25	25	100	25
15	8	8	8	8	8	8	8	8	8	8	8	8
16	25	25	25	25	25	25	25	25	25	100	25	25
17	8	8	8	8	8	8	8	8	8	8	8	8
18	8	8	8	8	8	8	8	8	8	8	8	8
19	25	25	25	25	25	25	25	25	25	25	100	66
20	250	250	250	250	250	250	250	250	250	250	250	250
21	250	250	250	72	250	250	250	250	250	250	250	250
22	25	25	25	25	25	25	25	25	25	100	96	100
23	25	25	25	25	25	25	25	25	25	25	100	25
24	200	200	200	50	200	200	200	200	200	200	200	200
25	200	200	200	50	200	200	200	200	200	200	200	200
26	25	25	25	25	25	25	25	25	25	25	100	25
27	420	420	340	100	132	420	420	420	420	420	420	420
28	420	420	420	100	100	420	420	420	420	420	420	420
29	80	80	80	80	80	80	80	80	284	300	300	300
30	30	30	30	30	30	30	30	30	30	30	30	30
31	10	10	10	10	10	10	10	10	10	10	10	10
32	5	5	5	5	5	5	5	5	5	5	5	5
33	5	5	5	5	5	5	5	5	5	5	5	5
34	25	25	25	25	25	25	25	25	25	25	100	25
35	25	25	25	25	25	25	25	25	25	100	25	25
36	300	300	150	150	150	202	300	300	300	300	300	300
37	25	25	25	25	25	25	25	25	25	100	25	25
38	10	10	10	10	10	10	10	10	10	10	10	10
39	300	300	300	300	300	300	300	300	300	300	300	300
40	200	200	200	50	200	200	200	200	200	200	200	20
41	8	8	8	8	8	8	8	8	8	8	8	8
42	20	20	20	20	20	20	20	20	20	20	20	20
43	100	100	100	100	100	100	112	300	300	300	300	300
44	300	248	100	100	100	100	300	300	300	300	300	300
45	100	100	100	100	100	100	100	240	300	300	300	300
46	25	25	25	25	25	25	25	25	25	100	25	25

Unit	**Hour 13**	**Hour 14**	**Hour 15**	**Hour 16**	**Hour 17**	**Hour 18**	**Hour 19**	**Hour 20**	**Hour 21**	**Hour 22**	**Hour 23**	**Hour 24**
1	5	5	5	5	5	5	5	5	5	5	5	5
2	5	5	5	5	5	5	5	5	5	5	5	5
3	5	5	5	5	5	5	5	5	30	5	5	5
4	300	300	300	300	300	300	300	300	300	300	300	300
5	300	300	300	300	300	300	300	300	300	300	300	300
6	10	10	10	10	10	10	10	30	30	10	10	10
7	25	25	100	100	25	25	100	100	100	25	100	25
8	5	5	5	5	5	5	5	5	30	5	5	5

9	5	5	5	5	5	5	5	5	27	5	5	5
10	300	300	300	300	300	300	300	300	300	300	300	300
11	350	350	350	350	350	350	350	350	350	350	350	350
12	8	8	8	8	8	8	8	17	30	8	8	8
13	8	8	8	8	8	8	8	30	30	8	8	8
14	25	25	25	100	25	100	100	100	100	100	25	25
15	8	8	8	8	8	8	8	30	30	8	8	8
16	25	25	25	87	25	100	100	100	100	87	25	25
17	8	8	8	8	8	8	8	30	30	8	8	8
18	8	8	8	8	8	8	8	8	30	8	8	8
19	25	25	25	100	25	25	100	100	100	100	25	25
20	250	250	250	250	250	250	250	250	250	250	250	250
21	250	250	250	250	250	250	250	250	250	250	250	250
22	25	25	30	25	100	96	100	100	100	100	39	25
23	25	25	100	25	25	100	100	100	100	25	100	25
24	200	200	200	200	200	200	200	200	200	200	200	200
25	200	200	200	200	200	200	200	200	200	200	200	200
26	25	25	100	100	100	25	100	100	100	100	25	25
27	420	420	420	420	420	420	420	420	420	420	420	420
28	420	420	420	420	420	420	420	420	420	420	420	420
29	152	80	300	300	300	300	300	300	300	300	300	284
30	30	30	30	30	30	30	30	80	80	30	30	30
31	10	10	10	10	10	10	10	30	30	10	10	10
32	5	5	5	5	5	5	5	5	30	5	5	5
33	5	5	5	5	5	5	5	5	5	5	5	5
34	25	25	25	25	25	25	100	100	100	25	100	25
35	25	25	100	100	25	100	100	100	100	100	25	25
36	300	300	300	300	300	300	300	300	300	300	300	300
37	25	25	100	25	57	100	51	10	100	25	25	25
38	10	10	10	10	10	10	10	10	30	10	10	10
39	300	300	300	300	300	300	300	300	300	300	300	300
40	200	200	200	200	200	200	200	200	200	200	200	200
41	8	8	8	8	8	8	8	8	8	8	8	8
42	20	20	20	20	20	20	20	50	50	20	20	20
43	300	300	300	300	300	300	300	300	300	300	300	300
44	300	108	300	300	300	300	300	300	300	300	300	300
45	300	300	300	300	300	300	300	300	300	300	300	300
46	25	25	25	100	25	25	100	100	100	100	100	25

The static results provided by IBM Ilog Cplex optimization studio during the unit commitment process, shows that the objective is 2169510.688 with 3382 constraints. Total variables executed are 3405 which consist of 1104 binary variables and 2301 other variables with non-zero coefficients that are 8878 . In addition of this 992 iterations are executed to converge to the optimal solution.

The tried aggregator is 2 times; MIP pre-solving eliminated 396 rows and 377 column, the aggregator did 850 substitutions. Reduced MIP has 2250 rows and 2989 columns with 1062 binaries; the model balance optimality and feasibility. The search method is a dynamic search.

The total saved cost during the thermal units commitment process is:

$$Total\ saved\ cost\ =\ 2208945.352 - 2169510.688 = 39434.664$$

The total running cost of these thermal generating units will be minimized by including the hydro plants in the system as it will be discussed in the next section.

Table 5-2: Hourly thermal unit commitment

Unit	Thermal Unit commitment											
	Hour 1	**Hour 2**	**Hour 3**	**Hour 4**	**Hour 5**	**Hour 6**	**Hour 7**	**Hour 8**	**Hour 9**	**Hour 10**	**Hour 11**	**Hour 12**
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	0	1	1	1	1	1	1	1	1
5	1	1	1	0	1	0	1	1	1	1	1	1
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	1	0	0	0	0	0	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	1	0
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	1	1	1	1
17	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1
22	0	0	0	0	0	0	0	0	0	1	1	0
23	0	0	0	0	0	0	0	0	0	1	1	1
24	1	1	1	1	1	1	1	1	1	1	1	1
25	1	1	1	1	1	1	1	1	1	1	1	1
26	0	0	0	0	0	0	0	0	1	1	1	1
27	1	1	1	1	1	1	1	1	1	1	1	1
28	1	1	1	1	1	1	1	1	1	1	1	1
29	0	0	0	0	0	0	0	1	1	1	1	1
30	0	0	0	0	0	0	0	0	0	0	0	0

31	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	1	1	1	1
35	0	0	0	0	0	0	0	1	1	1	1	1
36	1	1	1	1	1	1	1	1	1	1	1	1
37	0	0	0	0	0	0	0	1	0	1	1	0
38	0	0	0	0	0	0	0	0	0	0	0	0
39	1	1	1	1	1	1	1	1	1	1	1	1
40	1	1	1	1	1	1	1	1	1	1	1	1
41	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0
43	1	1	1	0	0	1	1	1	1	1	1	1
44	1	1	1	0	0	1	1	1	1	1	1	1
45	1	1	0	0	0	1	1	1	1	1	1	1
46	0	0	0	0	0	0	0	0	0	1	0	0

Unit	**Thermal Unit commitment**											
	Hour 13	**Hour 14**	**Hour 15**	**Hour 16**	**Hour 17**	**Hour 18**	**Hour 19**	**Hour 20**	**Hour 21**	**Hour 22**	**Hour 23**	**Hour 24**
1	0	0	0	0	0	0	0	1	1	0	0	0
2	0	0	0	0	0	0	0	0	1	0	0	0
3	0	0	0	0	0	0	0	0	1	0	0	0
4	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	1	1	0	0	0
7	0	0	0	1	1	0	1	1	1	1	1	0
8	0	0	0	0	0	0	0	0	1	0	0	0
9	0	0	0	0	0	0	0	1	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	1	0	0	0
13	0	0	0	0	0	0	0	1	1	0	0	0
14	0	0	1	1	1	1	1	1	1	1	0	0
15	0	0	0	0	0	0	0	1	1	0	0	0
16	0	0	1	1	0	1	1	1	1	1	1	1
17	0	0	0	0	0	0	0	1	1	0	0	0
18	0	0	0	0	0	0	0	1	1	0	0	0
19	0	0	1	1	0	0	1	1	1	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1
22	0	0	1	1	0	1	1	1	1	1	1	0
23	0	0	1	1	1	1	1	1	1	1	1	0
24	1	1	1	1	1	1	1	1	1	1	1	1
25	1	1	1	1	1	1	1	1	1	1	1	1

26	1	1	1	1	1	1	1	1	1	1	1	1
27	1	1	1	1	1	1	1	1	1	1	1	1
28	1	1	1	1	1	1	1	1	1	1	1	1
29	1	1	1	1	1	1	1	1	1	1	1	1
30	0	0	0	0	0	0	1	1	1	0	0	0
31	0	0	0	0	0	0	0	1	1	0	0	0
32	0	0	0	0	0	0	0	0	1	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0
34	1	0	1	1	1	1	1	1	1	1	1	1
35	1	0	1	1	1	1	1	1	1	1	1	1
36	1	1	1	1	1	1	1	1	1	1	1	1
37	0	0	1	1	1	1	1	1	1	1	0	0
38	0	0	0	0	0	0	0	1	1	0	0	0
39	1	1	1	1	1	1	1	1	1	1	1	1
40	1	1	1	1	1	1	1	1	1	1	1	1
41	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	1	1	1	0	0	0
43	1	1	1	1	1	1	1	1	1	1	1	1
44	1	1	1	1	1	1	1	1	1	1	1	1
45	1	1	1	1	1	1	1	1	1	1	1	1
46	0	0	0	0	0	1	1	1	1	1	1	1

Table 5-3: Hourly thermal generated power during the unit commitment

Unit	Thermal Generated power											
	Hour 1	**Hour 2**	**Hour 3**	**Hour 4**	**Hour 5**	**Hour 6**	**Hour 7**	**Hour 8**	**Hour 9**	**Hour 10**	**Hour 11**	**Hour 12**
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	300	300	300	0	300	300	300	300	300	300	300	300
5	300	300	238	0	110	0	300	300	300	300	300	300
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	25	25	84	54
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	300	0	0	0	0	0	300	300	300	300	300	300
11	350	350	350	350	350	350	350	350	350	350	350	350
12	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	100	0
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	97	93	100	100
17	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0

19	0	0	0	0	0	0	0	0	0	0	0	0
20	250	250	250	250	250	250	250	250	250	250	250	250
21	250	250	250	250	250	250	250	250	250	250	250	250
22	0	0	0	0	0	0	0	0	0	100	100	0
23	0	0	0	0	0	0	0	0	0	100	100	100
24	200	200	200	200	200	200	200	200	200	200	200	200
25	200	200	200	200	200	200	200	200	200	200	200	200
26	0	0	0	0	0	0	0	0	100	100	100	100
27	420	420	420	320	420	420	420	420	420	420	420	420
28	420	420	420	420	420	420	420	420	420	420	420	420
29	0	0	0	0	0	0	0	300	300	300	300	300
30	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	100	100	100	100
35	0	0	0	0	0	0	0	58	100	100	100	100
36	300	300	300	150	300	300	300	300	300	300	300	300
37	0	0	0	0	0	0	0	100	0	100	100	0
38	0	0	0	0	0	0	0	0	0	0	0	0
39	300	300	300	300	300	300	300	300	300	300	300	300
40	200	200	200	200	200	200	200	200	200	200	200	200
41	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0
43	230	300	300	0	0	300	300	300	300	300	300	300
44	300	266	100	0	0	300	300	300	300	300	300	300
45	300	300	0	0	0	170	230	300	300	300	300	300
46	0	0	0	0	0	0	0	0	0	100	0	0

Unit	**Thermal Generated power**											
	Hour 13	**Hour 14**	**Hour 15**	**Hour 16**	**Hour 17**	**Hour 18**	**Hour 19**	**Hour 20**	**Hour 21**	**Hour 22**	**Hour 23**	**Hour 24**
1	0	0	0	0	0	0	0	10	20	0	0	0
2	0	0	0	0	0	0	0	0	30	0	0	0
3	0	0	0	0	0	0	0	0	30	0	0	0
4	300	300	300	300	300	300	300	300	300	300	300	300
5	300	300	300	300	300	300	300	300	300	300	300	300
6	0	0	0	0	0	0	0	30	30	0	0	0
7	0	0	0	100	95	0	100	100	100	100	52	0
8	0	0	0	0	0	0	0	0	30	0	0	0
9	0	0	0	0	0	0	0	30	0	0	0	0
10	300	300	300	300	300	300	300	300	300	300	300	300
11	350	350	350	350	350	350	350	350	350	350	350	350
12	0	0	0	0	0	0	0	0	30	0	0	0
13	0	0	0	0	0	0	0	28	30	0	0	0

14	0	0	93	100	100	100	100	100	100	50	0	0
15	0	0	0	0	0	0	0	30	30	0	0	0
16	0	0	100	100	0	100	100	100	100	100	100	97
17	0	0	0	0	0	0	0	30	30	0	0	0
18	0	0	0	0	0	0	0	30	30	0	0	0
19	0	0	100	100	0	0	100	100	100	0	0	0
20	250	250	250	250	250	250	250	250	250	250	250	250
21	250	250	250	250	250	250	250	250	250	250	250	250
22	0	0	100	100	0	100	100	100	100	100	100	0
23	0	0	100	50	25	100	100	100	100	100	100	0
24	200	200	200	200	200	200	200	200	200	200	200	200
25	200	200	200	200	200	200	200	200	200	200	200	200
26	100	26	100	100	100	100	100	100	100	100	100	100
27	420	420	420	420	420	420	420	420	420	420	420	420
28	420	420	420	420	420	420	420	420	420	420	420	420
29	300	300	300	300	300	300	300	300	300	300	300	300
30	0	0	0	0	0	0	80	80	80	0	0	0
31	0	0	0	0	0	0	0	30	30	0	0	0
32	0	0	0	0	0	0	0	0	30	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0
34	100	0	100	100	100	100	100	100	100	100	100	100
35	90	0	100	100	100	100	100	100	100	100	100	100
36	300	300	300	300	300	300	300	300	300	300	300	300
37	0	0	25	100	100	100	100	100	100	100	0	0
38	0	0	0	0	0	0	0	30	30	0	0	0
39	300	300	300	300	300	300	300	300	300	300	300	300
40	200	200	200	200	200	200	200	200	200	200	200	200
41	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	34	50	50	0	0	0
43	300	300	300	300	300	300	300	300	300	300	300	300
44	300	300	300	300	300	300	300	300	300	300	300	300
45	300	300	300	300	300	300	300	300	300	300	300	300
46	0	0	0	0	0	84	100	100	100	100	100	25

5.2 Scheduling of thermal generating units with hydropower plant

By connecting the hydropower plant units in the system that was made of thermal units, the expensive generating units output power have been lowered tending toward their minimum power bound limits while the least expensive unit does not run to their maximum power capacity as it is considered in the previous case without hydropower plant. The production cost of thermal generating units being a function of the power output; this will lead to minimize their production cost which is our main goal of this case study. The Table 5-4 and Table 5-5 show hourly thermal unit commitment and output power generated respectively to supply the demand when the hydropower plant is turned on in this power system.

The generation scheduling of hydrothermal problem statistics results provide the optimal solution of 2133675.603. Constraints are 5758 and 4273 variables that consist of 1296 binary variables and 2977 other variables with 42982 non-zero coefficients. 139072 iterations are executed to converge to the optimal solution.

The tried aggregator is 2 times. MIP pre-solved eliminated 849 rows and 281 columns. The aggregator did 1225 substitutions furthermore, the reduced MIP has 1285 binaries. The mathematical model balance optimality and feasibility; the MIP search method is a dynamic search.

$$Total\ saved\ cost = 2169510.688 - 2133675.603 = 35835.085$$

Table 5-4: Hourly unit commitment of thermal generators

Unit	**Thermal Unit commitment**											
	Hour 1	**Hour 2**	**Hour 3**	**Hour 4**	**Hour 5**	**Hour 6**	**Hour 7**	**Hour 8**	**Hour 9**	**Hour 10**	**Hour 11**	**Hour 12**
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	0	1	1	1	1	1	1	1	1
5	1	0	0	0	1	0	1	1	1	1	1	1
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	1	1	0	0	0	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	1	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	1	0
17	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1
22	0	0	0	0	0	0	0	0	0	1	1	1
23	0	0	0	0	0	0	0	0	0	1	1	0
24	1	1	1	1	1	1	1	1	1	1	1	1
25	1	1	1	1	1	1	1	1	1	1	1	1
26	0	0	0	0	0	0	0	0	0	1	1	1
27	1	1	1	1	1	1	1	1	1	1	1	1
28	1	1	1	1	1	1	1	1	1	1	1	1
29	0	0	0	0	0	0	0	1	1	1	1	1

30	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	1	1	0
35	0	0	0	0	0	0	0	1	1	1	1	1
36	1	1	1	0	1	1	1	1	1	1	1	1
37	0	0	0	0	0	0	0	0	1	1	1	1
38	0	0	0	0	0	0	0	0	0	0	0	0
39	1	1	1	1	1	1	1	1	1	1	1	1
40	1	1	1	1	1	1	1	1	1	1	1	1
41	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0
43	1	1	0	0	0	0	1	1	1	1	1	1
44	1	1	1	0	0	1	1	1	1	1	1	1
45	1	1	1	0	0	1	1	1	1	1	1	1
46	0	0	0	0	0	0	0	0	1	1	1	0

Unit	**Thermal Unit commitment**											
	Hour 13	**Hour 14**	**Hour 15**	**Hour 16**	**Hour 17**	**Hour 18**	**Hour 19**	**Hour 20**	**Hour 21**	**Hour 22**	**Hour 23**	**Hour 24**
1	0	0	0	0	0	0	0	1	0	0	0	0
2	0	0	0	0	0	0	0	1	1	0	0	0
3	0	0	0	0	0	0	0	0	1	0	0	0
4	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	1	1	0	0	0
7	1	0	1	1	1	1	1	1	1	1	1	1
8	0	0	0	0	0	0	0	1	1	0	0	0
9	0	0	0	0	0	0	0	1	1	0	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	1	0	0	0
13	0	0	0	0	0	0	0	0	1	0	0	0
14	0	0	0	1	1	1	1	1	1	1	0	1
15	0	0	0	0	0	0	0	0	1	0	0	0
16	0	0	1	1	1	1	1	1	1	1	0	0
17	0	0	0	0	0	0	0	0	1	0	0	0
18	0	0	0	0	0	0	0	0	1	0	0	0
19	0	0	1	0	1	1	1	1	1	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1
22	0	0	0	0	0	0	1	1	1	1	0	0
23	0	0	1	1	1	1	1	1	1	1	1	1
24	1	1	1	1	1	1	1	1	1	1	1	1

25	1	1	1	1	1	1	1	1	1	1	1	1
26	0	0	1	1	0	1	1	1	1	1	1	0
27	1	1	1	1	1	1	1	1	1	1	1	1
28	1	1	1	1	1	1	1	1	1	1	1	1
29	1	1	1	1	1	1	1	1	1	1	1	1
30	0	0	0	0	0	0	1	1	1	0	0	0
31	0	0	0	0	0	0	0	1	1	0	0	0
32	0	0	0	0	0	0	0	0	1	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0
34	1	0	1	1	0	1	1	1	1	1	1	0
35	0	0	1	1	0	1	1	1	1	0	1	0
36	1	1	1	1	1	1	1	1	1	1	1	1
37	0	0	0	1	0	1	1	1	1	1	1	0
38	0	0	0	0	0	0	0	1	1	0	0	0
39	1	1	1	1	1	1	1	1	1	1	1	1
40	1	1	1	1	1	1	1	1	1	1	1	1
41	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	1	1	0	0	0
43	1	1	1	1	1	1	1	1	1	1	1	1
44	1	1	1	1	1	1	1	1	1	1	1	1
45	1	1	1	1	1	1	1	1	1	1	1	1
46	1	0	1	1	1	1	1	1	1	1	1	1

Table 5-5: Hourly thermal generation during the Unit Commitment

Unit	**Thermal Generated power**											
	Hour 1	**Hour 2**	**Hour 3**	**Hour 4**	**Hour 5**	**Hour 6**	**Hour 7**	**Hour 8**	**Hour 9**	**Hour 10**	**Hour 11**	**Hour 12**
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	300	300	300	0	300	300	300	300	300	300	300	300
5	300	0	0	0	100	0	300	300	300	300	300	300
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	100	97.506	100	100
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	300	300	0	0	0	100	300	300	300	300	300	300
11	350	350	350	350	350	350	350	350	350	350	350	350
12	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	25	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	25	0
17	0	0	0	0	0	0	0	0	0	0	0	0

18	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0
20	250	250	250	250	250	250	250	250	250	250	250	250
21	250	250	250	250	250	250	250	250	250	250	250	250
22	0	0	0	0	0	0	0	0	0	100	88.5	0
23	0	0	0	0	0	0	0	0	0	100	100	0
24	200	200	200	200	200	200	200	200	200	200	200	200
25	200	200	200	200	200	200	200	200	200	200	200	200
26	0	0	0	0	0	0	0	0	0	100	100	100
27	420	420	420	420	420	420	420	420	420	420	420	420
28	420	420	399.51	420	420	420	420	420	420	420	420	420
29	0	0	0	0	0	0	0	300	300	300	300	300
30	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	100	100	0
35	0	0	0	0	0	0	0	87.5	100	100	100	100
36	300	300	300	0	239.5	300	300	300	300	300	300	300
37	0	0	0	0	0	0	0	0	51.5	100	100	100
38	0	0	0	0	0	0	0	0	0	0	0	0
39	300	300	300	300	300	300	300	300	300	300	300	300
40	200	200	200	200	200	200	200	200	200	200	200	200
41	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0
43	159.5	159.5	0	0	0	0	300	300	300	300	300	300
44	300	300	267.5	0	0	299.5	300	300	300	300	300	300
45	300	300	300	0	0	300	159.5	300	300	300	300	300
46	0	0	0	0	0	0	0	0	100	25	100	0

Unit	**Thermal Generated power**											
	Hour 13	**Hour 14**	**Hour 15**	**Hour 16**	**Hour 17**	**Hour 18**	**Hour 19**	**Hour 20**	**Hour 21**	**Hour 22**	**Hour 23**	**Hour 24**
1	0	0	0	0	0	0	0	30	0	0	0	0
2	0	0	0	0	0	0	0	5	25	0	0	0
3	0	0	0	0	0	0	0	30	0	0	0	0
4	300	300	300	300	300	300	300	300	300	300	300	300
5	300	300	300	300	300	300	300	300	300	300	300	300
6	0	0	0	0	0	0	0	10	27.011	0	0	0
7	94.506	0	100	100	100	63.506	100	100	100	100	100	100
8	0	0	0	0	0	0	0	17.506	12.494	0	0	0
9	0	0	0	0	0	0	0	25	5	0	0	0

10	300	300	300	300	300	300	300	300	300	300	300	300
11	350	350	350	350	350	350	350	350	350	350	350	350
12	0	0	0	0	0	0	0	0	30	0	0	0
13	0	0	0	0	0	0	0	0	30	0	0	0
14	0	0	0	100	100	25	100	100	100	100	0	100
15	0	0	0	0	0	0	0	0	30	0	0	0
16	0	0	47.5	100	100	25	100	100	100	100	0	100
17	0	0	0	0	0	0	0	0	30	0	0	0
18	0	0	0	0	0	0	0	0	30	0	0	0
19	0	0	100	0	100	100	100	100	100	0	0	0
20	250	250	250	250	250	250	250	250	250	250	250	250
21	250	250	250	250	250	250	250	250	250	250	250	250
22	0	0	0	0	0	100	100	100	100	100	0	0
23	0	0	100	100	49.5	100	100	100	100	100	100	51.5
24	200	200	200	200	200	200	200	200	200	200	200	200
25	200	200	200	200	200	200	200	200	200	200	200	200
26	0	0	100	100	0	100	100	100	100	100	100	0
27	420	420	420	420	420	420	420	420	420	420	420	420
28	420	420	420	420	420	420	420	420	420	420	420	420
29	300	255.51	300	300	300	300	300	300	300	300	300	300
30	0	0	0	0	0	0	43.5	80	80	0	0	0
31	0	0	0	0	0	0	0	30	30	0	0	0
32	0	0	0	0	0	0	0	0	30	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0
34	100	0	100	100	0	100	100	100	100	100	81.5	0
35	0	0	100	100	0	100	100	100	100	0	100	0
36	300	300	300	300	300	300	300	300	300	300	300	300
37	0	0	0	100	0	100	100	100	100	100	100	0
38	0	0	0	0	0	0	0	30	30	0	0	0
39	300	300	300	300	300	300	300	300	300	300	300	300
40	200	200	200	200	200	200	200	200	200	200	200	200
41	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	50	50	0	0	0
43	300	300	300	300	300	300	300	300	300	300	300	300
44	300	300	300	300	300	300	300	300	300	300	300	300
45	300	300	300	300	300	300	300	300	300	300	300	300
46	25	0	100	79.5	100	100	100	100	100	100	100	100

The hydro-plants generated power are depicted in Table 5-6, the corresponding results are shown in Fig. 5-1 where the hydro unit 1 supply the load demand constantly from the 2nd hour up to 13th hour and then decrease its output capacity from 14th hour to 21st hour due to the availability of water storage. Those four hydro power plants being HCP meaning that the output water from the upper plant will be the input water of the next cascaded plant. The output power of hydro units being a function of water discharge and water content in the storage system these highly have impact on their output generated power and the results show that when the hydro unit 3 output power decreases, the hydro unit 4 output power increases.

Table 5-6: Hourly Hydropower plant generation

Hours	Hydro plants generated power			
	Unit 1	Unit 2	Unit 3	Unit 4
1	21.12	20.182	25.335	3.858
2	28.62	7.0068	14.683	20.185
3	28.62	2.37	25.774	13.73
4	28.62	4.6718	24.24	12.963
5	28.62	23.858	11.449	6.5676
6	28.62	6.0157	31.344	4.5151
7	28.62	13.759	10.182	17.934
8	28.62	21.145	16.091	4.6383
9	28.62	1.896	29.775	10.204
10	28.62	1.896	17.207	22.772
11	28.62	1.896	24.843	15.136
12	28.62	1.896	32.479	7.4992
13	28.62	1.896	17.207	22.772
14	25.53	1.896	34.446	8.6228
15	25.53	5.4222	24.565	14.977
16	25.53	1.896	18	25.068
17	25.53	1.896	32.4	10.668
18	25.53	1.896	18	25.068
19	25.53	1.896	25.2	17.868
20	25.53	1.896	32.4	10.668
21	25.53	2.781	17.469	24.714
22	22.44	21.646	15.204	11.204
23	22.44	21.646	15.204	11.204
24	21.972	26.797	15.894	5.8321

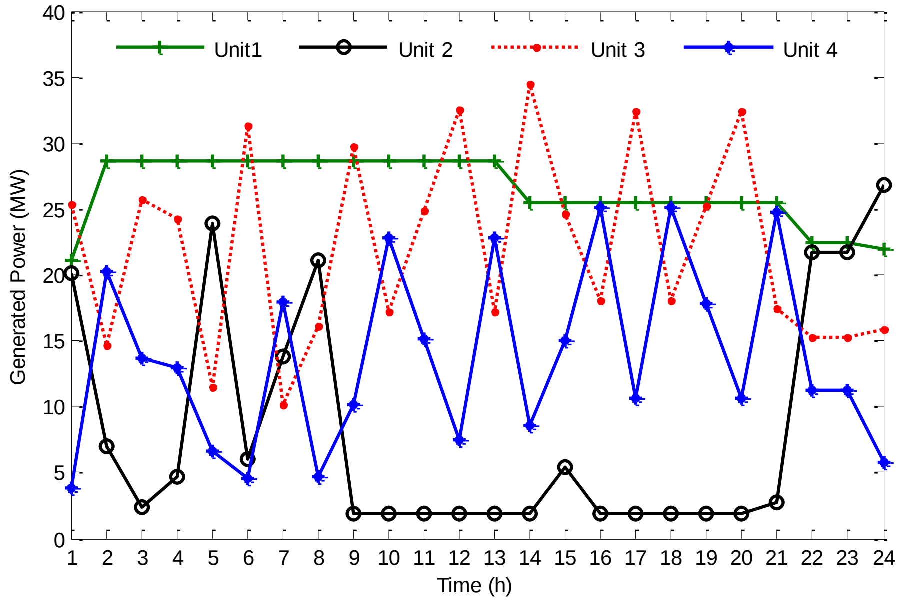

Fig. 5-1: Hydro plants generated power

The hourly hydro water discharge are shown in Table 5-7 with its corresponding Fig. 5-2. The water discharge characteristics curve are approximately inversely proportionally to the hydro power output curve characteristics shown in Fig 5-1.

Table 5-7: Hydro plants water discharge

Hours	Hydro plants water discharge			
	Unit 1	**Unit 2**	**Unit 3**	**Unit 4**
1	2	69.279	84.949	19
2	2	12.728	40.582	100.63
3	3.612	5	68.31	68.361
4	2	9.616	64.474	64.525
5	2	44.5	32.497	32.548
6	2	13.239	82.234	22.285
7	2	28.726	29.329	89.38
8	2	43.499	44.102	22.902
9	2	5	89.427	50.729
10	2	5	53.519	113.57
11	2	5	75.337	75.388
12	2	5	97.155	37.206
13	2	5	53.519	113.57
14	17.878	5	102.77	42.824
15	2	12.052	74.544	74.595
16	2	5	65.001	125.05
17	10.826	5	113	53.052
18	2	5	65.001	125.05

19	2	5	89.001	89.052
20	9.654	5	113	53.052
21	2	6.77	63.23	123.28
22	62	44.5	55.681	55.732
23	26.884	44.5	55.681	55.732
24	57.319	57.377	57.98	58.031

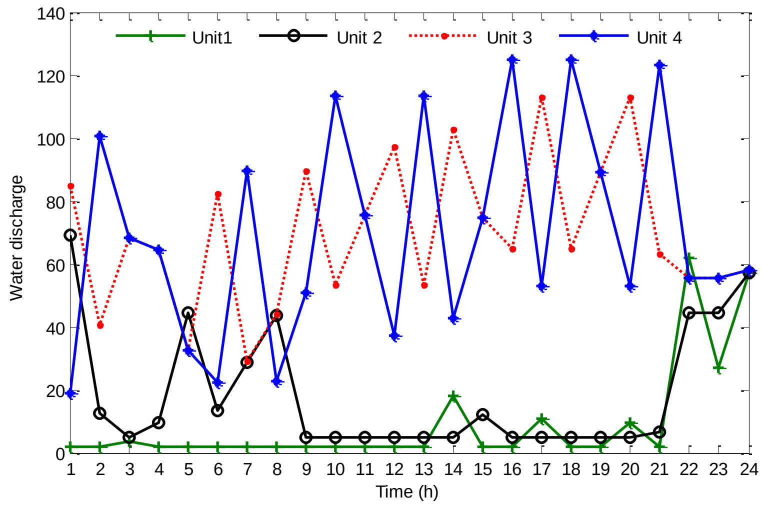

Fig. 5-2: Hydro power plants water discharge

The Table 5-8 with its corresponding Fig. 5-3 show that the reservoir volume during the scheduling period of hydropower plants is decreasing with time; the characteristic of the reservoir condition are taken into account during the scheduling process. The natural water spillage discharge rates are not assumed to occur in this case study and time delay of water between two successive plants are assumed to be zero.

Table 5-8: Hourly Hydropower plant volume

Hours	**Hydro power plants water volume**			
	Unit 1	**Unit 2**	**Unit 3**	**Unit 4**
1	225	162	1200	66
2	223.05	151.33	1172.7	6
3	219.49	150	1110	6
4	217.54	142.44	1055.8	6
5	215.59	100	1068.4	6
6	213.64	88.819	1000	66
7	211.69	62.151	1000	6
8	209.74	20.71	1000	27.251
9	207.8	17.768	916.18	66
10	205.85	14.826	868.26	6
11	203.9	11.884	798.53	6

12	201.95	8.942	706.97	66
13	200	6	659.06	6
14	182.17	18.936	561.89	66
15	180.22	8.942	500	66
16	178.27	6	440.6	6
17	167.5	11.884	333.2	66
18	165.55	8.942	273.81	6
19	163.6	6	190.41	6
20	154	10.712	83.012	66
21	152.05	6	27.155	6
22	90.101	23.558	16.578	6
23	63.268	6	6	6
24	6	6	6	6

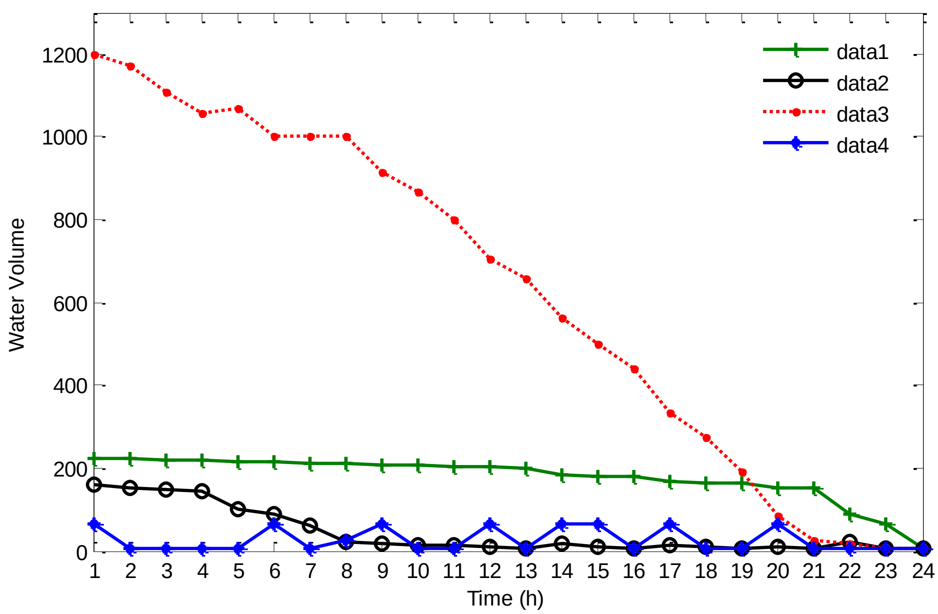

Fig. 5-3: Hydro power plants water volume

CHAPTER 6. CONCLUSIONS AND THE SCOPE OF FUTURE WORK

6.1 Conclusion

In this thesis, we present the GSAHTPS; the MILP based approach has been adopted to solve the short term generation scheduling problem; this has been executed by using IBM Ilog Cplex optimization software programming. Since both hydro and thermal characteristics are not linear, the piecewise linear approximation is used during this scheduling process, The effectiveness of the developed program is tested for a system having forty six thermal generating units and four hydropower plants connected in cascade with 24 hours load demand. The proposed MILP models allow to accurately representing most of hydro and thermal system characteristics proving high efficiency of modern MILP software tools particularly IBM Ilog Cplex optimization Studio both in term of solution accuracy and computing time.

The objective of coordination optimization include reduce fuel cost and number of thermal generating units to startup or shutdown during the scheduling process.

The test results and numerical experiences show that the proposed solution technique can give a near optimal or optimal solution. The advantage of the use of combined modeling language and optimization tools for generation scheduling approach of hydrothermal can be easily formulated in detail and updated or fully changed with less programming effort and the solution algorithm is computationally efficient.

6.2 Scope of future work

Despite all the achievements mentioned above, much more profound work still needs to be carried out, which can be broadly suggested. In coordinating constraint model, the stochastic nature of electricity demand, probabilistic requirement constraints and more details electrical transmission network representation including transmission losses as a non linear function of generator output and inter-regional transmission capacity limitations, interchange contacts and other export/ import constraints for large multi- area power system can be included for future research work in this area in order to generate more feasible operating schedules.

A number of features and transmission network constraints characteristics were omitted in this scheduling model of hydrothermal power system; conceptually, the mathematical model applied should have the ability of taking into account more security constraints and environment considerations. The technique should allow a broad range of operating constraints to be incorporated in the model and handled with more flexibility and computational difficulty.

ACKNOWLEDGEMENT

Before I get into thick of things, first, I would like to thank the Government of Rwanda and the People of Republic of China through North China Electric Power University (NCEPU) for this golden opportunity and financial support contributed to my postgraduate study.

I wish to express my profound gratefulness and appreciation to my supervisor Prof. Zhou Ming, power grid research institute, for her patient guidance and support throughout the report. She has been my mentor for help, she encourages me during my studies and during the phase of my research, she has been always there with kindness and fruitful technical advices and assistance.

I shall be failing in my duty, if I don't place on record my thanks to all lecturers who taught engineering courses related to my major, electrical power system and its automation to their efforts provided during this period of my postgraduate study.

In addition of this, I would like to thank my schoolmates for their kind assistance through the completion of this work.

Thanks are due to my family for their motivation and inspiration which has made me to come on this platform.

In the last but not the least, thanks to the Almighty God who gave me the opportunity and strength to carry out my postgraduate study.

To conclude, my best regards and thanks are due to whom I have not mentioned names but directly or indirectly helped me in the preparation and completion of this research thesis.

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