

ON THE PERFORMANCE OF UNDERLAY COGNITIVE RADIO NETWORKS WITH INTERFERENCE CONSTRAINTS AND RELAYING

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Blekinge Institute of Technology
Doctoral Dissertation Series No. 2015:02
Department of Communication Systems



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Abstract

Efficiently allocating the scarce and expensive radio resources is a key challenge for advanced radio communication systems. To this end, cognitive radio (CR) has emerged as a promising solution which can offer considerable improvements in spectrum utilization. Furthermore, cooperative communication is a concept proposed to obtain spatial diversity gains through relays without requiring multiple antennas. To benefit from both CR and cooperative communications, a combination of CR networks (CRNs) with cooperative relaying referred to as cognitive cooperative relay networks (CCRN) has recently been proposed. CCRNs can better utilize the radio spectrum by allowing the secondary users (SUs) to opportunistically access spectrum, share spectrum with primary users (PUs), and provide performance gains offered by cooperative relaying. In this thesis, a performance analysis of underlay CRNs and CCRNs in different fading channels is provided based on analytical expressions, numerical results, and simulations. To allocate power in the CCRNs, power allocation policies are proposed which consider the peak transmit power limit of the SUs and the outage probability constraint of the primary network. Thus, the impact of multiuser diversity, peak transmit power, fading parameters, and modulation schemes on the performance of the CRNs and CCRNs can be analyzed.

The thesis is divided into an introduction and five research parts based on peer-reviewed conference papers and journal articles. The introduction provides fundamental background on spectrum sharing systems, fading channels, and performance metrics. In the first part, a basic underlay CRN is analyzed where the outage probability and the ergodic capacity of the network over general fading channels are derived. In the second part, the outage probability and the ergodic capacity of an underlay CRN are assessed capturing the effect of multiuser diversity on the network subject to Nakagami- m fading. Considering the presence of a PU transmitter (PU-Tx), a power allocation policy is derived and utilized for CRN performance analysis under Rayleigh fading. In the third part, the impact of multiple PU-Txs and multiple PU receivers (PU-Rxs) on the outage probability of an underlay CCRN is studied. The outage constraint at the PU-Rx and the peak transmit power constraint of the SUs are taken into account to derive the power allocation policies for the SUs. In the fourth part, analytical expressions for the outage probability and symbol error probability for CCRNs are derived where signal combining schemes at the SU receiver (SU-Rx) are compared. Finally, the fifth part applies a sleep/wake-up strategy and the $\min(N, T)$ policy to an underlay CRN. The SUs of the network operate as wireless sensor nodes under Nakagami- m fading. A power consumption function of the CRN is derived. Further, the impact of M/G/1 queue and fading channel parameters on the power consumption is assessed.

Preface

This thesis summarizes my work within the field of performance analysis of spectrum sharing networks with cooperative relaying over fading channels. The work has been conducted within the Faculty of Engineering and the Faculty of Computing, Blekinge Institute of Technology, Karlskrona, Sweden. The thesis consists of an introduction followed by five research parts as follows:

Introduction

Part I

- A** Outage Analysis of Cognitive Relay Networks over α - μ Fading Channels
- B** Ergodic Capacity of Cognitive Relay Networks over α - μ Fading Channels

Part II

- A** Outage Probability and Ergodic Capacity of a Spectrum Sharing System with Multiuser Diversity
- B** Outage Probability and Ergodic Capacity of Underlay Cognitive Radio Systems with Adaptive Power Transmission

Part III

Outage Probability of a Cognitive Cooperative Relay Network with Multiple Primary Users Under Primary Outage Constraint

Part IV

- A** On the Performance of Cognitive Radio Networks with DF Relay Assistance Under Primary Outage Constraint Using SC and MRC
- B** Symbol Error Probability of Cognitive Cooperative Relay Networks Under Primary Outage and Secondary Peak Transmit Power Constraints

Part V

On Power Consumption of Wireless Sensor Nodes with Min(N,T) Policy in Spectrum Sharing Systems

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Karlskrona, March 2015

To my family

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Acronyms

AF	Amplify-and-Forward
AWGN	Additive White Gaussian Noise
BEE	Berkeley Emulation Engine
CCI	Co-Channel Interference
CCRN	Cognitive Cooperative Relay Network
CDF	Cumulative Distribution Function
CF	Compress-and-Forward
CR	Cognitive Radio
CRN	Cognitive Radio Networks
CSI	Channel State Information
D2D	Device-to-Device Communication
DF	Decode-and-Forward
DLC	Delay Limited Capacity
FCC	Federal Communications Commission
FPGA	Field-Programmable Gate Array
GP	Gel'fand-Pinsker
ISI	Intersymbol Interference
ITU	International Telecommunications Union
LOS	Line-of-Sight
MGF	Moment Generating Function
MRC	Maximal Ratio Combining
PDF	Probability Density Function
PSK	Phase Shift Keying
PU	Primary User
PU-Tx	Primary User Transmitter
QoS	Quality of Service
RF	Radio Frequency
SC	Selection Combining
SEP	Symbol Error Probability
SINR	Signal-to-Interference-plus-Noise Ratio
SNR	Signal-to-Noise Ratio
SPTF	Spectrum Policy Task Force
SR	Secondary Relay
SU-Tx	Secondary User Transmitter
USRP	Universal Software Radio Peripheral
WARP	Wireless Open Access Research Platform

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Introduction

1 Motivation

The continuing technological evolution of wireless communications is driven by an increasing demand on high-rate multimedia services and high spectral efficiency. Examples include mobile networks that support services such as video teleconferencing, gaming, and high-rate multimedia streaming. Such mobile networks require technical advances, not only in the development of the physical layer but also for the whole protocol stack.

Achieving high spectral efficiency and providing reliable communications in wireless systems under severe fading conditions are main technical challenges in a radio spectrum that is nowadays becoming exhausted. Promising solutions that help to overcome these challenges have been proposed recently. Cognitive radio (CR) [1], a technology that offers considerable improvements to the spectral efficiency, has attracted the interest of many researchers. Measurements have shown that the spectrum scarcity problem is caused by inefficiency in spectrum utilization rather than the spectrum shortage [2]. For instance, most of the spectrum is occupied by primary users (PUs) which own a spectrum license but do not transmit at all locations all the time. Making more spectra available and open to secondary users (SUs) may enhance the growth of new services. An example is the large increase of innovations that have resulted from the exploitation of unlicensed bands with WiFi and Bluetooth technologies.

CR networks (CRNs) distinguish three spectrum access paradigms based on the type of available network information and regulatory constraints. Those spectrum access paradigms are underlay, overlay, and interweave. For the underlay spectrum access paradigm, the SU transmitters (SU-Txs) send their signals such that the interference caused to the PU receivers (PU-Rxs) is kept below a given threshold. The SU-Tx and the PU transmitter (PU-Tx) transmit their signals concurrently. The overlay spectrum access paradigm also allows the SU-Tx and PU-Tx to transmit concurrently. In this approach, the SU-Tx has knowledge of the PU's codebooks and its messages. The codebook information could be obtained, for example, if PUs broadcast their codebooks

periodically or PUs employ a uniform standard for communication based on a publicized codebook. The knowledge of PU messages can be achieved by decoding the message at the SU receiver (SU-Rx). However, it is commonly assumed in the overlay model that the PU message is known at the SU-Tx when the PU begins its transmission [3]. This is not practical for an initial transmission. However, the assumption holds for a message retransmission where the SU listens to the first transmission and decodes it, while the intended receiver cannot decode the initial transmission due to fading or interference [3]. For the interweave spectrum access paradigm, the SUs opportunistically exploit spectrum holes (potential opportunities for non-interfering use of spectrum) to communicate without disrupting the PU communications [4, 3]. These spectrum holes can be considered as multidimensional within frequency, time, code, and space.

Cooperative communication, on the other hand, has shown benefits for allowing reliable communications with an increase of radio coverage. The idea behind cooperative communications is that the direct communication between the source and the destination can be supported by a relay channel. With cooperative relaying, the end-to-end transmission in the time domain, is divided into two phases, namely, broadcasting and relaying phases. In the broadcasting phase, the source sends data to both relay and destination. In the relaying phase, the relay processes the received data and then forwards it to the destination. The signals received at the destination are combined into one signal to recover the source data. Practical relaying protocols and coding designs have been extensively studied in order to achieve cooperative diversity [5]. Early works on cooperative communications and information theoretic properties of the relay channel can be tracked back to the 1970's [6, 7].

Due to the benefits offered by both concepts, CR and relaying, many research works have focused on their combination to improve the performance of the secondary network. The networks resulting from this combination, in this thesis, are referred to as cognitive cooperative relay networks (CCRN). The performance of CRNs and CCRNs using different performance metrics in different fading environments have been widely discussed in the literature. For instance, the channel capacity for a basic underlay CRN under different fading conditions has been investigated in [8]. For delay-sensitive applications, in order to support quality of service (QoS) requirements for the secondary network, other notions of capacity have become attractive. For example, effective capacity is considered as a link-layer model supporting QoS metrics such as data rate, packet loss rate, and service delay. It can be regarded as the maximum constant data rate that can be supported while satisfying the packet loss rate target with a specific delay. In [9], the effective capacity of a

CCRN under Rayleigh fading has been analyzed. In [10], from a cross-layer design perspective, under Rayleigh fading, the authors analyzed the effective capacity and proposed a scheme aiming at maximizing the supported arrival rate subject to a given statistical delay QoS constraint in CCRNs.

Various studies involving CCRNs have generally characterized the wireless fading channels by statistical distributions of the faded signals such as Rayleigh, Nakagami- m , and Weibull distributions. In particular, Rayleigh and Nakagami- m distributions have received large attention due to their wide range of applicability. On the other hand, the α - μ distribution has recently been proposed in [11] as a general model that includes important distributions such as Nakagami- m and Weibull distributions. The α - μ distribution considers the non-linearity of the propagation environment and the number of multipath clusters, two important phenomena inherent to radio propagation. Other distributions such as one-sided Gaussian, Rayleigh, or negative exponential distribution can be obtained as special cases.

Given the above background, this thesis analyzes, under different fading channels, the performance of CRNs and CCRNs in terms of metrics such as outage probability, ergodic capacity, and symbol error probability. The thesis focuses on the underlay approach of CRNs and CCRNs due to its main advantage of allowing the SU-Txs and PU-Txs to transmit their signals concurrently. The power consumption of an underlay CRN and the adaptive power allocation for different underlay CRNs and CCRNs are also derived.

The remainder of this introduction is organized as follows. Section 2 gives an overview of the radio spectrum. Section 3 provides fundamentals of CRNs. Section 4 discusses spectrum sensing methods for CRNs. Section 5 introduces the use of relaying with CRNs to establish CCRNs. Section 6 gives the basics of radio propagation mechanisms and fading channels. Section 7 highlights some important performance metrics. Section 8 provides an overview of this thesis. Finally, Section 9 suggests some topics for future works.

2 Radio Spectrum

The radio spectrum is a range of frequencies, part of the electromagnetic spectrum, from approximately 3 kHz to around 300 GHz. Radio communication and many other applications find use in this range also called radio frequency (RF). At frequencies ranging beyond 300 GHz, there is high absorption of the electromagnetic radiation so that the atmosphere becomes effectively opaque. The atmosphere becomes transparent again in the frequency ranges of the near-infrared and optical window. Frequencies are often grouped in ranges of small sections called bands. In practice, those bands are frequencies set aside or used for the same purpose.

2.1 Radio Spectrum Management

The radio spectrum is a key resource for wireless communication. It is regulated by the government in most countries and coordinated by the International Telecommunications Union (ITU). An important role of radio spectrum management consists of regulating the use of radio frequencies to promote their efficient use. Different radio frequency bands are allocated for different radio transmission technologies and applications. Frequency bands are often allocated to operators of radio systems such as cellular mobile communication systems or broadcast television stations.

There is a high cost associated with this allocation which makes the spectrum affordable only for larger telecommunications and media companies. However, some frequency bands are designated by the ITU as unlicensed. The user terminals operating within these bands are subject to power and antenna gain limitations that restrict their range.

According to the ITU, the radio spectrum is subdivided into eight assigned bands as shown in Table 1. The frequency range of different radio bands, including the last unassigned band is provided in Table 2. The number of each band is the logarithm of the approximate geometric mean of the upper and lower band limits in Hz [13]. For example, the approximate geometric mean of Band 6 is 1 MHz, or 10^6 Hz.

Differently to the current traditional fixed allocation policy, a tendency is towards a near zero regulation of the radio spectrum, evoked by Paul Baran [14], and the open spectrum by Eli Noam [15]. Many ideas and approaches to reform spectrum allocation suggest that spectrum should be accessed dynamically in time and space [16]. The motivation behind this reform is the shortage paradox lying between the spectrum management policy and the physical scarcity of usable frequencies.

Table 1: International Telecommunication Union radio bands [12]

Band name	Abbreviation	Example use
Very Low Frequency	VLF	Long distance radio
Low Frequency	LF	Naval broadcast
Medium Frequency	MF	Aeronautical communications
High Frequency	HF	Sound broadcasting
Very High Frequency	VHF	Private business radio
Ultra High Frequency	UHF	TV broadcasting
Super High Frequency	SHF	Radar
Extremely High Frequency	EHF	Broadband wireless access

Table 2: Frequency range of different radio bands

Band	Abbreviation	Frequency range	Wave subdivision
4	VLF	[3 kHz, 30 kHz[	Myriameter
5	LF	[30 kHz, 300 kHz[	Kilometer
6	MF	[300 kHz, 3000 kHz[	Hectometer
7	HF	[3 MHz, 30 MHz[	Decameter
8	VHF	[30 MHz, 300 MHz[	Meter
9	UHF	[300 MHz, 3000 MHz[	Decimeter
10	SHF	[3 GHz, 30 GHz[	Centimeter
11	EHF	[30 GHz, 300 GHz[	Millimeter
12	-	[300 GHz, 3000 GHz[	Decimillimeter

2.2 Inefficiency in Spectrum Utilization

Spectrum allocation throughout the world has been based on a fixed allocation for which a particular band is specified and dedicated to a primary network. For example, broadcasters and commercial cellular service providers operate in specified bands under restricted use licenses. This type of spectrum management has started being perceived as inefficient. Measurement campaigns conducted by the Federal Communications Commission (FCC) through the Spectrum Policy Task Force (SPTF) have shown that frequency bands are under-utilized, indicating many portions of unused or lightly used spectrum for significant periods of time [2]. Some spectrum measurement campaigns that have been recently conducted are briefly reported in Section 2.3.

It has been essential to define a design paradigm for adaptive algorithms that enable much higher spectrum utilization. Such a paradigm should provide more reliable and personal radio services, reduce harmful interference, and facilitate the inter-operability of different wireless communication networks [17]. To this end, CRs have gained much attention by the research community as a powerful solution. CRs should handle seamless adaptation of radio link parameters, make opportunistic use of under-utilized spectrum, and provide increased flexibility in modulation and waveform selection to better adapt to the varying wireless environment [1, 18].

2.3 Spectrum Measurement Campaigns

Early works in spectrum measurements have characterized variations in spectral occupancy of HF radio communications in terms of frequency, time, bandwidth, threshold level of the received signal, type of user allocation, antenna, and geographical location [19]. To review the occupancy of spectrum, sev-

eral measurement campaigns have been conducted in the last two decades. Spectrum occupancy for a channel can be defined as the fraction of time that the received power in the channel exceeds a threshold level [20]. The study in [21] characterizes the influence of the spatial dimension on the spectrum occupancy in a given area by introducing the directional spectrum occupancy metric. Therein, the directional spectrum occupancy is defined as the fraction of time that the received power in a channel exceeds a threshold in a given measurement direction.

For the United States, the study in [22] provides broadband measurement surveys of spectrum usage in several metropolitan areas across the frequency range from 108 MHz to 19.7 GHz. The study in [23] reports an extensive measurement campaign conducted in Aachen, Germany, comparing indoor and outdoor measurement results. The results have shown high spectrum occupancy in the outdoor scenario in the band from 20 MHz up to 3 GHz, whereas less occupancy was measured in the indoor scenario. The measurements have shown that the spectrum occupancy in the band 3-6 GHz is very low. A spectral occupancy measurement campaign has been conducted in the frequency range between 806 MHz and 2.75 GHz in urban Auckland, New Zealand [24]. The results indicate that the actual spectral usage in that band is only about 6.2%. In Singapore, a 24-hour spectrum usage pattern in the frequency band ranging from 80 MHz to 5.85 GHz is reported in [25]. With spectrum measurements taken over 12 weekdays, the results have revealed that a significant amount of spectrum in Singapore has very low occupancy all the time. An average spectrum occupancy of 4.54% is reported. In an urban area, the study in [26] reports the spectrum occupancy measurements conducted in the frequency range from 75 MHz to 3 GHz in an outdoor environment in Barcelona, Spain. The study reports an overall average spectrum occupancy of 22.57% over the whole frequency range considered. In [27], the spectrum usage pattern is explored in the frequency bands ranging from 20 MHz to 3 GHz in Ho Chi Minh City and the Long An province of Vietnam. The measurement results indicate that a spectrum occupancy, on average, in all bands is 13.74% for Ho Chi Minh City and 11.19% for the Long An province. An average spectrum occupancy of 12.5% in the city of San Luis Potosi, Mexico has been reported as a result of a spectrum occupancy campaign [28]. The frequency band of interest considered in that study ranges from 30 MHz to 910 MHz.

3 Fundamentals of Cognitive Radio Networks

The scarcity of energy and bandwidth, two fundamental resources for communications, severely affects the service quality and channel capacity in wireless

systems. A huge number of research works in wireless networks are currently focusing on new communication and networking paradigms that can intelligently and efficiently utilize these scarce resources [29]. Cognitive radio is one critical enabling technology that utilizes the limited network resources in a more efficient and flexible way by allowing the radios/devices to adapt their operating parameters to the variations of the surrounding radio environment. Such parameters are, for example, transmission power, frequency, and modulation type. In order to adapt to the variations of the surrounding radio environment, a cognitive radio, differently to conventional radio devices, equips devices with a cognitive capability and reconfigurability [30]. The cognitive capability enables the device to be aware of its surrounding environment, that is, sensing and gaining necessary information about, for example, transmission power, frequency, or modulation type. Such capability allows the cognitive user to select the available spectrum. Reconfigurability means that the cognitive radio can rapidly adapt the transmission parameters according to the sensed information so as to reach the optimal performance. Important functions of a CR can be summarized mainly into the following tasks:

- Spectrum sensing

SUs are supposed to regularly monitor the activity of PUs. Appropriate and accurate sensing along with signalling mechanisms to cope with PU activity is very essential in CRs.

- Channel identification

This function is carried out in the SU-Rx. It encompasses the estimation of channel state information (CSI) and the prediction of channel capacity for use by the SU-Tx.

- Transmit power control with respect to PU

This task is carried out in the SU-Tx. The power control should be effective so that the tradeoff between the QoS of both primary and secondary users is satisfied.

3.1 Typical Candidate Bands for Cognitive Radios

UHF bands, cellular bands, and fixed wireless access bands are the typical candidate bands for CR systems as follows.

UHF band

The UHF band is currently used for TV broadcasting. In the broadcast television spectrum, the regulatory agency FCC has adopted policies [2] that allow the SU-Txs to operate at locations where the spectrum is not used by PUs. The unused TV spectrum has been predicted to be one of the first spectrum ranges where innovative products and services using CR systems may appear [31].

Cellular bands

Many cellular bands are centered around 800/900 MHz, 1.8/1.9 GHz, 2.1 GHz, 2.3 GHz, and 2.5 GHz. They are characterized by an ubiquitous coverage and the communication is bidirectional. The cellular customers are mobile and the cell sites are generally in the same region as the CRs.

Fixed wireless access bands

The fixed wireless access bands provide bidirectional broadband services and are centered around 2.5 GHz and 3.5 GHz. Wireless systems operating in these bands are very similar to cellular networks, except that the devices are not mobile in fixed wireless access bands. The devices are at fixed locations such as homes or businesses.

3.2 Cognitive Radio Network Paradigms

CRNs have focused on three main spectrum access paradigms, namely, underlay, overlay, and interweave. Each of these paradigms requires a different level of cognition about the surrounding environment and a different level of sophistication which leads to different challenges. This thesis mainly focuses on underlay CR systems due to the main advantage of underlay spectrum access of allowing simultaneous transmissions of SUs and PUs. Other reasons include, for example, compared to the overlay scheme, less difficulties in implementation and less assumptions in the analysis. In particular, the attention is on scenarios where the PU is willing to share its spectrum with the SU, given that the PU rate requirements are satisfied or a certain threshold of interference to the PU-Rx is respected.

3.2.1 Underlay Paradigm

The underlay approach, as illustrated in Fig. 1, allows SUs and PUs to transmit concurrently subject to the constraint that the interference caused by the SU stays below an acceptable threshold.

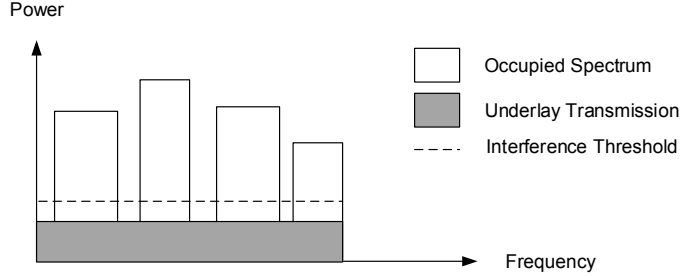


Figure 1: Spectrum occupancy and interference threshold for an underlay CRN.

A basic model of an underlay CRN is shown in Fig. 2. In this model, the SU-Tx communicates with the SU-Rx through a wireless link with channel power gain g_1 . The channel power gain for the interfering link SU-Tx $\rightarrow$ PU-Rx is denoted by g_0 . The PU-Tx is assumed to be located far away from the secondary network and hence does not interfere with the SU communication. Given this model, the study in [8] derived the capacity of a basic underlay CRN subject to average and peak interference power constraints. Different studies, e.g., [32, 33, 34, 35] have analyzed the performance of underlay CRNs

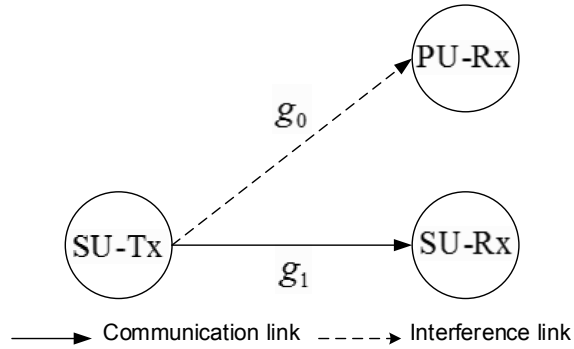


Figure 2: An example of a basic underlay CRN where PU-Tx is far away from the secondary network.

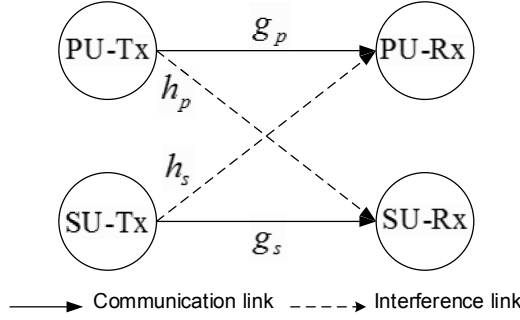


Figure 3: A simple model of a CRN where PU-Tx is in proximity of the secondary network.

subject to the peak interference power constraint with different performance metrics. For example, the outage probability and some queuing performance metrics are analyzed in [32, 33] for the basic underlay CRN presented in Fig. 2. The outage capacity and ergodic capacity are analyzed in [34, 35].

Fig. 3 shows an underlay CRN where the PU-Tx is assumed to be in the range of the secondary network. The interfering channel from PU-Tx to SU-Rx is modeled with the channel power gain h_p . The channel power gain between PU-Tx and PU-Rx is denoted by g_p . For this type of model with PU-Tx in the proximity of the secondary network, a system outage analysis is conducted in [36].

Underlay CRN subject to average interference power constraint

For a network such as that shown in Fig. 2, the average interference power constraint can be expressed as

$$\mathbb{E}[P(g_0, g_1)g_0] \leq Q_{avg} \quad (1)$$

where $\mathbb{E}[\cdot]$ denotes the expectation operator, $P(g_0, g_1)$ denotes the SU transmit power as a function of channel power gains g_0 and g_1 , and Q_{avg} is the average interference power tolerated by the PU-Rx. The average interference power constraint is also called long-term interference power constraint. As the transmit power of the SU-Tx cannot be infinite in practice, an additional

constraint may be invoked such as

$$\mathbb{E}[P(g_0, g_1)] \leq P_{avg} \quad (2)$$

where P_{avg} denotes the average transmit power limit of the SU-Tx.

Underlay CRN subject to peak interference power constraint

Similarly, the peak interference power constraint, also called short-term interference power constraint, can be expressed as

$$P(g_0, g_1)g_0 \leq Q_{peak} \quad (3)$$

where Q_{peak} denotes the peak interference power tolerated at the PU-Rx. Again, an additional constraint accounting for the practical limitations on the transmit power is expressed as

$$P(g_0, g_1) \leq P_{peak} \quad (4)$$

where P_{peak} denotes the peak transmit power of the SU-Tx [37].

3.2.2 Overlay Paradigm

The overlay approach also allows SU and PU to transmit concurrently. However, for this approach, the SU-Tx requires knowledge of the PU codebooks and its messages. The SUs exploit PU information to either cancel or mitigate the induced interference. The knowledge of the PU messages allows the SU-Tx to apply a suitable encoding scheme that will improve both its own rate and that of the PU.

Different strategies for encoding/decoding in overlay CRNs are reviewed in [3]. Some of the encoding strategies are rate-splitting [38], Gel'fand-Pinsker (GP) binning [39], cooperation, and superposition coding. These techniques are mostly derived from encoding strategies for the interference, broadcast, and multi-access channels. The rate-splitting, which is the best known encoding technique for interfering channels, improves rates by enabling (partial) interference cancelation at the decoders. For the GP binning and binning against a codebook, the SU encoder improves its rate by precoding against interference. Specifically, for Gaussian channels, dirty paper coding [40] can be applied. For the encoding cooperation, the SU encoder increases the rate of the PU by (partially) relaying the PU message. Finally, superposition coding allows a combination of some of the above techniques.

3.2.3 Interweave Paradigm

The interweave approach is based on the idea of opportunistic communication that allows SUs to transmit only over spectrum holes that arise in space, time, code, or frequency [30]. In fact, this was the original motivation for cognitive radio [1]. The approach requires knowledge of the activity of the PUs. This means that the SU-Tx should regularly monitor the spectrum area of interest and detect the spectrum holes before transmitting to the SU-Rx. Spectrum sensing techniques have been developed for spectrum hole detection [41]. Basically, the performance of the interweave approach is determined by the applied spectrum sensing technique.

3.2.4 Comparison of the Spectrum Access Paradigms

Apparently, there is no best solution among the three mentioned approaches, as each of them has its advantages and disadvantages [42].

For instance, in the underlay approach the main advantage is that SUs can transmit simultaneously with PUs as long as the interference caused is below an acceptable limit. The disadvantage is that this limit is hard to estimate and depends on the relative position between the SU-Tx and interfered PU-Rx. Also, due to the imposed interference power limit by primary networks, the SUs have always to transmit at low power. Clearly, this drawback has a negative impact on the CRNs coverage. As a result, CCRNs have been proposed as a solution of utmost importance, especially, for coverage improvement, and hence improving the overall system performance of CRNs.

The main advantage of the overlay approach is the possibility to have concurrent transmissions of SU and PU within the same interference region. The interference caused to the PU can be offset by relaying the PU's message. However, a critical point of the overlay approach is whether the primary messages are allowed to be known by the secondary network and how to obtain this knowledge. As drawback, a tight interaction between PUs and SUs is required. This leads to a higher degree of sophistication of the SU terminals and requires PU system flexibility.

In the interweave spectrum access paradigm, an advantage is that an SU can transmit at any power once a spectrum hole has been detected. The disadvantage happens when no spectrum hole is available, or when SUs have to wait until a spectrum hole is available. This can cause long delays for SU transmissions. If the PU has its own transmissions, the SU has to vacate the occupied spectrum hole and the PU reoccupies the spectrum hole. In a highly fluctuating radio environment, spectrum hole detection may become difficult. This can lead to missed detection, causing severe interference to the PU.

The three approaches can also be compared by taking into account the required information and the SU transmit power as follows.

Comparison based on the information required

Regarding the required information for the underlay approach, SUs require knowledge of the interference caused by the SU-Txs to the PU-Rxs. That is, CSI of the interfering channels to the PU-Rxs should be obtained by the SUs. For the overlay approach, the SUs require knowledge of the PUs codebook and its messages. Finally, in the interweave approach, the knowledge of the PU activity is required which can be obtained by spectrum sensing.

Comparison based on the SU transmit power

From an SU transmit power point of view, it is worth noting that for the underlay approach, the SU transmit power is not only limited by the device constraints, but also by the interference constraint of the primary network. The SU transmit power is only limited by device constraints for the overlay systems. For the interweave approach, the SU transmit power is essentially limited by the device constraints and the range of sensing.

3.2.5 Hybrid Schemes

Hybrid solutions that combine the advantages of different approaches can be a promising solution to overcome the current spectrum under-utilization. For instance, the study in [43] integrates the overlay scheme with the interweave scheme to form a protocol called credit-based overlay and interweave dynamic spectrum access. The use of this protocol has mainly focused on multi-hop wireless networks subject to two challenging design constraints: (1) PUs cannot be modified, and (2) the performance of PUs cannot degrade. Specifically, the study uses a notion of credits where SUs first help with the traffic delivery for PUs. In return, SUs are allowed to access the spectrum in a manner disruptive to PUs.

Regarding other studies that combine different spectrum access paradigms, the interested reader is referred, for instance, to the results reported in [44, 45]. Therein, the authors investigate the combination of the underlay approach with the interweave approach. Specifically, with the goal of maximizing the average service rate and minimizing the overall average delay of the SUs multiple queues, a hybrid CR system is proposed in [44]. The proposed model combines the conventional interweave and underlay paradigms for enhancing the chances of the SU to access the spectrum. Importantly, an optimal weighting coefficient of the hybrid interweave-underlay scheme that maximizes the

average service rate of the SU is determined. The results reported in [45] show that the hybrid approach, combining interweave-underlay spectrum access systems, outperforms the conventional underlay spectrum access. The study considers amplify-and-forward relaying and uses a continuous-time Markov chain to model and analyze the proposed hybrid spectrum access. Therein, the performance is assessed in terms of outage probability, symbol error rate, and outage capacity over Nakagami- m fading. The study in [46] proposes a combination of the overlay and underlay approaches in a multiple primary operators environment. The authors make use of the relaying capability of the CR base station and linear programming with interference cancelation techniques to maximize the SU capacity. Again, with a hybrid overlay-underlay spectrum sharing method, fair access opportunities for SUs and throughput improvement for CRNs are the aims of a resource allocation investigation for downlink transmissions in [47]. Assuming mobility of SUs, an algorithm based on maximum weighted bipartite matching is proposed which outperforms, with a small reduction in network throughput, the channel greedy algorithm.

4 Spectrum Sensing for Cognitive Radio Networks

Spectrum sensing is a key enabling function of CRs. In order to provide more spectrum access opportunities to SUs without interfering to PUs, sensors that detect spectrum holes, i.e., under-utilized subbands of the radio spectrum are required [48]. They provide high spectral efficiency, estimate the average power in each subband of the spectrum, and identify the unknown directions of interfering signals [49]. By considering different dimensions of the spectrum space, the study in [41] reviewed several aspects of spectrum opportunity and spectrum sensing concepts. In the following, spectrum sensing methods and challenges for cognitive radios are briefly discussed.

4.1 Multidimensional Spectrum Opportunities

Spectrum sensing can be thought of as a multidimensional problem with different dimensions such as location, angle of arrival, frequency, and time. Each dimension has its own parameters that have to be sensed. Spreading code or hopping pattern can also be called dimension even though they can bring new challenges.

Opportunity in frequency and time

The available spectrum is divided into chunks of narrow bands. Spectrum opportunity in the frequency domain means that not all bands are used simultaneously, so that some bands are available for opportunistic usage. As can be seen in Fig. 4, the dimension in frequency goes hand in hand with the dimension in time.

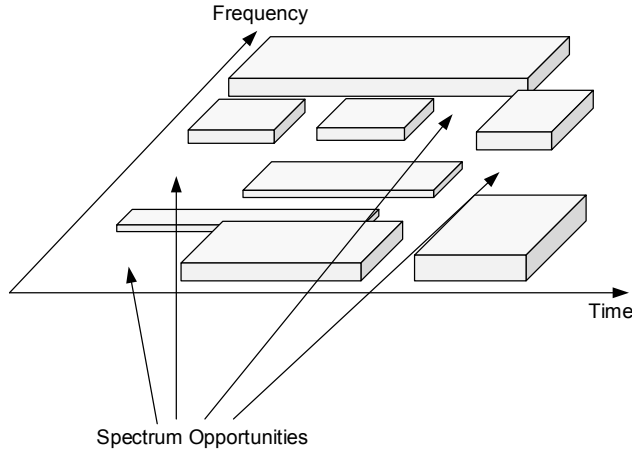


Figure 4: Opportunity in frequency and time.

The opportunity in time occurs when a specific band is available at certain times. The band is not continuously used, and hence, there will be times where it will be available for opportunistic usage.

Opportunity in geographical area

In some geographical areas, the spectrum can be available while it is occupied in some other parts of the area at a given time. An example of geographical separation between a PU and SUs communicating at the same frequency is illustrated in Fig. 5. In the geographical dimension, an SU makes use of path loss. If the SU does not detect any interference from the PU communications, then, it is assumed that there is no PU transmission in a given local area.

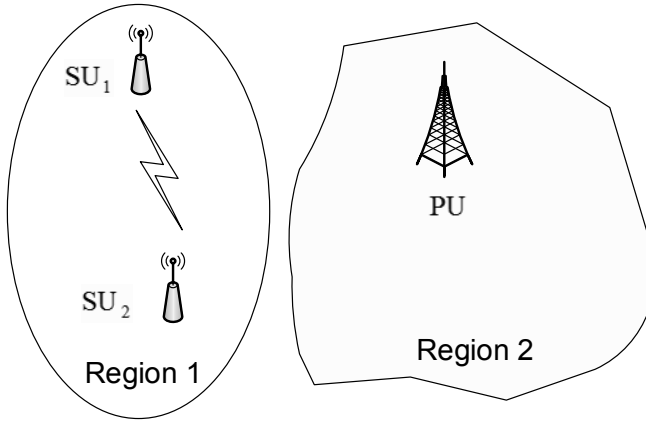


Figure 5: Geographical location of PU and SUs.

Opportunity in code domain

In order to make use of the opportunity in the code domain (see Fig. 6), it is necessary for an SU to not only detect the general usage of the spectrum, but also to determine the type of code used. If the PU code is available at the secondary network, SUs could simultaneously transmit with PUs with no interference to PUs communications by using orthogonal codes, through spread spectrum techniques.

Nevertheless, if PUs at a given time use the spectrum over a wide band through spread spectrum techniques, there is a difficulty of obtaining the necessary PU code information by using conventional sensing techniques. Here, the SUs require timing information so that they can synchronize their transmissions with PUs.

Opportunity in angle domain

The location/position and direction of an RF beam of the PU can be used by the SU as spectrum opportunity. As illustrated in Fig. 7, if a PU is transmitting in a specific direction, an SU can transmit in other directions without causing harmful interference to the PU. This can be feasible when PUs are using directional antennas and beamforming.

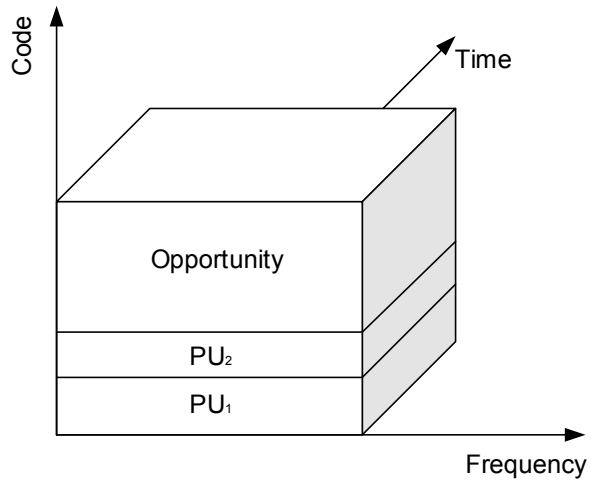


Figure 6: Opportunity in code domain.

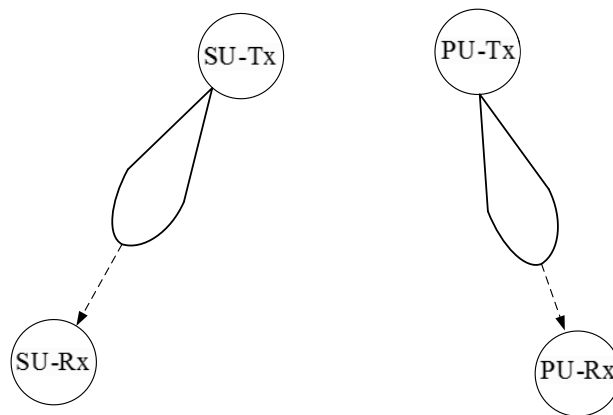


Figure 7: Opportunity in angle domain.

4.2 Spectrum Sensing Techniques

Through sensing, SUs can learn the presence of PUs and the availability of spectrum holes. Accordingly, an SU can adapt its transmit parameters such as transmit power, carrier frequency, and modulation schemes to achieve efficient spectrum utilization. Fig. 8 compares main sensing methods with respect to their accuracy and complexity. As can be seen from the figure, waveform-based sensing and matched filtering are more accurate than energy detection and cyclostationarity-based methods. In this section, we briefly discuss the spectrum sensing techniques shown in Fig. 8. For details on other sensing methods including multitaper spectral estimation, wavelet transform based estimation, and adaptive beamforming, the interested reader is referred to [30, 50, 51, 52].

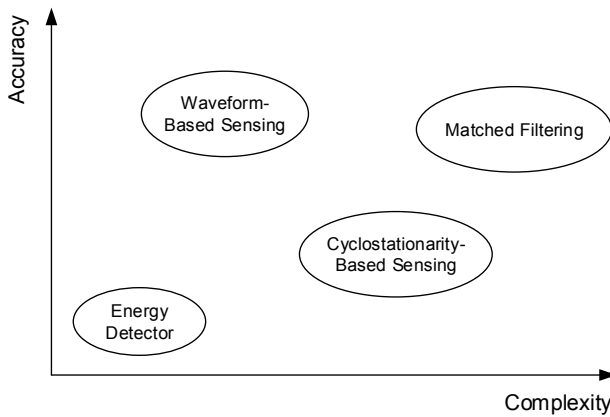


Figure 8: Main sensing methods [41]: Accuracy versus complexity.

4.2.1 Energy Detector Sensing

The most common type of spectrum sensing is the energy detector due to its simple implementation. It does not require *a priori* knowledge about the primary signal, and the detector requires only a short detection time. The signal is detected from a comparison of the energy detector output with a threshold which depends on the noise floor [53]. Without loss of generality, let $s(k)$ be the signal to be detected, and $n(k)$ be the additive white Gaussian noise (AWGN) sample, where k is the sample index. The received signal can

be written as

$$y(k) = s(k) + n(k) \quad (5)$$

For $s(k) = 0$, the PU is not transmitting. The decision for the energy detector can be simply realized with the following metric:

$$Y = \sum_{k=1}^K |y(k)|^2 \quad (6)$$

where K is the size of the observation vector. By comparing the metric Y with a fixed threshold λ_E , the decision of the energy detector is based on the following hypothesis model of the received signal:

$$\begin{aligned} \mathcal{H}_0: & \quad y(k) = n(k), & Y < \lambda_E \\ \mathcal{H}_1: & \quad y(k) = s(k) + n(k), & Y \geq \lambda_E \end{aligned} \quad (7)$$

where hypothesis $\mathcal{H}_0$ assumes that the PU is silent and hypothesis $\mathcal{H}_1$ assumes that the PU is active. The performance of the detector is characterized by two probabilities: the probability of false alarm P_F and the probability of detection P_D . The probability P_F accounts for the case where the hypothesis test decides $\mathcal{H}_1$ while it is actually $\mathcal{H}_0$, i.e.,

$$P_F = \Pr(Y \geq \lambda_E | \mathcal{H}_0). \quad (8)$$

The probability P_D , accounting for the case where the test correctly decides $\mathcal{H}_1$, is expressed as

$$P_D = \Pr(Y \geq \lambda_E | \mathcal{H}_1). \quad (9)$$

From the above probabilities, a well chosen energy detector should guarantee a high probability of detection P_D and a low false alarm probability P_F .

Selection of the threshold λ_E for detecting primary users, inability to differentiate interference from primary users and noise, and poor performance for low signal-to-noise ratio (SNR) are some of the challenges of the energy detector. Also, energy detectors do not work efficiently for detecting spread spectrum signals. The performance of energy detector based sensing over various fading channels has been investigated in [54]. Therein, it is shown that there is no significant improvement in the probability of detection when either P_F exceeds 0.1 or the average SNR exceeds 20 dB. Concerning the detection time and other timing related parameters, the interested reader is referred to [55].

4.2.2 Waveform-Based Sensing

In the presence of a known signal pattern such as preamble, midamble, regularly transmitted pilot pattern, and spreading sequence, sensing can be performed by correlating the received signal with a known copy of itself. This

method is also termed coherent sensing [56]. Using the same model as in (5), the waveform-based sensing metric can be expressed as

$$Y = \mathcal{R}e \left[\sum_{k=1}^K y(k) s^*(k) \right] \quad (10)$$

where $(\cdot)^*$ denotes the complex conjugate of its argument. In the presence of a PU signal, the sensing metric can be written as [41]

$$Y = \sum_{k=1}^K |s(k)|^2 + \mathcal{R}e \left[\sum_{k=1}^K n(k) s^*(k) \right] \quad (11)$$

In the absence of a PU signal, the metric is expressed as [56]

$$Y = \mathcal{R}e \left[\sum_{k=1}^K n(k) \right] \quad (12)$$

The decision on the presence of a PU signal can be obtained by comparing the metric Y with a fixed threshold λ_W . Waveform-based sensing exhibits short measurement time [57], but sensing is susceptible to synchronization errors.

4.2.3 Cyclostationarity-Based Sensing

The statistics of the transmitted signals in many communication systems are periodic because of inherent periodicities such as modulation rate and carrier frequency. Generally, cyclostationary features result from the periodicity in the signal or in its statistics like mean and autocorrelation [58]. They can be intentionally induced to assist spectrum sensing. From cyclostationary features [59, 60], a detector can distinguish cyclostationary signals from stationary noise. This is because the noise is wide-sense stationary with no correlation while modulated signals are cyclostationary with spectral correlation due to the redundancy of signal periodicities. The cyclic spectral density function of a received signal (5) can be obtained as [58]

$$\tilde{S}_y^\alpha(f) = \sum_{l=-\infty}^{\infty} \tilde{R}_y^\alpha(lT_s) e^{-j2\pi lT_s f} \quad (13)$$

where α is the cyclic frequency and T_s is the sampling period. The cyclic autocorrelation function is defined as [58]

$$\tilde{R}_y^\alpha(lT_s) = \langle y(kT_s + lT_s) y^*(kT_s) e^{-j2\pi\alpha kT_s} \rangle e^{-j\pi\alpha lT_s} \quad (14)$$

where $\langle \cdot \rangle$ denotes the discrete-time averaging over k . Cyclostationary detectors can differentiate noise from PU signals and have better detection robustness in the low SNR regime. Different to cyclostationary detectors, energy detectors cannot detect weak signals and can cause a high false alarm probability due to noise uncertainty.

4.2.4 Matched Filtering

From a signal processing point of view, a matched filter correlates a known signal (PU signal), or template, with an unknown received signal to detect the presence of the PU [61]. Matched filtering requires perfect knowledge of the PU signal, such as the operating frequency, bandwidth, modulation type and order, pulse shape, and packet format. One of the important advantages of matched filtering is the short time required to achieve a certain probability of detection. Matched filtering is more robust to noise uncertainty and presents a better detection in the low SNR regime than feature detectors. Moreover, it requires less signal samples to achieve good detection. However, the matched filtering has some disadvantages such as complex implementation and high power consumption [57]. Also, matched filtering requires precise prior information about certain waveform patterns of PU signals. Otherwise, if such information is wrongly provided, the sensing performance degrades rapidly.

4.3 Challenges of Spectrum Sensing in Cognitive Radio Networks

One of the challenges of spectrum sensing in cognitive radio networks is the sensing duration/frequency as SUs continuously monitor the presence of PUs. If an SU already occupies a PU frequency band, the PU detection when PU reappears should be done as quickly as possible. The SU has to vacate the occupied band immediately in order to avoid interference to the PU communications.

From this point of view, there should be sensing methods that are capable of detecting the presence of PUs within a certain duration. For example, the sensing period is selected as 30 seconds in the IEEE 802.22 draft standard [55]. Such a requirement implies performance limitations on sensing algorithms and poses a challenge for cognitive radio design. Sensing frequency, on the other hand, expresses how often an SU should perform spectrum sensing. It is a design parameter to be chosen carefully. The optimum value of sensing frequency depends upon the hardware capabilities of the SU and temporal and spatial characteristics of the PUs. For details on timing related parameters such as sensing frequency, channel detection time, and channel move time, the interested reader is referred to the IEEE 802.22 draft standard [55].

For a hidden PU, the problem is quite similar to the hidden node problem in carrier sense multiple access. It is due to many factors including severe multipath fading or shadowing observed by SUs while scanning for primary user transmissions. Cooperative sensing is proposed in the literature for handling the hidden PU problem [57, 62]. Other challenges of spectrum sensing in cognitive radio networks include the problem of detecting spread spectrum PUs, hardware requirements, decision fusion in cooperative sensing, and security.

4.4 Sensing Testbed

An accessible testbed for implementing sensing algorithms is a combination of GNU Radio [63] for software development and the Universal Software Radio Peripheral (USRP) for the hardware that transmits and receives the data. GNU Radio is a free software toolkit for building and deploying software radios. It provides a library of signal processing blocks and the connection to tie them all together. GNU Radio is open source which provides complete source code. GNU Radio uses C and C++ for signal processing blocks writing, and mainly Python for constructing signal flow graphs and visualization tools. The USRP was developed by Ettus Research LLC as a result of the GNU

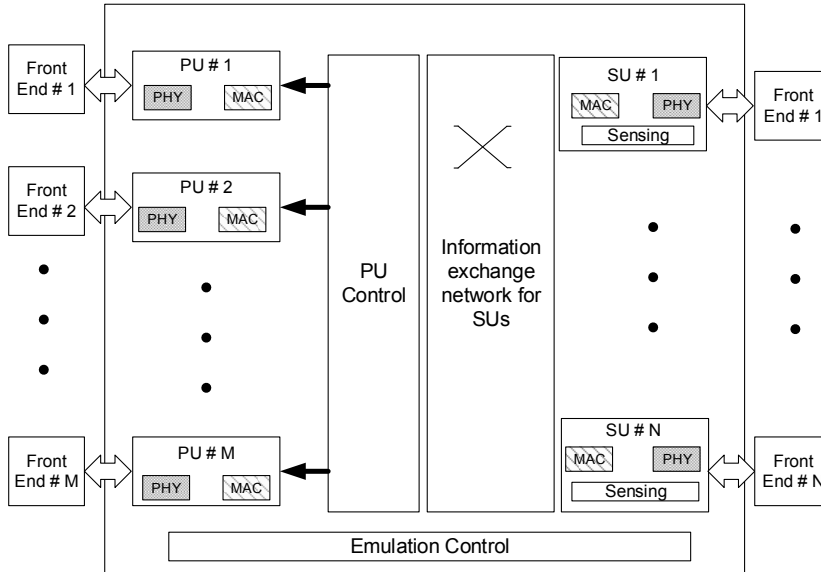


Figure 9: Testbed for cognitive radios [64].

Radio project [65]. The USRP is designed to allow general-purpose computers to function as high-bandwidth software radios. It serves as a digital baseband and intermediate frequency section of a radio communication system. The basic design philosophy behind the USRP has been to do all the waveform-specific processing, like modulation and demodulation, on the host central processing unit. All the high-speed general-purpose operations, like digital up- and down-conversion, decimation, and interpolation, are executed on a field-programmable gate array (FPGA). The USRP is very similar in hardware to the Wireless Open-Access Research Platform (WARP) from Rice University, Houston, Texas. A comparison of three types of sensor devices providing a wide range of sensing and processing capabilities, namely, USRP2, WARP, and TelosB is given in [66]. Fig. 9 shows a block diagram of a cognitive radio testbed. Such a platform should have the following features:

- The ability to support multiple radios, which can serve as PUs or SUs.
- The ability for PHY/MAC layer adaptation and fast information exchange between multiple radios for sensing and cooperation.
- The ability to connect various different front-ends in order to be able to test in different frequency ranges (and thus with different PUs).
- The ability to perform rapid prototyping in order to experiment with different sensing algorithms.

More details on reconfigurable software/hardware platforms such as Iris, a dynamically reconfigurable software radio framework developed by the University of Dublin, Trinity College, and the Berkeley Emulation Engine (BEE), a hardware platform developed by the University of California at Berkeley Wireless Research Center can be found in [65].

5 Cognitive Cooperative Relay Networks

CCRNs resulting from a combination of cooperative relay and CRNs have recently appeared as a promising technology that can improve significantly the spectrum efficiency while adhering to the QoS requirements of the PUs [67, 68].

An extensive study on three traditional relay protocols, i.e., fixed relaying, selection relaying, and incremental relaying has been conducted in [69]. Depending upon the method used by the relay to process and forward the received signal from the source to the destination, important protocols such as amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF) can be implemented. In this thesis, we focus on the DF protocol. The DF protocol has an advantage over AF protocol in reducing the effects

of channel interferences and additive noise at the relay. Its use over the AF protocol helps to avoid the noise amplification that happens with the AF protocol. However, due to the possibility of forwarding erroneously decoded signals to the destination, the system complexity may increase with the DF protocol in order to guarantee the correct signal detection. Otherwise, the system performance can significantly degrade due to error propagation.

5.1 Decode-and-Forward Protocol in CRNs

With the DF protocol, the relay attempts to decode the received signals from the SU-Tx. If successful, the relay re-encodes the signal and forwards it to the SU-Rx.

Let us consider a secondary user supported by a cooperative relay as illustrated in Fig. 10. In the figure, g_{sr} , g_{rd} , and g_{sd} denote the channel power gains of the links SU-Tx→SR, SR→SU-Rx, and SU-Tx→SU-Rx, respectively.

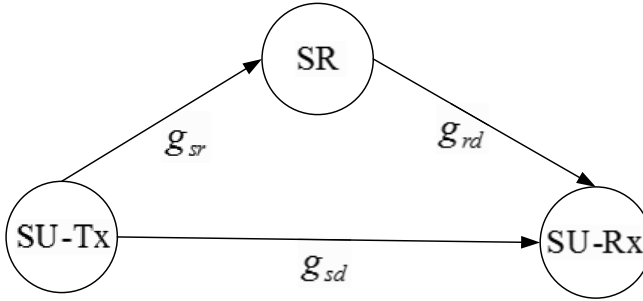


Figure 10: A simple cognitive cooperative relay network.

Generally, the communication of a DF relay network occurs over two time slots. For a fixed DF relay, in the first time slot, the SU-Tx broadcasts its signal to the relay SR and the destination SU-Rx. If the relay SR successfully decodes the SU-Tx signal, it will re-encode and forward the signal to the SU-Rx, in the second time slot. Otherwise, the relay keeps silent. Note that the SU-Tx remains silent during the second time slot. At the end of the transmission cycle, the SU-Rx attempts to decode the received signal. The maximum average mutual information of DF relaying can be expressed as [69]

$$I = \frac{1}{2} \min \left\{ \log \left(1 + \frac{P_s g_{sr}}{N_0} \right), \log \left(1 + \frac{P_s g_{sd}}{N_0} + \frac{P_r g_{rd}}{N_0} \right) \right\} \quad (15)$$

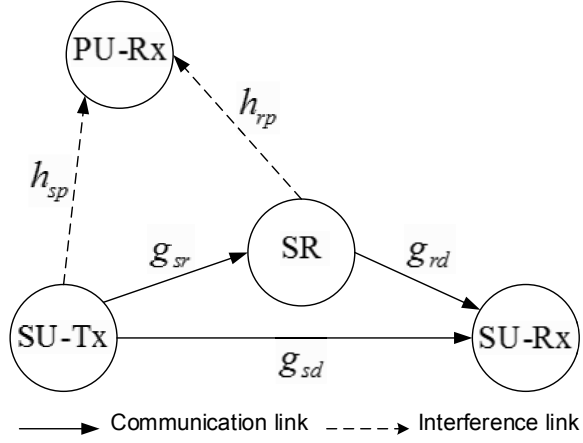


Figure 11: Example of a basic cognitive cooperative relay network.

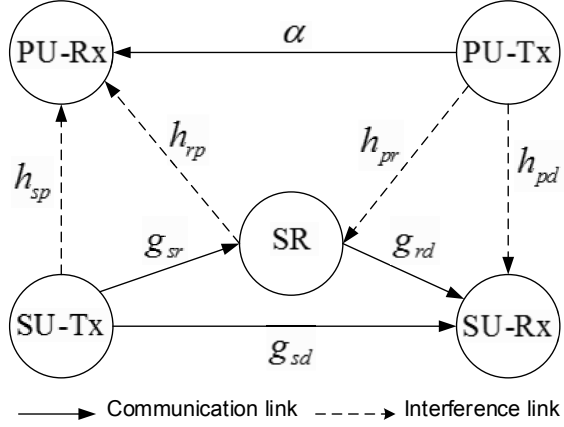


Figure 12: Example of a cognitive cooperative relay network with interference from SUs to PU and interference from PU to SUs.

where N_0 is the noise power, and P_s and P_r are the transmit power of the SU-Tx and SR, respectively.

A basic scenario of a CCRN is shown in Fig. 11 where the interference from the PU-Tx to the secondary network is absent. Fig. 12 illustrates a CCRN where the interference from the PU-Tx is considered. Specifically, in Fig. 11 and Fig. 12, both SU-Tx and SR interfere, during their respective transmission phases with the PU-Rx communications. In Fig. 12, the PU-Tx interferes with the SR and SU-Rx transmissions.

5.2 Relaying with Interference Aided Energy Harvesting

In traditional relaying, co-channel interference (CCI) within the same bandwidth as the transmitted signal deteriorates the system performance. In this case, CCI has to be eliminated by applying an interference alignment approach [70, 71] or by decoding the interfering signals when they are much stronger than the signal. The joint effect of outdated CSI and CCI on the performance of opportunistic DF cooperative networks is studied in [72]. Therein, the results indicate that the diversity order of the system is equal to one for the case of outdated CSI, and zero for the case where the ratio of the interference energy to the signal energy is constant.

Different to the traditional relaying, CCI signals can be leveraged as a new source of power for relay recharging [73, 74]. In this way, relays harvest energy from both the information signal [75] and the CCI signals [74, 76, 77, 78]. The harvested energy is used to decode the source signal and forward it to the destination node. In other words, the interference acts as useful energy in the energy harvesting phase and as noise in the information decoding phase.

Fig. 13 illustrates the energy harvesting at the relay from CCI signals with channel power gains β_i , $i \in \{1, 2, \dots, K\}$. The channel power gains of the links SU-Tx→SR and SR→SU-Rx are denoted by g_1 and g_2 , respectively.

In general, there exist two important schemes that can be used for energy harvesting and decoding information separately. The time-switching based protocol as in Fig. 14, allows the receiver to switch over time between decoding information and harvesting energy. On the other hand, the power-splitting scheme [78, 79] enables a portion of the received power to be used for energy harvesting and the rest of the power is used for information processing.

In Fig. 14, T is the block time in which a certain block of information is transmitted from the SU-Tx to SU-Rx, and δ , $0 \leq \delta \leq 1$, is the fraction of the block time in which the SR harvests energy from the received interference and information signal. Alternatively, a power splitting factor can be used in the case of the power-splitting scheme [78].

During the first phase, the source transmits signal s with power P_s to the

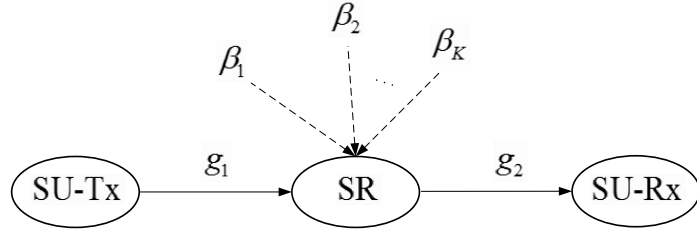


Figure 13: Relaying with interference aided energy harvesting.

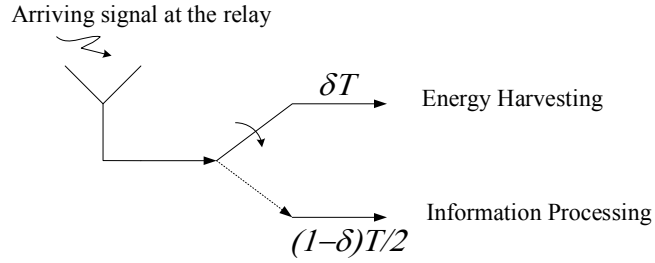


Figure 14: Time switching scheme for energy harvesting and information processing at the relay.

relay. Accordingly, the received signal at the relay is given by

$$y_R = \sqrt{g_1 P_S} s + \sum_{i=1}^K \sqrt{\beta_i P_i} s_i + n_R \quad (16)$$

where s_i and P_i denote the signal and the corresponding power, respectively, from the i^{th} interferer. The AWGN at the relay is denoted by n_R and is assumed to have zero mean and variance N_0 . The harvested energy that results from the received information signal and the interference signal for a duration of δT at each block of information can be expressed as

$$E_H = \eta \left(P_S g_1 + \sum_{i=1}^K P_i \beta_i \right) \delta T \quad (17)$$

where η , $0 \leq \eta \leq 1$, is the energy conversion efficiency depending upon the harvesting circuitry. Generally, the processing power required by the transmit/receive circuitry at the relay is negligible compared to the power used for signal transmission. Hence, it is reasonable to assume that all the energy harvested from the received signal is consumed by the relay for forwarding the information to the destination. Therefore, from (17), the transmit power of the relay is expressed by

$$\begin{aligned} P_R &= \frac{E_H}{(1-\delta)T/2} \\ &= \frac{2\delta\eta}{1-\delta} \left(P_S g_1 + \sum_{i=1}^K P_i \beta_i \right) \end{aligned} \quad (18)$$

During the second phase, the received signal at the destination is given by

$$y_{RD} = \sqrt{g_2 P_R} s_R + n_D \quad (19)$$

where s_R is the transmitted signal from the relay, and n_D is the AWGN at the destination which is assumed to be the same as n_R with zero mean and variance N_0 .

6 Radio Propagation and Fading Models

In wireless systems, radio waves are attenuated due to the effect of the transmitter-receiver separation distance and large objects such as hills and buildings. The radio channel, contrary to the wired channel which is more predictable, is a complex system due to its random behavior. To understand the radio channel and to have accurate prediction models of channel characteristics are key factors for designing reliable wireless systems. In the following, basic propagation mechanisms, path loss, and fading models are summarized.

6.1 Basic Propagation Mechanisms

Reflection, diffraction, and scattering are the three main propagation mechanisms that affect the transmission of radio signals [80].

Reflection

Reflection occurs when plane waves are incident upon surfaces such as ground, walls, or furniture that have very large dimensions compared to the wavelength. When reflection happens, the electromagnetic wave may also be partially refracted. The coefficients of reflection and refraction are functions of the material properties of the media [80], and generally depend on the wave polarization, the angle of incidence, and the frequency of the propagating wave.

Diffraction

Diffraction occurs when a radio wave encounters an obstacle or a slit comparable in size to its wavelength. The secondary waves resulting from diffraction can be found throughout space and behind the obstacle. Diffraction occurs not only with radio waves, but with all waves, including sound waves, water waves, and electromagnetic waves such as visible light, and X-rays. At high frequencies, diffraction like reflection depends on the geometry of the object, as well as on the amplitude, phase, and polarization of the incident wave at the point of diffraction.

Scattering

Scattered waves are produced when plane waves are incident upon rough surfaces, small objects, or other irregularities whose dimensions are in the order of a wavelength or less. Scattering causes the energy to be redirected in many directions.

6.2 Path Loss Models

In practice, the free-space conditions in a radio channel are rarely met. This is often due to the non-availability of a line-of-sight (LOS) path. The non-LOS paths are numerous due to multiple obstructions, reflections, and scattering that make up the received signal. The path loss characterizes how the field strength in radio propagation varies as a function of distance and propagation environment. Different path loss models have been developed and some well-known path loss models for the cellular environment can be found in [80]. A comprehensive survey of path loss models that are used in CRNs is provided in [81].

According to [80], the simplest path loss model is the free-space propagation for which the path loss in decibels (dB) is expressed as [82]

$$L[dB] = 32.44 + 20 \log_{10}(f_c) + 20 \log_{10}(d) \quad (20)$$

where f_c is the carrier frequency in MHz and d is the transmitter-receiver separation distance in km.

In a multipath environment, however, path loss can be deduced from a generic description of received power P_r [81]

$$P_r = P_t \left[a \left(\frac{d}{d_0} \right)^{-\nu} \zeta s \right] \quad (21)$$

where P_t is the transmit power; a is a constant depending on frequency, antenna heights, and other factors; d and d_0 are the transmitter-receiver separation distance and reference distance, respectively; ν is the path loss exponent; and ζ and s represent the small-scale and large-scale fading, respectively. Some path loss exponents for different radio environments are given in Table 3.

Table 3: Path loss exponents for different radio environments [80]

Environment	Path Loss Exponent, ν
Free Space	2.0
Urban Area Cellular Radio	2.7–3.5
Shadowed Urban Cellular Radio	3.0–5.0
Line-of-Sight in Buildings	1.6–1.8
Obstructed in Buildings	4.0–6.0
Obstructed in Factories	2.0–3.0

6.3 Large-Scale Fading

Large-scale fading expresses the average signal power attenuation due to the move of the mobile through a large distance, for example, a distance of the order of the cell size which is easily determined based on the cell radius. Large-scale fading is characterized by average path loss and shadowing. It is greatly affected by large objects such as buildings, hills, and vegetation. It is common to approximate large-scale fading by a lognormal distribution $X \sim \mathcal{LN}(\mu_X, \sigma_X^2)$ [83] with probability density function (PDF) [84]

$$p_X(x) = \frac{1}{\sqrt{2\pi}\sigma_X} \frac{1}{x} \exp \left[-(\ln x - \mu_X)^2 / (2\sigma_X^2) \right] \quad (22)$$

where X is a random variable with mean μ_X and standard deviation σ_X such that $-\infty < \mu_X < \infty$, $\sigma_X > 0$, and $0 < x < \infty$.

6.4 Small-Scale Fading

Small-scale fading describes the rapid fluctuation of the radio signal amplitude over a short period of time or travel distance. It is due to the constructive and destructive interference of multiple signal paths.

For example, a movement of a fraction of a wavelength can induce a variation of signal levels in the order of 30 dB to 40 dB [85]. Small-scale fading may be classified into two categories depending upon the relative extent of multipath and time variation in the channel due to mobile speed as shown in Fig. 15. The first category can be expressed in terms of time spreading and the second one in terms of time variance.

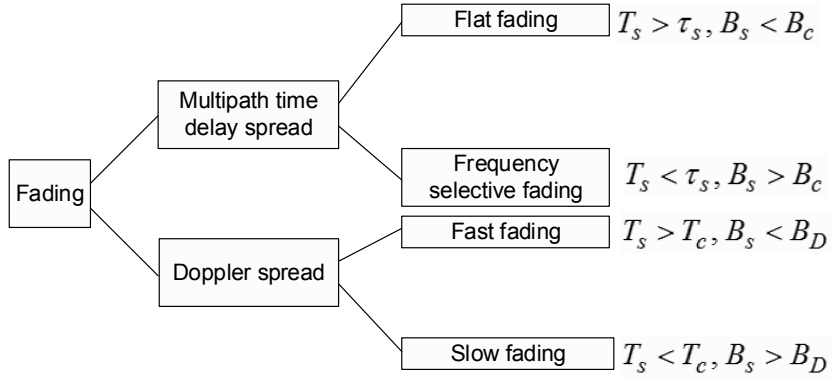


Figure 15: Characteristics for small-scale fading [80].

The symbols used in the relationships shown in Fig. 15 are defined as follows:

- τ_s : Multipath time delay spread
- B_c : Channel coherence bandwidth
- T_s : Symbol duration
- T_c : Channel coherence time
- B_s : Bandwidth of the baseband signal
- B_D : Doppler spread of the channel

Fading based on multipath time delay spread

Multipath time delay spread τ_s is the time between the first and the last arriving echo component with respect to the signal power above a given threshold.

In the frequency domain, the channel coherence bandwidth B_c is the bandwidth over which the channel transfer function remains virtually constant. This means that the channel may be considered relatively constant over the transmit bandwidth. The channel coherence bandwidth can be approximated as the reciprocal of the multipath time delay spread as [82, 86]

$$B_c \simeq \frac{1}{\tau_s} \quad (23)$$

Within B_c the spectral components of the transmitted signal pass through the channel with approximately equal gain and linear phase. In this case, the channel is considered as flat. All multipath components of a transmitted symbol arrive within the symbol duration T_s . Radio signals that experience flat fading are used for narrowband transmission. Under these conditions, the intersymbol interference (ISI) is avoided.

On the other hand, the fading is qualified to be frequency selective if the signal's spectral components are not affected equally by the channel. In this case, the channel has a constant gain and a linear phase response over a bandwidth which is smaller than the bandwidth of the transmitted signal. Thus, the spectrum of the transmitted signal has a bandwidth which is greater than the coherence bandwidth B_c of the channel. In these conditions, ISI is induced.

Fading based on Doppler spread

Depending on how rapidly the channel impulse response varies, the channel can be considered as fast fading or slow fading. For fast fading (or time selective fading), the channel impulse response varies rapidly within a symbol duration. In this case, the coherence time of the channel T_c is smaller than the symbol period T_s of the transmitted signal. This effect causes frequency dispersion due to Doppler spreading which leads to signal distortion. In the frequency domain, the Doppler spread B_D of the channel is greater than the bandwidth B_s of the baseband signal for fast fading. The Doppler spread is approximately equal to the inverse of the channel coherence time [82, 86]:

$$B_D \simeq \frac{1}{T_c} \quad (24)$$

On the other hand, slow fading (or time flat fading) happens when the symbol duration T_s is less than the coherence time T_c . The channel is invariant

over one or several reciprocal bandwidth intervals which leads to the absence of distortion. In the frequency domain, the Doppler spread B_D is much less than the bandwidth B_s of the baseband signal.

6.5 Models for Small-scale Fading

The most commonly used distributions to describe the severity of small-scale fading are Rayleigh and Rician distributions. Other more or less general models exist and some of them have been utilized to model the fading channels of CRNs in this thesis. In the following, the PDFs of different small-scale fading models are described.

Rayleigh Distribution

The Rayleigh distribution is usually used to model multipath fading with non-LOS between the transmitter and receiver antennas. The PDF of the Rayleigh distribution is expressed as [80]

$$f_R(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), \quad r \geq 0 \quad (25)$$

where r is the envelope of the received signal and σ^2 denotes the time-average power of the received signal before the envelope detection. The envelope, r , of the sum of two quadrature Gaussian noise signals [87] is also called channel fading amplitude [86]. Let $g = R^2$ be the channel power gain or simply channel gain. According to [86, eq.(2.3)], we can write

$$f_g(x) = \frac{f_R(\sqrt{x})}{2\sqrt{x}} \quad (26)$$

and hence, following (25) and (26), we have

$$f_g(x) = \frac{1}{\Omega} \exp\left(-\frac{x}{\Omega}\right), \quad x \geq 0 \quad (27)$$

where $\Omega = 2\sigma^2$. Note that $f_g(x)$ represents the PDF of the instantaneous SNR per symbol of the channel and hence, the expression in (26) also applies to other types of fading.

Rician Distribution

If a LOS path exists between the transmitter and receiver antennas, the small-scale fading can be described by the Rician distribution as [87]

$$f_R(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + s^2}{2\sigma^2}\right) I_0\left(\frac{rs}{\sigma^2}\right), \quad r \geq 0 \quad (28)$$

where s denotes the peak amplitude of the dominant signal path and $I_0(\cdot)$ represents the modified Bessel function of the 0-th order. The Rician distribution can be characterized by its K -factor defined as the ratio between the power of the LOS component and the average power in the non-LOS multipath components and is expressed as

$$K = \frac{s^2}{2\sigma^2} \quad (29)$$

A small K implies severe fading and a large K implies less fading. For $K = 0$, we obtain Rayleigh fading, and for $K = \infty$, there is no fading, i.e., a channel without multipath with only a LOS component.

From (28), the average received power in the Rician fading can be obtained as

$$\int_0^\infty r^2 f_R(r) dr = s^2 + 2\sigma^2 = \Omega \quad (30)$$

Let us define $s^2 = K\Omega/(K+1)$ and $2\sigma^2 = \Omega/(K+1)$, and substitute these expressions into (28). Then, the Rician distribution can be rewritten as [87]

$$f_R(r) = \frac{2r(K+1)}{\Omega} \exp\left(-K - \frac{(K+1)r^2}{\Omega}\right) I_0\left(2r\sqrt{\frac{K(K+1)}{\Omega}}\right), \quad r \geq 0 \quad (31)$$

Using (26), it can be shown that the power distribution of Rician fading is given by [86]

$$f_g(x) = \frac{(K+1)}{\Omega} \exp\left(-K - \frac{(K+1)x}{\Omega}\right) I_0\left(2\sqrt{\frac{K(K+1)x}{\Omega}}\right), \quad x \geq 0 \quad (32)$$

Nakagami- m Distribution

Although Rayleigh and Rician distributions can capture the underlying physical properties of many fading channels, some experimental data does not fit

well into either of these distributions. Thus, more general fading distributions whose parameters can be adjusted to fit a variety of empirical measurements have been developed. An example is the Nakagami fading distribution given by [87]

$$f_R(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(\frac{-mr^2}{\Omega}\right), \quad m \geq 0.5 \quad (33)$$

where m is the fading severity parameter, Ω is the averaged received power, and $\Gamma(\cdot)$ is the Gamma function. For $m \rightarrow \infty$, the channel does not experience any fading. For $m = 1$, Nakagami fading reduces to Rayleigh fading, and reduces to one-sided Gaussian fading [88] for $m = 0.5$. For $m = (K+1)^2/(2K+1)$, the distribution in (33) is approximately equivalent to Rician fading with parameter K . The fading severity of the channel increases as m decreases. Using (26), it can be shown that the power distribution of Nakagami- m fading is given by [86]

$$f_g(x) = \left(\frac{m}{\Omega}\right)^m \frac{x^{m-1}}{\Gamma(m)} \exp\left(\frac{-mx}{\Omega}\right) \quad (34)$$

The Nakagami- m distribution is mostly applicable for modeling land-mobile and indoor-mobile multipath propagation [89, 86].

Weibull Distribution

The Weibull distribution [90, 91] characterizes the amplitude fading in a multipath environment, especially that associated with mobile radio systems operating in the 800/900 MHz bands [92, 93]. The Weibull PDF is given by

$$f_R(r) = a \left(\frac{\Gamma(1 + \frac{2}{a})}{\Omega}\right)^{a/2} r^{a-1} \exp\left[-\left(\frac{r^2}{\Omega}\Gamma\left(1 + \frac{2}{a}\right)\right)^{a/2}\right], \quad r \geq 0 \quad (35)$$

where a is the shape parameter. For $a = 2$, the expression in (35) describes the Rayleigh distribution, and for $a = 1$, it describes the exponential distribution. Applying (26), the expression in (35) is rewritten as

$$f_g(x) = \frac{a}{2} \left(\frac{\Gamma(1 + \frac{2}{a})}{\Omega}\right)^{a/2} x^{\frac{a}{2}-1} \exp\left[-\left(\frac{x}{\Omega}\Gamma\left(1 + \frac{2}{a}\right)\right)^{a/2}\right], \quad x \geq 0 \quad (36)$$

which is the power distribution for Weibull fading.

α - μ Distribution

A general model comprising important distributions such as Nakagami- m and Weibull distributions has recently been proposed in [11]. It considers the

non-linearity of the propagation environment and the number of multipath clusters, two important phenomena inherent to radio propagation.

Other distributions such as one-sided Gaussian, Rayleigh, or negative exponential distribution can be obtained as special cases. The PDF of the channel amplitude, R , for α - μ fading is given by [11, 94]

$$f_R(r) = \frac{\alpha \mu^\mu r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu} \Gamma(\mu)} \exp\left(-\mu \frac{r^\alpha}{\hat{r}^\alpha}\right), \quad r > 0, \quad (37)$$

where $\hat{r} = (\mathbb{E}[R^\alpha])^{\frac{1}{\alpha}}$ is the α -th root mean value, $\alpha > 0$ is an arbitrary fading parameter, $\mu > 0$ is the inverse of the normalized variance of R^α calculated as $\mu = \mathbb{E}^2[R^\alpha]/\mathbb{V}[R^\alpha]$ in which $\mathbb{E}[\cdot]$ and $\mathbb{V}[\cdot]$ denote the expectation and variance, respectively. Furthermore, the expression $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ defines the Gamma function.

The k -th moment of R can be expressed as

$$\mathbb{E}[R^k] = \hat{r}^k \frac{\Gamma(\mu + \frac{k}{\alpha})}{\mu^{\frac{k}{\alpha}} \Gamma(\mu)} \quad (38)$$

Applying (26), the PDF of the channel power gain is given by

$$f_g(x) = \frac{\alpha x^{\frac{\alpha\mu}{2}-1}}{2\zeta^{\frac{\alpha\mu}{2}} \Gamma(\mu)} \exp\left[-\left(\frac{x}{\zeta}\right)^{\frac{\alpha}{2}}\right] \quad (39)$$

where

$$\zeta = \frac{\Gamma(\mu)}{\Gamma(\mu + \frac{2}{\alpha})}, \quad (40)$$

and $\Omega = \mathbb{E}[R^2]$ is the average received power of this fading channel.

The α - μ fading channel is considered as a general fading model comprising other PDFs as special cases. By varying the fading parameters, α and μ , well-known fading models such as Rayleigh fading ($\alpha = 2, \mu = 1$), Nakagami- m fading ($\alpha = 2, \mu = m$), and Weibull fading ($\mu = 1$) can be obtained. Also, the exponential fading ($\alpha = 1; \mu = 1$) and one-sided Gaussian fading ($\alpha = 1; \mu = 0.5$), a worst fading case, can be easily obtained.

Cumulative Distribution Function and Moment Generating Function

For a random variable X , other important statistics useful in the modeling of fading channels are related as follows. The cumulative distribution function (CDF) of X is defined as

$$F(x) = \Pr[X \leq x] \quad (41)$$

where $\Pr[\cdot]$ denotes probability. The PDF of X is related to its CDF as

$$f_X(x) = \frac{d}{dx}F(x) \quad (42)$$

The moment generating function (MGF) is defined as

$$\phi(s) = E[e^{-sX}] = \int_{-\infty}^{\infty} e^{-sx} f_X(x) dx \quad (43)$$

where $E[X]$ is the expected value of X . Clearly, the expression in (43) is the Laplace transform of the PDF $f_X(x)$. From the derivative property of the Laplace transform, the MGF can be rewritten in terms of the CDF as

$$\phi(s) = \int_{-\infty}^{\infty} e^{-sx} F'(x) dx = s \int_{-\infty}^{\infty} e^{-sx} F(x) dx - F(0) \quad (44)$$

where $F'(x)$ is the derivative of $F(x)$.

7 Performance Metrics

Let us consider a basic underlay CRN where an SU is allowed to transmit within channel bands allocated to the PU as long as the SU transmission does not cause harmful interference to the PU. Let g_1 and g_0 , respectively, denote the instantaneous channel power gains for the communication link between the SU-Tx and the SU-Rx, and the interference link between the SU-Tx and PU-Rx. It is assumed that both links are independent flat fading channels. The channel coefficients follow a certain channel model (Rayleigh, Nakagami- m , etc.). The noise is assumed to be AWGN with zero mean and power spectral density N_0 . It is also assumed that perfect CSI about g_0 and g_1 is available at the SU-Tx.

For delay-insensitive services, the average interference power constraint Q_{avg} can be expressed as $\mathbb{E}[g_0 P(g_0, g_1)] \leq Q_{avg}$ where $P(g_0, g_1) \geq 0$ is the instantaneous transmit power at the SU-Tx for the channel gain pair (g_0, g_1) . For services with an instantaneous QoS requirement, the peak transmit power limit is considered. The peak interference power constraint Q_{peak} is expressed as $g_0 P(g_0, g_1) \leq Q_{peak}$.

7.1 Outage Capacity

The outage capacity is one of the capacity notions of fading channels. The outage capacity is defined as the maximum constant rate that can be maintained for a specified outage probability. This is equivalent to minimizing

the outage probability for a given transmission rate r_0 [95]. Given a peak interference power constraint Q_{peak} , the problem reduces to

$$\min_{g_0 P(g_0, g_1) \leq Q_{peak}} \Pr \left\{ \log_2 \left[1 + \frac{g_1 P(g_0, g_1)}{N_0} \right] < r_0 \right\} \quad (45)$$

As the expression in (45) is minimized for $P(g_0, g_1) = Q_{peak}/g_0$, the outage probability can be expressed as

$$P_{out} = \Pr \left\{ \frac{g_1}{g_0} < \frac{N_0(2^{r_0} - 1)}{Q_{peak}} \right\} \quad (46)$$

In this regards, the statistical distribution of the ratio g_1/g_0 is a key factor in calculating (46).

7.2 Delay-Limited Capacity

The delay limited capacity (DLC) is defined as the maximum constant transmission rate achievable in all fading states under finite long-term power constraints. For the average interference power constraint Q_{avg} , the DLC can be obtained by solving the following problem [95]:

$$\max_{\mathbb{E}[g_0 P(g_0, g_1)] \leq Q_{avg}} \log_2 \left[1 + \frac{g_1 P(g_0, g_1)}{N_0} \right] \quad (47)$$

It has been shown that the optimal power allocation for this problem is given by [95]

$$P(g_0, g_1) = \frac{Q_{avg}}{g_1 \mathbb{E}[g_0/g_1]} \quad (48)$$

In fact, for Rayleigh fading, the channel power gains g_0 and g_1 are exponentially distributed. By assuming g_0 and g_1 to be unit-mean and mutually independent, $\mathbb{E}[g_0/g_1]$ can be shown to be $+\infty$. Thus, for the single-input single-output Rayleigh fading channel, the DLC is zero. However, the DLC is greater than zero in multiple channels depending on the properties of the fading channel, e.g., on the spatial correlation [95].

7.3 Ergodic Capacity

The ergodic capacity is defined as the maximum achievable rate averaged over all fading states (long-term average). For the case of the average interference power constraint Q_{avg} , the ergodic capacity can be obtained by solving the following optimization problem [95]:

$$\max_{\mathbb{E}[g_0 P(g_0, g_1)] \leq Q_{avg}} \mathbb{E} \left[\log_2 \left(1 + \frac{g_1 P(g_0, g_1)}{N_0} \right) \right] \quad (49)$$

The ergodic capacity under the average interference power constraint can be obtained as [87]

$$C_{erg} = \int_{\frac{1}{\gamma_0}}^{\infty} B \log_2(\gamma_0 x) f_X(x) dx \quad (50)$$

where B is the total available bandwidth, $\gamma_0 = 1/(\psi_0 N_0 B)$, and ψ_0 is calculated such that the average interference power in (50) equals Q_{avg} , i.e.,

$$\int_{g_0} \int_{g_1} \max\left(0, \frac{1}{\psi_0} - N_0 B \frac{g_0}{g_1}\right) f_{g_0}(g_0) f_{g_1}(g_1) dg_1 dg_0 = Q_{avg} \quad (51)$$

Here, $f_{g_0}(g_0)$ and $f_{g_1}(g_1)$ are the PDFs associated with the channel power gains g_0 and g_1 , respectively.

7.4 Effective Capacity

The effective capacity is defined as a dual concept to the bandwidth [96]. Assuming that the transmission technique of the secondary user must satisfy a statistical delay QoS constraint, it has been shown that the probability for the queue length of the transmit buffer exceeding a certain threshold, x , decays exponentially as a function of x [97]. Let us define θ as the delay QoS exponent such that

$$\theta = - \lim_{x \rightarrow \infty} \frac{\ln(\Pr\{q(\infty) > x\})}{x} \quad (52)$$

where $q(n)$ indicates the transmit buffer length at time n . A smaller θ corresponds to a loose QoS constraint, whereas a larger θ corresponds to strict delay constraints. That is, for $\theta \rightarrow 0$, the system is able to tolerate infinite long packet delay and $\theta \rightarrow \infty$ means that no delay is allowed in the system.

Let $\{R[n], n = 1, 2, \dots\}$ be a stochastic service process assumed to be stationary and ergodic. Assume that the capacity function $\Lambda(-\theta)$ exists, such as

$$\Lambda(-\theta) = \lim_{N \rightarrow \infty} \frac{1}{N} \ln \left(\mathbb{E} \left\{ e^{-\theta \sum_{n=1}^N R[n]} \right\} \right) \quad (53)$$

Then, the effective capacity can be expressed as [96]

$$E_c(\theta) = -\frac{\Lambda(-\theta)}{\theta} = - \lim_{N \rightarrow \infty} \frac{1}{N\theta} \ln \left(\mathbb{E} \left\{ e^{-\theta \sum_{n=1}^N R[n]} \right\} \right) \quad (54)$$

where θ is the QoS exponent interpreted as the delay constraint, and $R[n]$ is the data rate with time index n . The effective capacity in (54) gives the

maximal data rate that can be supported by the channel under the delay constraint θ .

It should be mentioned that the effective capacity given in (54) can be simplified for block-fading channels as

$$E_c(\theta) = -\frac{1}{\theta} \ln \left(\mathbb{E} \left\{ e^{-\theta R[n]} \right\} \right) \quad (55)$$

where the sequence $R[n]$, $n = 1, 2, \dots$, is uncorrelated.

The data rate $R[n]$ can be related to the SNR as

$$R[n] = T_f B \ln(1 + \gamma) \quad (56)$$

where

$$\gamma = \frac{g_1 Q_{pk}}{g_0 N_0}. \quad (57)$$

7.5 Symbol Error Probability

The symbol error probability (SEP) is another important performance metric. The SEP generally depends upon the modulation/detection scheme employed by the system. Making use of the results of [98], the SEP can be expressed as

$$P_e = \frac{\eta \sqrt{\theta}}{2\sqrt{\pi}} \int_0^\infty F_X(x) \frac{e^{-\theta x}}{\sqrt{x}} dx \quad (58)$$

where η and θ are constants depending on a specific modulation scheme [86]. For M -PSK, these parameters are given as $\eta = 2$ and $\theta = \sin^2(\pi/M)$. The function $F_X(\cdot)$ denotes the CDF of the SNR random variable X .

8 Thesis Overview

This doctoral thesis studies the performance of CRNs and CCRNs in different fading environments. Analytical expressions for different performance metrics such as outage probability, ergodic capacity, and symbol error probability are derived. Numerical results are provided to illustrate the effect of multiuser diversity, peak transmit power, different fading parameters, and modulation schemes on the system performance. The thesis consists of five research parts based on two journal articles and five peer-reviewed conference papers as follows.

In the first part, a basic underlay CRN is analyzed. The network is composed of one source, one relay, and one destination. The network is subject

to the peak interference power constraint at two PUs. In particular, an analytical expression of the outage probability at the destination is derived as a function of α - μ fading parameters. Also, an analytical expression for the ergodic capacity of the CRN is derived. From the obtained results, the CRN subject to Nakagami- m and Weibull fading is analyzed. The CRN analysis can also be extended to other fading conditions with related distributions.

In the second part, the outage probability and the ergodic capacity of an underlay CRN are analyzed. The analyzed CRN demonstrates the multiuser diversity effect on the outage probability and the ergodic capacity while the CRN undergoes Nakagami- m fading. Considering multiuser diversity, the presence of a PU-Tx, and a predefined outage constraint of the PU network, a power allocation policy for the CRN subject to Rayleigh fading is derived. The derived power allocation policy is utilized to analyze the outage probability of the CRN and to approximate the corresponding ergodic capacity.

The third part reports the effect of multiple PU-Txs and multiple PU-Rxs on the outage probability of an underlay CCRN. The outage constraint at the PU-Rx and the peak transmit power constraint of the SUs are taken into account to derive the power allocation policies for the SUs. Utilizing the derived power allocation policies, the analytical expression for the outage probability of the CCRN is also derived.

In the fourth part, considering the PU outage and SU peak transmit power constraints, a power allocation policy is derived and used to analyze the performance of single relay underlay CCRNs with selection combining (SC) and maximal ratio combining (MRC) at the SU-Rx. With and without the availability of a direct link between the SU-Tx and the SU-Rx, analytical expressions of the outage probability are derived and used to analyze the effect of PU transmit SNR on the SU network. Then, the SEP of single relay and multiple relay CCRNs with SC are derived. Based on the derived SEP expressions, the impact of PU transmit SNR, number of secondary relays, and different modulation schemes on the SU SEP is analyzed.

Finally, the fifth part applies a sleep/wake-up strategy with $\min(N, T)$ policy to a basic underlay CRN to derive the power consumption function. In this case, the SUs operate as wireless sensor nodes subject to Nakagami- m fading. The obtained results illustrate the impact of M/G/1 queue and fading parameters on the power consumption.

PART I-A - Outage Analysis of Cognitive Relay Networks over α - μ Fading Channels

In this part, we investigate the performance of an underlay cognitive relay network over α - μ fading channels. In particular, we assume non-identical fading parameters and derive an analytical expression for the outage probability of a dual-hop underlay CCRN subject to a peak interference power constraint. Numerical and simulation results are presented to illustrate the effect of fading parameters on the outage probability of the system. In the following, important contributions of this part are provided.

- Assuming the fading channels to be independent non-identically distributed α - μ random variables, an analytical expression of the outage probability for a CCRN is derived.
- The derived expressions take into account the peak interference power constraint at two distinct PUs.
- An observation from the numerical results is that regardless of different fading parameters adopted, the outage probability is the same at a certain SNR; for example, $Q/N_0 = 5$ dB for the analyzed scenario.

PART I-B - Ergodic Capacity of Cognitive Relay Networks over α - μ Fading Channels

In this part, we investigate the ergodic capacity of an underlay cognitive relay network over α - μ fading channels. In particular, we assume that the channels are independent non-identically distributed α - μ random variables and analyze the system performance subject to a peak interference power constraint. Numerical results are presented to study the effect of fading parameters on the ergodic capacity of the system. The contributions of this part are provided as follows:

- With non-identical μ parameters, we make use of the ratio of independent α - μ random variables to derive the PDF and CDF for the instantaneous SNR at the SU-Rx of a dual-hop DF CCRN over α - μ fading channels.
- From the PDF expression of the SNR at the SU-Rx, an analytical expression of the ergodic capacity in its integral form is obtained and analyzed.

- We investigate scenarios consisting of varying fading parameters ($\alpha; \mu$) in order to obtain insights on the ergodic capacity of the underlay CCRN under several fading models such as Rayleigh ($\alpha = 2; \mu = 1$), Nakagami- m ($\alpha = 2; \mu = m$), Weibull ($\mu = 1$), exponential ($\alpha = 1; \mu = 1$) and one-sided Gaussian ($\alpha = 1; \mu = 0.5$).

Part II-A - Outage Probability and Ergodic Capacity of a Spectrum Sharing System with Multiuser Diversity

In this part, we derive analytical expressions for the outage probability and the ergodic capacity of a spectrum sharing system with multiuser diversity under Nakagami- m fading. In particular, several secondary users utilize the licensed spectrum of the primary user and they are selected based on the best and worst channel quality. Numerical results and simulations are shown in order to reveal the effect of multiuser diversity and fading parameters on the considered system. The contributions of this part are summarized as follows:

- The effect of multiuser diversity on the spectrum sharing system under Nakagami- m fading is analyzed.
- The PDFs of the SNRs at the SU-Rx corresponding to the best and worst channel quality scenarios, respectively, are derived.
- From the derived PDFs, analytical expressions for the outage probability corresponding to the best and worst channel quality scenarios are obtained.
- The ergodic capacity of the secondary network where the SU-Rx selects an SU-Tx with best channel condition is obtained. The secondary network performance in terms of ergodic capacity is analyzed versus the tolerable peak interference power at the PU-Rx.
- The numerical results indicate that a significant performance improvement can be achieved by increasing the number of SU-Txs for the considered scenarios.

Part II-B - Outage Probability and Ergodic Capacity of Underlay Cognitive Radio Systems with Adaptive Power Transmission

In this part, we consider a spectrum sharing system in which a primary transmission coexists with a secondary network composed of a secondary base station and multiple SU-Rxs. The SU can access the spectrum licensed to the PU as long as the SU does not cause any harmful interference to the PU. Among all available SU-Rxs, only the best SU-Rx with the highest signal-to-interference-plus-noise ratio (SINR) is selected to receive the transmit signals from the secondary base station. We derive an analytical expression for the outage probability of the SU-Rx with the highest SINR. Moreover, we derive an approximation of the ergodic capacity at the SU-Rx. In order to meet the requirements at the PU-Rx, at the same time, satisfying the quality of the SU link, an adaptive power allocation strategy is developed. The power allocation policy allows the SU-Tx to adapt its transmit power in order to avoid harmful interference at the PU-Rx, with a PU predefined outage constraint. Below are the contributions of this part:

- An analytical expression for the outage probability in closed form is derived for a CRN with multiuser diversity.
- The multiuser diversity effect on the CRN under Rayleigh fading is investigated.
- The presence of the PU-Tx is considered on the CRN performance analysis.
- A power allocation policy is derived considering the minimum required PU outage probability.
- The ergodic capacity of the CRN is approximated and evaluated with respect to the PU transmit power.
- Generally, it has been observed that the performance degradation for both the outage probability and the ergodic capacity is mostly sensitive to the transmit power limit of the secondary base station.

Part III - Outage Probability of a Cognitive Cooperative Relay Network with Multiple Primary Users Under Primary Outage Constraint

In this part, the impact of multiple PU-Txs and multiple PU-Rxs on the outage probability of an underlay CCRN with DF relaying is studied. Specifically, a power allocation policy is derived, subject to an outage constraint at the PU-Rxs and a peak transmit power constraint of the secondary transmitters. An analytical expression for the outage probability of the CCRN is derived. Numerical examples are provided to illustrate the impact of fading parameters, multiple PU-Txs, and multiple PU-Rxs on the outage probability of the CCRN. Below are the main contributions of this part:

- An expression for the outage probability of an underlay CCRN with DF relaying subject to Rayleigh fading is derived.
- A power allocation policy is derived taking multiple PU-Txs and multiple PU-Rxs into consideration.
- An outage constraint at the PU-Rx and peak transmit power constraint of the secondary transmitters contribute to the derivation of the power allocation policy.
- For given examples, a degradation in terms of outage probability due to multiple PU-Txs and multiple PU-Rxs is quantified.

Part IV-A - On the Performance of Cognitive Radio Networks with DF Relay Assistance Under Primary Outage Constraint Using SC and MRC

In this part, we analyze the performance of a CRN that is assisted by a single relay. In particular, the SU-Tx and the SR utilize the licensed frequency band of the PU. To protect the PU from harmful interference, the SU-Tx and SR must regulate their transmit power to satisfy the outage probability constraint of the PU. System performance in terms of outage probability is analyzed for SC and MRC. Specifically, a power allocation policy and analytical expressions for the outage probability with SC and MRC are derived. Our results show that the upper bound of the outage probability corresponding to MRC is

equal to the exact expression for the outage probability for SC. The main contributions of this part are summarized as follows:

- Analytical expressions for the outage probability of the underlay CCRN are derived.
- Two combining schemes SC and MRC applied to the DF based underlay CCRN are compared in terms of outage probability.
- An adaptive transmit power policy for the analyzed CCRN is derived subject to the peak transmit power constraint of the SU-Tx and the outage constraint of the PU-Rx.
- The upper bound of the outage probability for MRC matches with the expression of the outage probability corresponding to SC.
- Furthermore, the numerical results show that the lower bound of the outage probability for MRC outperforms that for SC for the scenario considered.

Part IV-B - Symbol Error Probability of Cognitive Cooperative Relay Networks Under Primary Outage and Secondary Peak Transmit Power Constraints

In this part, we analyze the performance of cognitive radio networks in terms of symbol error probability that utilizes decode-and-forward relaying. The communication in the secondary network is subject to the peak transmit power constraint of the secondary user transmitter and the outage constraint given by the primary user receiver. Given these constraints, an adaptive transmit power policy is formulated and the SEP of the secondary network is derived for different CCRN topologies. In particular, a single relay CCRN with and without direct link as well as a multiple relay CCRN are considered. Numerical results of the SEP for CCRNs with different modulation schemes and different number of relays are provided. These results illustrate that the availability of a direct link gives small performance improvements while an increase of the number of relays improves performance more significantly for the scenarios considered. Below are the main contributions of this part:

- Analytical expressions of the symbol error probability for different underlay CCRN topologies with and without direct link are derived.

- For different modulation schemes, the impact of having a single relay with direct link, single relay without direct link, and multiple relays without direct link on the derived SEP is analyzed.
- The numerical results show that having a direct link provides only minor improvements in SEP performance for the analyzed scenarios.
- Furthermore, the numerical results indicate that the increase of the number of secondary relays in combination with selection combining is a powerful option for improving SEP performance of CCRNs.

Part V - On Power Consumption of Wireless Sensor Nodes with $\min(N, T)$ Policy in Spectrum Sharing Systems

In this part, we analyze the power consumption of wireless sensor nodes with $\min(N, T)$ policy and M/G/1 queue in the presence of Nakagami- m fading. In particular, this system setting is applied to a wireless sensor node operating in a cognitive radio system as secondary user in the presence of a primary user. As such, not only the queue parameters influence the power consumption but also the interference power constraint imposed on the wireless sensor node by the primary user. Thus, a queued sleep/wake-up strategy is analyzed in the context of a spectrum sharing environment in the presence of signal fading. The average power consumption of a sensor node using the $\min(N, T)$ policy and M/G/1 queue is analyzed. Numerical examples are presented to illustrate the impact of queuing parameters and fading channel parameters on the power consumption of the wireless sensor node. Important contributions of this part are summarized as follows:

- We apply a sleep/wake-up strategy with the $\min(N, T)$ policy to a basic underlay CRN in order to derive the power consumption function.
- While several works dealing with the $\min(N, T)$ policy assume ideal transmission channels, in reality, channels are far from being ideal as the signals transmitted by a user arrive at the receiver with different power levels due to fading, shadowing, and multipath. Such fading environment is captured in the power consumption function derived in this part.
- The packet transmission time at the secondary network is expressed in terms of system bandwidth, number of bits per packet, and SNR at the

SU-Rx. Then, the first and second moments of the packet transmission time are derived.

- From the derived moments of the packet transmission time, the impact of M/G/1 queue and fading parameters on the power consumption is analyzed.
- For given examples, the results show that if only a maximum of a single packet is queued, i.e. $N = 1$, then the threshold T has no influence on the power consumption and the largest consumption is observed.
- For a given scenario and a given threshold of waiting time T in the queue, the power consumption decreases steeply with the number of packets N increasing from 1 to 6 and then reaches a constant floor.
- For the considered scenario, a maximum of $N = 6$ packets in the queue appears to be sufficient for reducing the power consumption of the wireless sensor node.

9 Future Works

An important topic for future work is the further development of multihop device-to-device (D2D) communications. Multihop D2D is regarded as a key component of next generation cellular networks that improves the spectral efficiency and also reduces the power consumption. However, D2D communication can cause severe interference to cellular networks due to spectrum sharing and multihop transmissions which typically depend on resource sharing and allocation schemes.

The CCI is regarded as a significant channel impairment in practical wireless systems. Nevertheless, the energy harvesting from CCI can be an important enabler for energy-efficient cognitive-based D2D communications. To exploit CCI in cognitive networks, energy harvesting-based D2D communications, and their power consumption gains are certainly interesting topics for further research works.

Finally, another important aspect for further investigations is on how much enhanced security D2D systems can achieve in terms of physical-layer secrecy capacity. This work may consist of exploiting wireless channel characteristics, modulation and coding schemes, and multiple antennas to reduce the ability of eavesdroppers to detect and intercept sensitive communications.

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Part I-A

PART I-A

Outage Analysis of Cognitive Relay Networks over α - μ Fading Channels

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Outage Analysis of Cognitive Relay Networks over α - μ Fading Channels

Charles Kabiri, Hans-Jürgen Zepernick, and Hung Tran

Abstract

In this paper, we investigate the performance of an underlay cognitive relay network over α - μ fading channels. In particular, we assume non-identical fading parameters and derive an analytical expression for the outage probability of a dual-hop underlay system subject to a peak interference power constraint. Numerical and simulation results are presented to evaluate the effect of fading parameters on the outage probability of the system.

1 Introduction

With the growth of wireless applications, the problem of spectrum shortage and utilization efficiency of the spectrum is becoming more challenging. Cognitive radio networks (CRNs) are regarded as a promising technology to alleviate such problems. In CRNs, the underlay model permits secondary users (SUs) to communicate provided that their transmitted signals do not cause harmful interference to the primary users (PUs). Cooperative communication using relaying, on the other hand, has been proposed to achieve transmission diversity and increase the radio coverage in wireless networks. The combination of CRNs with relaying, therefore, have gained significant interest due to the remarkable enhancement of system performance. Several works address cognitive relay networks in additive white Gaussian noise (AWGN) and traditional fading environments such as Rayleigh and Nakagami- m fading channels. Recently, the authors in [1] examined under Nakagami- m fading the outage

probability of dual-hop cognitive amplify-and-forward (AF) relay networks subject to independent non-identically distributed (i.n.i.d.) fading channels. The same authors in [2] investigated the impact of multiple primary transmitters and receivers in cognitive relay networks. There also exist works [3], [4] addressing issues of spectrum sharing such as outage probability or outage capacity in a general fading model, the α - μ fading model. However, more results on the performance of CRNs subject to α - μ fading channels are still thought to be needed. The distribution associated with the α - μ fading model is a general case of traditional important distributions, such as Nakagami- m and Weibull. Other distributions such as one-sided Gaussian, Rayleigh, or negative exponential can be obtained from the α - μ distribution as special cases. The α - μ physical parameters are, respectively, linked to the non-linearity of the propagation environment and the number of multipath clusters [5]. In [3], analytical expressions for the minimum outage probability and delay-limited capacity are derived for a spectrum sharing system undergoing α - μ fading with independent and identically distributed (i.i.d.) parameters. With non-identical parameter μ , [4] investigated, under the peak interference power constraint, analytical expressions for the outage probability, amount of fading, and approximate ergodic capacity for a spectrum sharing system in α - μ fading channel. In [6], the ratio of i.n.i.d. α - μ random variables has been derived and applied to a spectrum sharing system composed of an SU in the presence of a single PU.

In this paper, we analyze the performance of an underlay cognitive relay network over α - μ fading channels under a peak interference power constraint. Different scenarios of non-identical μ parameters and non-identical α parameters at the first hop and the second hop are investigated to provide insights on the outage performance of the system. We derive the probability density function (PDF) and the cumulative distribution function (CDF) of the signal-to-noise ratio (SNR) of the SU in dual-hop transmission with decode-and-forward (DF) relaying. The obtained results are used to derive an analytical expression of the outage probability for the considered system.

The remainder of this paper is organized as follows. Section II provides the system and channel model. Section III derive the PDF and CDF of the SNR at both relay and SU receiver (SU-Rx) along with an analytical expression of the outage probability of the secondary system. Numerical results are presented in Section IV. Finally, conclusions are given in Section V.

2 System Model

Let us consider the system model as shown in Fig. 1 where an SU transmitter (SU-Tx) communicates with an SU-Rx by the help of a secondary relay (SR).

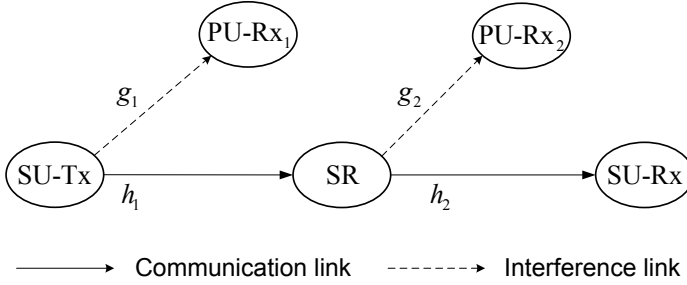


Figure 1: Model of a relay-based spectrum sharing system.

The SU-Tx and the SR share the spectrum with the PU receivers PU-Rx₁ and PU-Rx₂, respectively. Furthermore, we denote the channel coefficients for the SU-Tx→SR, SU-Tx→PU-Rx₁, SR→SU-Rx, and SR→PU-Rx₂ links by X_1 , Y_1 , X_2 and Y_2 , respectively. For convenience, let $\mathcal{S} = \{X_1, Y_1, X_2, Y_2\}$ denote the set of the above channel coefficients. Moreover, let R_i , $i = \{1, 2\}$, express any of the elements in $\mathcal{S}$ and be distributed following the α - μ distribution [7]. According to [5], [7], the PDF of R_i is given by

$$f_{R_i}(r) = \frac{\alpha_{R_i} \mu_{R_i}^{\mu_{R_i}} r^{\alpha_{R_i} \mu_{R_i} - 1}}{\hat{r}_{R_i}^{\alpha_{R_i} \mu_{R_i}} \Gamma(\mu_{R_i})} \exp\left(-\mu_{R_i} \frac{r^{\alpha_{R_i}}}{\hat{r}_{R_i}^{\alpha_{R_i}}}\right) \quad (1)$$

where $\hat{r}_{R_i} = (\mathbb{E}[R_i^{\alpha_{R_i}}])^{\frac{1}{\alpha_{R_i}}}$ is the α_{R_i} -th root mean value, $\alpha_{R_i} > 0$ is an arbitrary fading parameter, $\mu_{R_i} > 0$ is the inverse of the normalized variance of $R_i^{\alpha_{R_i}}$ and is calculated as $\mu_{R_i} = \mathbb{E}^2[R_i^{\alpha_{R_i}}]/\mathbb{V}[R_i^{\alpha_{R_i}}]$ in which $\mathbb{E}[\cdot]$ and $\mathbb{V}[\cdot]$ denote the expectation and variance, respectively. Furthermore, the expression $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ defines the gamma function.

The k -th moment of R_i can be expressed as

$$\mathbb{E}[R_i^k] = \hat{r}_{R_i}^k \frac{\Gamma\left(\mu_{R_i} + \frac{k}{\alpha_{R_i}}\right)}{\frac{k}{\mu_{R_i}^{\alpha_{R_i}} \Gamma(\mu_{R_i})}} \quad (2)$$

According to [7], the α - μ fading channel is considered as a general fading model comprising other PDFs as special cases. By varying the fading parameters, α_{R_i} and μ_{R_i} , the well-known models such as Rayleigh ($\alpha_{R_i} = 2, \mu_{R_i} = 1$), Nakagami- m ($\alpha_{R_i} = 2, \mu_{R_i} = m$), and Weibull ($\mu_{R_i} = 1$) can be obtained.

Let $\mathcal{K} = \{h_1, g_1, h_2, g_2\}$ denote the set of instantaneous channel power gains that are assumed to be of unit mean, and $h_1 = |X_1|^2$, $g_1 = |Y_1|^2$, $h_2 = |X_2|^2$, $g_2 = |Y_2|^2$. According to [8], the PDF of $k_i \in \mathcal{K}$ can be expressed by

$$f_{k_i}(x) = \frac{\alpha_{k_i} x^{\frac{\alpha_{k_i} \mu_{k_i}}{2} - 1}}{2 \Omega_{k_i}^{\frac{\alpha_{k_i} \mu_{k_i}}{2}} \Gamma(\mu_{k_i})} \exp \left[- \left(\frac{x}{\Omega_{k_i}} \right)^{\frac{\alpha_{k_i}}{2}} \right] \quad (3)$$

where

$$\Omega_{k_i} = \frac{\Gamma(\mu_{k_i})}{\Gamma(\mu_{k_i} + \frac{2}{\alpha_{k_i}})}. \quad (4)$$

Furthermore, we assume that perfect channel state information (CSI) is available at all SUs. Therefore, the SU-Tx and the SR can regulate their power based on the joined CSI to guarantee that the interference caused by the SU at the PU-Rx does not exceed a threshold. Accordingly, the SNR of the SR and the SU-Rx together with the transmission power conditions at PU-Rx₁ and PU-Rx₂ can be expressed as

$$\gamma_i = \frac{P(g_i, h_i) h_i}{N_0} \quad (5a)$$

$$\text{s.t. } P(g_i, h_i) g_i \leq Q_i \quad (5b)$$

where $P(g_i, h_i)$ is the instantaneous transmission power at SU-Tx and SR, N_0 denotes the noise power and Q_i denotes the peak interference power constraint at the PU-Rx_i. If we assume that there are no limitations on the transmission power of the SU-Tx and the SR, the SU-Tx and the SR can transmit at maximal instantaneous power, Q_i/g_i . Therefore, (5a) can be rewritten, assuming $Q_i = Q$, $i = \{1, 2\}$, as

$$\gamma_i = \frac{h_i Q}{g_i N_0}. \quad (6)$$

With respect to system capacity, as the SU-Tx transmits with the help of the relay in dual-hop transmission, the capacity of the SU-Rx, assuming unit bandwidth, can be formulated as follows [9], [10]:

$$C = \frac{1}{2} \min \{ \log_2 (1 + \gamma_1), \log_2 (1 + \gamma_2) \} \quad (7)$$

where $1/2$ is due to the transmission in two time slots. In the sequel, an analytical expression for the outage probability is derived.

3 Performance Analysis

In this section, we derive the CDF of the instantaneous SNR, γ_s , at the SU-Rx which supports the calculation of the outage probability of the system.

3.1 CDF of γ_s

In dual-hop transmission, the SNR at the SU-Rx can be written as

$$\gamma_s = \min \{ \gamma_1, \gamma_2 \} \quad (8)$$

Given the CDF of γ_s , we are able to investigate the outage probability of the considered cognitive system. The PDF of γ_i , in (6), assuming $\alpha_{k_i} = \alpha_i$, $i = \{1, 2\}$, can be given as in [4] by

$$f_{\gamma_i}(\gamma) = \frac{\alpha_i \left(\frac{N_0 \Omega_{g_i}}{Q \Omega_{h_i}} \right)^{\frac{\alpha_i \mu_{h_i}}{2}}}{2\mathcal{B}(\mu_{h_i}, \mu_{g_i})} \frac{\gamma^{\frac{\alpha_i \mu_{h_i}}{2} - 1}}{\left[1 + \left(\frac{N_0 \Omega_{g_i}}{Q \Omega_{h_i}} \right)^{\frac{\alpha_i}{2}} \gamma^{\frac{\alpha_i}{2}} \right]^{\mu_{h_i} + \mu_{g_i}}} \quad (9)$$

where $\mathcal{B}(\cdot, \cdot)$ is the Beta function [11, eq.(8.384.1)]. We can obtain the CDF of γ_i by integrating (9) with respect to γ as

$$\begin{aligned} F_{\gamma_i}(\gamma) &= \left(\frac{N_0 \Omega_{g_i}}{Q \Omega_{h_i}} \right)^{\frac{\alpha_i \mu_{h_i}}{2}} \frac{\gamma^{\frac{\alpha_i \mu_{h_i}}{2}}}{\mu_{h_i} \mathcal{B}(\mu_{h_i}, \mu_{g_i})} \\ &\quad \times {}_2F_1 \left(\mu_{h_i} + \mu_{g_i}, \mu_{h_i}; 1 + \mu_{h_i}; - \left(\frac{BN_0 \Omega_{g_i}}{Q \Omega_{h_i}} \right)^{\frac{\alpha_i}{2}} \gamma^{\frac{\alpha_i}{2}} \right) \end{aligned} \quad (10)$$

where ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ stands for the hypergeometric function [11, eq. (3.194.5)].

Given (10), the CDF of γ_s can be found by the order statistics theory as follows:

$$F_{\gamma_s}(\gamma) = 1 - (1 - F_{\gamma_1}(\gamma))(1 - F_{\gamma_2}(\gamma)) \quad (11)$$

An analytical expression of $F_{\gamma_s}(\gamma)$ can be readily obtained by substituting $F_{\gamma_1}(\gamma)$ and $F_{\gamma_2}(\gamma)$ from (10) into (11) as

$$\begin{aligned}
 F_{\gamma_s}(\gamma) = & 1 - \left[1 - \frac{\gamma^{\frac{\alpha_1 \mu_{h_1}}{2}}}{\mathcal{B}(\mu_{h_1}, \mu_{g_1}) \mu_{h_1}} \right. \\
 & \times {}_2F_1 \left(\mu_{h_1}, \mu_{g_1} + \mu_{h_1}; 1 + \mu_{h_1}; -\gamma^{\alpha_1/2} \left(\frac{N_0 \Omega_{g_1}}{Q \Omega_{h_1}} \right)^{\alpha_1/2} \right) \\
 & \times \left(\frac{N_0 \Omega_{g_1}}{Q \Omega_{h_1}} \right)^{\frac{\alpha_1 \mu_{h_1}}{2}} \left. \right] \left[1 - \frac{\gamma^{\frac{\alpha_2 \mu_{h_2}}{2}}}{\mathcal{B}(\mu_{h_2}, \mu_{g_2}) \mu_{h_2}} \right. \\
 & \times {}_2F_1 \left(\mu_{h_2}, \mu_{g_2} + \mu_{h_2}; 1 + \mu_{h_2}; -\gamma^{\alpha_2/2} \left(\frac{N_0 \Omega_{g_2}}{Q \Omega_{h_2}} \right)^{\alpha_2/2} \right) \\
 & \times \left(\frac{N_0 \Omega_{g_2}}{Q \Omega_{h_2}} \right)^{\frac{\alpha_2 \mu_{h_2}}{2}} \left. \right] \quad (12)
 \end{aligned}$$

3.2 Outage Probability

The outage probability is defined as the probability that the SNR of a received signal is less than a predefined threshold γ_{th} . In other words, it is the probability that the capacity of the received signal is below a threshold. For the considered secondary relay network, the outage probability is obtained as

$$\begin{aligned}
 P_{out} &= \Pr \{C < C_0\} \\
 &= \Pr \left(\min [\gamma_1, \gamma_2] < 2^{2C_0} - 1 \triangleq \gamma_{th} \right) \\
 &= F_{\gamma_s}(\gamma_{th}) \quad (13)
 \end{aligned}$$

4 Numerical Results

The outage performance of the considered cognitive relay network is evaluated numerically by setting the outage threshold $C_0 = 1$ bit/sec. The *wggamrnd* function provided in the WAFO Matlab toolbox [12] is used, like in [4], to simulate the α - μ fading channels. Fig. 2 plots the outage probability versus peak interference power to noise ratio for different fading parameters α_i , μ_{h_i} and μ_{g_i} , $i = 1, 2$. The Nakagami- m and Rayleigh fading models, as well as some other more and less severe cases are simulated and numerically analyzed. For very small values of Q/N_0 , all the curves tend to overlapping with each other. For $Q/N_0 > 5$ dB, less severe fading channels result in low outage probability. Moreover, one can easily observe that regardless of the fading parameters, the outage probability is the same at $Q/N_0 = 5$ dB.

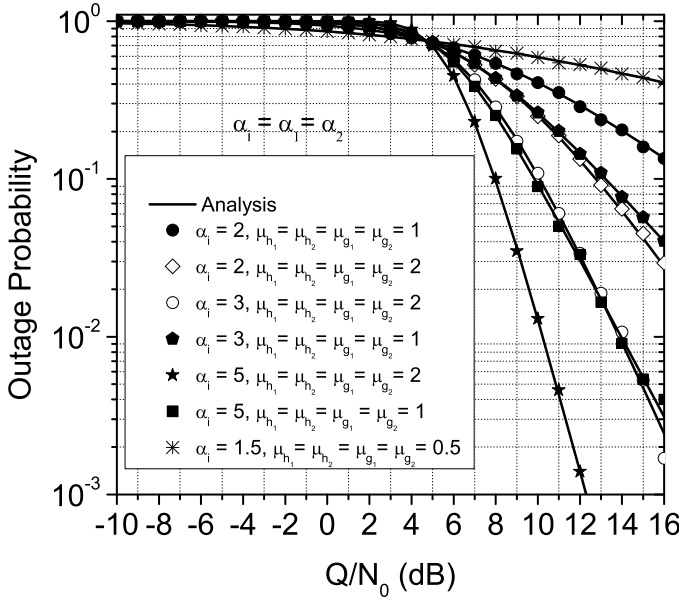


Figure 2: Outage probability of underlay cognitive system employing DF relaying for identical μ fading parameter: Nakagami- m , Rayleigh and other different distributions (Solid line: Analysis, Markers: Simulation).

These results are in line with those in [3] and [4]. It is interesting to consider the difference in outage performance when we increase α or μ parameter by one unit. Increasing μ by one unit provides a steeper progression than when one unit is increased in the α parameter. In Fig. 3, α_i is set to 2 for all channels. Then, μ_{h_i} and μ_{g_i} are varied while keeping the channels identical in terms of the μ parameter. With this setting, the Nakagami- m distribution is obtained. From Nakagami- m distribution, by setting μ_{h_i} and μ_{g_i} to 1, we obtain the Rayleigh distribution. The performance becomes worse for $\mu_{h_i} = \mu_{g_i} = 0.5$ corresponding to one-sided Gaussian distribution. In Fig. 4, by setting $\mu_{h_i} = \mu_{g_i} = 1$, the Weibull distribution is obtained. From Weibull distribution, the Rayleigh and the negative exponential distributions are obtained by, respectively, setting α to 2 and 1. It can be easily observed that the negative exponential distribution closely performs as the one-sided

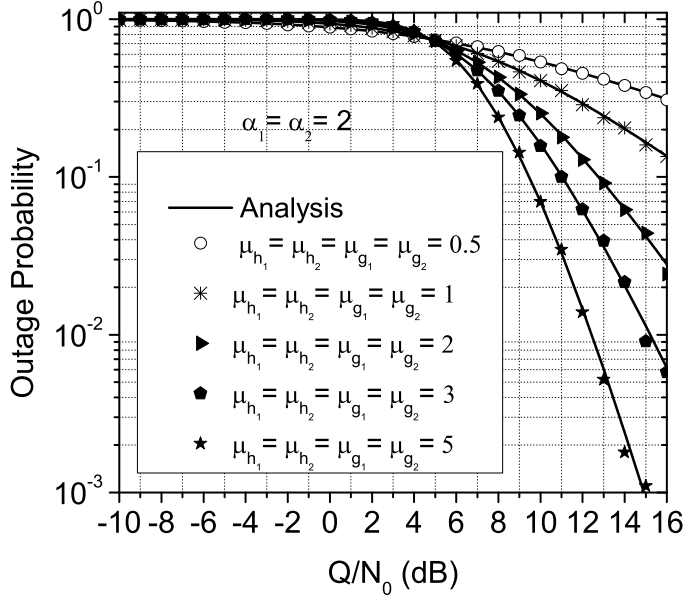


Figure 3: Outage probability of underlay cognitive system employing DF relaying for identical μ fading parameter: Nakagami- m and its special cases (Solid line: Analysis, Markers: Simulation).

Gaussian distribution plotted in Fig. 3. Fig. 5 shows the outage probability of the system with non-identical parameters μ_{h_i} and μ_{g_i} . We compare the setting by hop (first hop: α_i , μ_{h_1} and μ_{g_1} ; second hop: α_i , μ_{h_2} and μ_{g_2}) with the setting by interference or communication link channels (α_i , μ_{g_1} and μ_{g_2} ; α_i , μ_{h_1} and μ_{h_2}). Our observations in Fig. 5 focus on the Nakagami- m fading channels with $\alpha_1 = \alpha_2 = 2$. Firstly, we observe that the curve with $\mu_{h_1} = \mu_{g_1} = 2, \mu_{h_2} = \mu_{g_2} = 1$ matches that of $\mu_{h_1} = \mu_{g_1} = 1, \mu_{h_2} = \mu_{g_2} = 2$. That is, the channel parameters at the first and the second hop can be interchanged without altering the performance of the system. This is due to the fact that only the minimum SNR is chosen in dual hop communication. Secondly, it can be observed that $\mu_{h_1} = \mu_{h_2} = 2, \mu_{g_1} = \mu_{g_2} = 1$ does not offer the same performance as $\mu_{h_1} = \mu_{h_2} = 1, \mu_{g_1} = \mu_{g_2} = 2$. The reason is that the performance of the system is most sensitive to interference channels. The

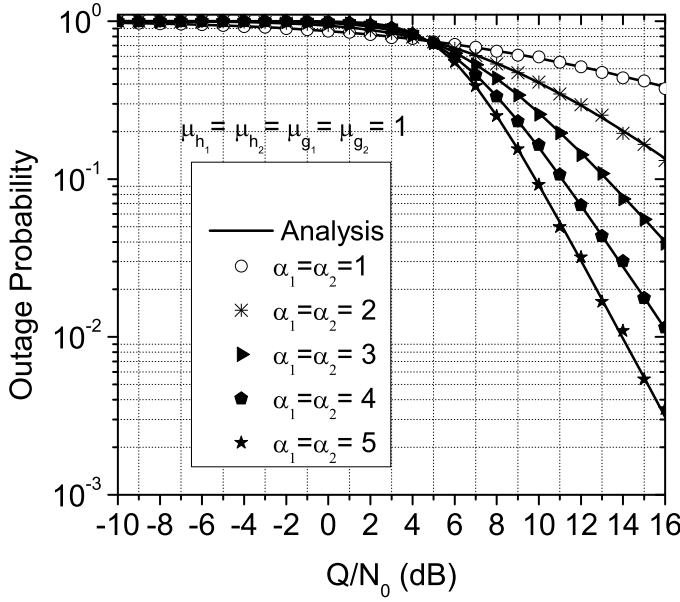


Figure 4: Outage probability of underlay cognitive system employing DF relaying for identical μ fading parameter: Weibull and its special cases (Solid line: Analysis, Markers: Simulation).

higher the fading severity of the interference channels, the lower the system outage probability. Fig. 6 plots the outage probability of the system with non-identical α parameter at the first hop and the second hop. It is interesting to observe that for $\alpha_1 = 4$ and $\alpha_2 = 2$ the outage probability is less than for the Rayleigh case at both channels, first and second hop, with $\alpha_1 = 2$ and $\alpha_2 = 2$, respectively. In this setting, we have also observed that a further increase of α_1 , only at the first hop, such that $\alpha_1 > 4$, does not offer a significant impact on the outage probability of the system.

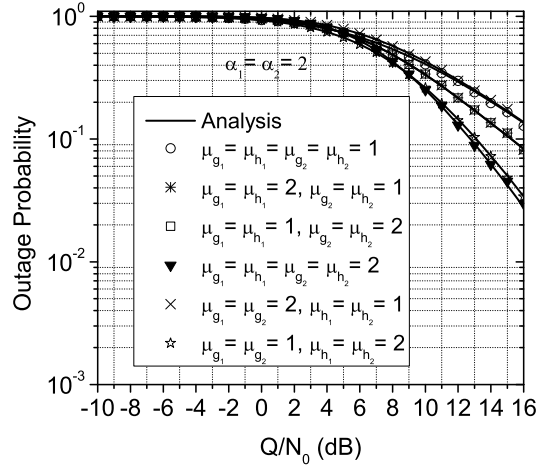


Figure 5: Outage probability of underlay cognitive system employing DF relaying for non-identical μ fading parameter (Solid line: Analysis, Markers: Simulation).

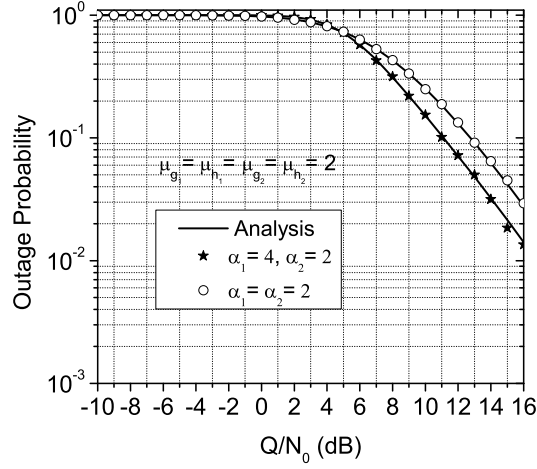


Figure 6: Outage probability of underlay cognitive system employing DF relaying for non-identical α fading parameter (Solid line: Analysis, Markers: Simulation).

5 Conclusions

In this paper, we have investigated the outage performance of an underlay cognitive relaying system under the peak interference power constraint for α - μ fading channels with non-identical parameters μ . We have derived an analytical expression for the outage probability of the dual-hop spectrum sharing system. The outage performance of the system in fading channels according to Nakagami- m and Weibull distributions, as well as some other related distributions, has been presented.

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Part I-B

PART I-B

Ergodic Capacity of Cognitive Relay Networks over α - μ Fading Channels

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Ergodic Capacity of Cognitive Relay Networks over α - μ Fading Channels

Charles Kabiri, Hans-Jürgen Zepernick, and Hung Tran

Abstract

In this paper, we investigate the ergodic capacity for an underlay cognitive relay network over α - μ fading channels. In particular, we assume the channels to be independent non-identically distributed α - μ random variables and analyze the system performance subject to a peak interference power constraint. Numerical results are presented to evaluate the effect of fading parameters on the ergodic capacity of the system.

1 Introduction

The rapid development of wireless applications has resulted in spectrum shortage and spectrum utilization inefficiency. To alleviate such problems, cognitive radio networks (CRNs) have gained significant research interests as a promising technique. In CRNs, the underlay scheme is one of the promising spectrum access paradigms, offering practical advantages. It allows the secondary (unlicensed) users (SUs) to communicate in the licensed spectrum of the primary users (PUs). On the other hand, due to signal fading in wireless channels, CRNs with relays can increase the radio coverage and enhance the secondary transmission performance.

In wireless communications, the fading channels have been generally characterized by statistical distributions including Rayleigh, Nakagami- m , and Weibull. In particular, Rayleigh and Nakagami- m distributions have received

special attention in CRNs due to the wide range of applicability. For example, a study on the channel capacity of a basic underlay spectrum-sharing system under different fading channels has been investigated in [1]. Recently, [2] examined under Nakagami- m fading the outage probability of dual-hop cognitive amplify-and-forward (AF) relay networks subject to independent non-identically distributed (i.n.i.d.) fading channels.

A general model of important distributions such as Nakagami- m and Weibull distributions has recently been proposed in [3], considering the non-linearity of the propagation environment and the number of multipath clusters, two important phenomena inherent to radio propagation. Other distributions such as one-sided Gaussian, Rayleigh, or negative exponential distribution can be obtained as special cases. Only few works in the literature, for example, [4] and [5] have addressed issues of spectrum-sharing such as outage probability or outage capacity for a general fading model, i.e. the α - μ fading. In [4], analytical expressions for the minimum outage probability and delay-limited capacity are derived for a spectrum sharing system undergoing α - μ fading with identical fading parameters. With non-identical parameter μ , [5] derived, under the peak interference power constraint, analytical expressions for the outage probability, amount of fading, and approximate ergodic capacity for a spectrum-sharing system with no relaying in the secondary network. The results reported in [6] analyze a single relay selection cooperative wireless network using decode-and-forward (DF) relaying and present the outage probability and symbol error probability of the network. There is no spectrum sharing considered in that network. Due to the relevance of the ratio of signal powers (quotient of the desired signal power and the interference power) in several analysis of wireless communication systems, the authors in [7] have derived the ratio of i.n.i.d. α - μ random variables with application to an underlay spectrum sharing system of a single hop composed of one SU in the presence of a single PU.

In this paper, we analyze the ergodic capacity of an underlay CRN with DF relaying under the peak interference power constraint over α - μ fading channels. In particular, we consider the ratio of independent α - μ random variables having non-identical μ parameters and derive the probability density function (PDF) and the cumulative distribution function (CDF) of the signal-to-noise ratio (SNR) of the dual-hop network. The obtained results are used to investigate the effect of fading parameters on the system capacity.

The remainder of this paper is organized as follows. Section II provides the system and channel model. In Section III, the PDF and CDF of the SNR at both secondary relay (SR) and SU receiver (SU-Rx) are derived. Numerical results are presented in Section IV. Finally, conclusions are given in Section V.

2 System Model

Consider a CRN as shown in Fig. 1 where the SU transmitter (SU-Tx) communicates with the SU-Rx only via the SR in a dual-hop fashion. There is no direct link between SU-Tx and SU-Rx. It is assumed that the SU-Tx is located far away from the SU-Rx and the SR is needed to assist the secondary transmission. The SU-Tx and the SR share the spectrum with the PU receivers, PU-Rx₁ and PU-Rx₂, respectively. We consider that the interference generated by the primary user transmitters (PU-Txs) operating in the secondary transmission area is modeled as additive zero-mean white Gaussian noise. We denote the channel coefficients for the links SU-Tx→SR, SU-Tx→PU-Rx₁, SR→SU-Rx, and SR→PU-Rx₂ by W , X , Y and Z , respectively. For conve-

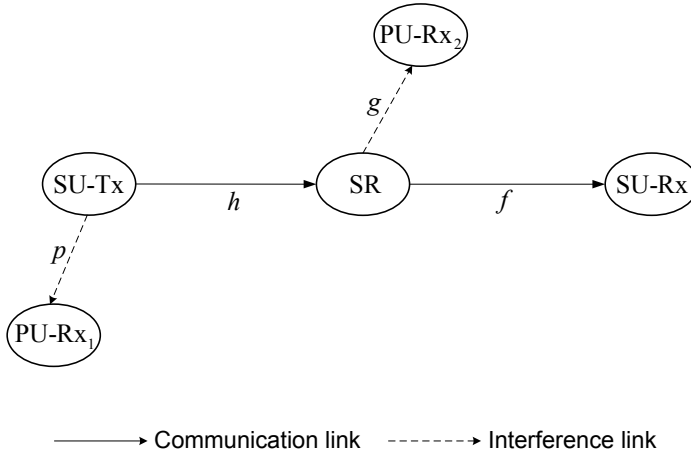


Figure 1: System model of a relay-based spectrum sharing system.

nience, let $\mathcal{S} = \{W, X, Y, Z\}$ denote the set of channel coefficients. Moreover, let R represent any of the elements in $\mathcal{S}$ and be distributed following the α - μ distribution [8]. According to [3], [8], the PDF of R is given by

$$f_R(r) = \frac{\alpha \mu^\mu \hat{r}^{\alpha\mu-1}}{\hat{r}^{\alpha\mu} \Gamma(\mu)} \exp\left(-\mu \frac{r^\alpha}{\hat{r}^\alpha}\right) \quad (1)$$

where $\hat{r} = (\mathbb{E}[R^\alpha])^{\frac{1}{\alpha}}$ is the α -th root mean value, $\alpha > 0$ is an arbitrary fading parameter, $\mu > 0$ is the inverse of the normalized variance of R^α calculated as

$\mu = \mathbb{E}^2[R^\alpha]/\mathbb{V}[R^\alpha]$ in which $\mathbb{E}[\cdot]$ and $\mathbb{V}[\cdot]$ denote the expectation and variance, respectively. Furthermore, the expression $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ defines the gamma function.

The k -th moment of R can be expressed as

$$\mathbb{E}[R^k] = \hat{r}^k \frac{\Gamma(\mu + \frac{k}{\alpha})}{\frac{k}{\mu\alpha} \Gamma(\mu)} \quad (2)$$

According to [8], the α - μ fading channel is considered as a general fading model comprising other PDFs as special cases. By varying the fading parameters, α and μ , well-known models such as Rayleigh ($\alpha = 2, \mu = 1$), Nakagami- m ($\alpha = 2, \mu = m$), and Weibull ($\mu = 1$) can be obtained. Also, note that from the above models, exponential ($\alpha = 1; \mu = 1$) and one-sided Gaussian ($\alpha = 1; \mu = 0.5$) can be easily obtained.

Let $\mathcal{K} = \{h, p, f, g\}$ denote the set of instantaneous channel power gains that are assumed to have unit mean, and $h = |W|^2$, $p = |X|^2$, $f = |Y|^2$, $g = |Z|^2$. According to [9], the PDF of $k \in \mathcal{K}$ can be expressed by

$$f_k(x) = \frac{\alpha_k x^{\frac{\alpha_k \mu_k}{2} - 1}}{2\Omega_k^{\frac{\alpha_k \mu_k}{2}} \Gamma(\mu_k)} \exp \left[- \left(\frac{x}{\Omega_k} \right)^{\frac{\alpha_k}{2}} \right] \quad (3)$$

where

$$\Omega_k = \frac{\Gamma(\mu_k)}{\Gamma(\mu_k + \frac{2}{\alpha_k})} \quad (4)$$

We assume that perfect channel state information (CSI) is available at SU-Tx, SR, and SU-Rx. Therefore, the SU-Tx and SR can regulate their power based on the CSI to guarantee that the interference caused by the secondary transmissions at the PU-Rxs does not exceed a certain threshold. Accordingly, the SNRs of the SR and the SU-Rx, together with the transmission power constraints at the PU-Rx₁ and the PU-Rx₂ can be expressed as follows:

$$\gamma_1 = \frac{P(h, p)h}{N_0} \quad (5a)$$

$$\text{s.t. } P(h, p)p \leq Q_1 \quad (5b)$$

$$\gamma_2 = \frac{P(f, g)f}{N_0} \quad (6a)$$

$$\text{s.t. } P(f, g)g \leq Q_2 \quad (6b)$$

where $P(h, p)$ and $P(f, g)$ are the instantaneous transmission powers at the SU-Tx and the SR, respectively. The noise power is denoted as N_0 , and Q_1 and Q_2 are the peak interference powers allowed at the PU-Rx₁ and the PU-Rx₂, respectively. If we assume that the SU-Tx and the SR can transmit at the maximal instantaneous powers allowed Q_1/p and Q_2/g , respectively, then, without loss of generality, assuming $Q_1 = Q_2 = Q$, the expressions (5a) and (6a) can be rewritten as

$$\gamma_1 = \frac{hQ}{pN_0} \quad (7)$$

$$\gamma_2 = \frac{fQ}{gN_0} \quad (8)$$

3 Performance Analysis

In this section, we derive the CDF and PDF of the instantaneous SNR at SU-Rx which supports the calculation of the ergodic capacity of the system.

3.1 CDF and PDF of the instantaneous SNR at SU-Rx

For a dual-hop transmission with DF mode, the SNR at the SU-Rx can be found as

$$\gamma_s = \min \{\gamma_1, \gamma_2\} \quad (9)$$

Let us assume that $\alpha_h = \alpha_p = \alpha_1$ and $\alpha_f = \alpha_g = \alpha_2$. Hence, according to [5], the PDF of γ_1 and γ_2 can, respectively, be given by

$$f_{\gamma_1}(\gamma) = \frac{\alpha_1 \left(\frac{N_0}{Q} \frac{\Omega_p}{\Omega_h} \right)^{\frac{\alpha_1 \mu_h}{2}}}{2\mathcal{B}(\mu_h, \mu_p)} \frac{\gamma^{\frac{\alpha_1 \mu_h}{2} - 1}}{\left[1 + \left(\frac{N_0 \Omega_p}{Q \Omega_h} \right)^{\frac{\alpha_1}{2}} \gamma^{\frac{\alpha_1}{2}} \right]^{\mu_p + \mu_h}} \quad (10)$$

$$f_{\gamma_2}(\gamma) = \frac{\alpha_2 \left(\frac{N_0}{Q} \frac{\Omega_g}{\Omega_f} \right)^{\frac{\alpha_2 \mu_f}{2}}}{2\mathcal{B}(\mu_f, \mu_g)} \frac{\gamma^{\frac{\alpha_2 \mu_f}{2} - 1}}{\left[1 + \left(\frac{N_0 \Omega_g}{Q \Omega_f} \right)^{\frac{\alpha_2}{2}} \gamma^{\frac{\alpha_2}{2}} \right]^{\mu_f + \mu_g}} \quad (11)$$

where $\mathcal{B}(\cdot, \cdot)$ is the Beta function [10, eq.(8.384.1)]. The CDF of γ_1 and γ_2 can, respectively, be obtained by integrating (10) and (11) with respect to γ

as

$$F_{\gamma_1}(\gamma) = \left(\frac{N_0 \Omega_p}{Q \Omega_h} \right)^{\frac{\alpha_1 \mu_h}{2}} \frac{\gamma^{\frac{\alpha_1 \mu_h}{2}}}{\mu_h \mathcal{B}(\mu_h, \mu_p)} \times {}_2F_1 \left(\mu_h + \mu_p, \mu_h; 1 + \mu_h; - \left(\frac{B N_0 \Omega_p}{Q \Omega_h} \right)^{\frac{\alpha_1}{2}} \gamma^{\frac{\alpha_1}{2}} \right) \quad (12)$$

$$F_{\gamma_2}(\gamma) = \left(\frac{N_0 \Omega_g}{Q \Omega_f} \right)^{\frac{\alpha_2 \mu_f}{2}} \frac{\gamma^{\frac{\alpha_2 \mu_f}{2}}}{\mu_f \mathcal{B}(\mu_f, \mu_g)} \times {}_2F_1 \left(\mu_f + \mu_g, \mu_f; 1 + \mu_f; - \left(\frac{B N_0 \Omega_g}{Q \Omega_f} \right)^{\frac{\alpha_2}{2}} \gamma^{\frac{\alpha_2}{2}} \right) \quad (13)$$

where ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ stands for the hypergeometric function [10, eq. (3.194.5)]. The CDF of γ_s in (9) can be found by the order statistics theory as follows:

$$F_{\gamma_s}(\gamma) = 1 - [1 - F_1(\gamma)][1 - F_2(\gamma)] \quad (14)$$

An analytical expression of $F_{\gamma_s}(\gamma)$ can be readily obtained by substituting $F_{\gamma_1}(\gamma)$ and $F_{\gamma_2}(\gamma)$ from (12) and (13) into (14) as

$$\begin{aligned} F_{\gamma_s}(\gamma) = 1 - & \left[1 - \frac{\gamma^{\frac{\alpha_1 \mu_h}{2}}}{\mathcal{B}(\mu_h, \mu_p) \mu_h} \times {}_2F_1 \left(\mu_h, \mu_p + \mu_h; 1 + \mu_h; \right. \right. \\ & \left. \left. - \gamma^{\alpha_1/2} \left(\frac{N_0 \Omega_p}{Q \Omega_h} \right)^{\alpha_1/2} \right) \left(\frac{N_0 \Omega_p}{Q \Omega_{h_i}} \right)^{\frac{\alpha_1 \mu_h}{2}} \right] \left[1 - \frac{\gamma^{\frac{\alpha_2 \mu_f}{2}}}{\mathcal{B}(\mu_f, \mu_g) \mu_f} \right. \\ & \left. \times {}_2F_1 \left(\mu_f, \mu_g + \mu_f; 1 + \mu_f; - \gamma^{\alpha_2/2} \left(\frac{N_0 \Omega_g}{Q \Omega_f} \right)^{\alpha_2/2} \right) \right. \\ & \left. \times \left(\frac{N_0 \Omega_g}{Q \Omega_f} \right)^{\frac{\alpha_2 \mu_f}{2}} \right] \end{aligned} \quad (15)$$

As such, the PDF of γ_s in (9) can be obtained by performing the first derivative of the CDF in (15) with respect to γ , i.e.

$$f_{\gamma_s}(\gamma) = \frac{dF_{\gamma_s}(\gamma)}{d\gamma} \quad (16)$$

which after appropriate substitutions and some algebraic manipulations, can be rewritten as in (17)

$$\begin{aligned}
f_{\gamma_s}(\gamma) = & \left\{ \beta_1^{-\mu_p - \mu_h} \beta_2^{-\mu_g - \mu_f} \left[\gamma^{\frac{\alpha_1 \mu_h}{2}} \alpha_1 \mu_h \mu_f \left(\frac{N_0 \Omega_p}{Q \Omega_h} \right)^{\frac{\alpha_1 \mu_h}{2}} \right. \right. \\
& \times \beta_2^{\mu_g + \mu_f} \mathcal{B}(\mu_f, \mu_g) + \gamma^{\frac{\alpha_2 \mu_f}{2}} \left(\frac{N_0 \Omega_g}{Q \Omega_f} \right)^{\frac{\alpha_2 \mu_f}{2}} \\
& \times \left[\alpha_2 \mu_h \mu_f \beta_1^{\mu_p + \mu_h} \mathcal{B}(\mu_h, \mu_p) - \gamma^{\frac{\alpha_1 \mu_h}{2}} \left(\frac{N_0 \Omega_p}{Q \Omega_h} \right)^{\frac{\alpha_1 \mu_h}{2}} \right. \\
& \times \left[\alpha_2 \mu_f \beta_1 \times {}_2F_1 \left(1, 1 - \mu_p; 1 + \mu_h; -\gamma^{\alpha_1/2} \left(\frac{N_0 \Omega_p}{Q \Omega_h} \right)^{\alpha_1/2} \right) \right. \\
& \left. \left. \left. + \alpha_1 \mu_h \beta_2 \times {}_2F_1 \left(1, 1 - \mu_g; 1 + \mu_f; -\gamma^{\alpha_2/2} \left(\frac{N_0 \Omega_g}{Q \Omega_f} \right)^{\alpha_2/2} \right) \right] \right] \right] \right\} \\
& \times \left\{ 2\gamma \mu_h \mu_f \mathcal{B}(\mu_h, \mu_p) \mathcal{B}(\mu_f, \mu_g) \right\}^{-1} \quad (17)
\end{aligned}$$

where

$$\beta_1 = 1 + \gamma^{\alpha_1/2} \left(\frac{N_0 \Omega_p}{Q \Omega_h} \right)^{\alpha_1/2} \quad (18)$$

$$\beta_2 = 1 + \gamma^{\alpha_2/2} \left(\frac{N_0 \Omega_g}{Q \Omega_f} \right)^{\alpha_2/2} \quad (19)$$

3.2 Ergodic capacity

The ergodic capacity is defined as the maximum rate achieved in the long-term over all channel states of the time-varying fading channel. According to the min-cut max-flow theorem [11], in dual-hop DF cooperative relaying transmission, the total system capacity cannot be larger than the capacity achieved by individual relaying link. That is, the overall system capacity is the minimum of the individual capacity achieved over the first and second hops [12], [13]. In this context, considering (9) and the PDF expression in (17), the ergodic capacity of the considered dual-hop DF relaying system is given by

$$C_{erg} = \frac{1}{2} \int_0^\infty B \log_2(1 + \gamma) f_{\gamma_s}(\gamma) d\gamma \quad (20)$$

where B (Hz) is the total available bandwidth. As a closed-form expression of (20) is not available due to calculation complexity of the integral, with the help of the computing softwares such as Mathematica and Matlab, the numerical results of the ergodic capacity of the system can be obtained.

4 Numerical Results

In this section, the ergodic capacity per unit bandwidth of the considered cognitive relay network is numerically evaluated. The *wggamrnd* function provided in the WAFO Matlab toolbox [14] is used to simulate the α - μ fading channels. Generally, the investigated scenarios consist of varying fading parameters (α ; μ) in order to get insights on the ergodic capacity of the cognitive relay system under several fading models such as Rayleigh ($\alpha = 2$; $\mu = 1$), Nakagami- m ($\alpha = 2$; $\mu = m$), Weibull ($\mu = 1$), exponential ($\alpha = 1$; $\mu = 1$) and one-sided Gaussian ($\alpha = 1$; $\mu = 0.5$).

Fig. 2 illustrates the ergodic capacity per unit bandwidth as a function of the peak interference power to noise ratio Q/N_0 for various fading scenarios. Clearly, the ergodic capacity increases as Q/N_0 grows. It can be seen that at

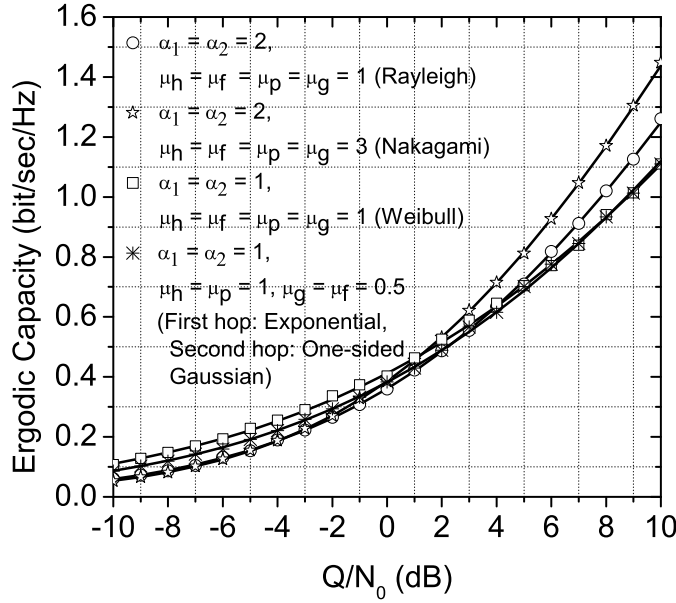


Figure 2: Ergodic capacity per unit bandwidth of an underlay cognitive system employing DF relaying over α - μ fading channels (solid line: Analysis; markers: Simulation).

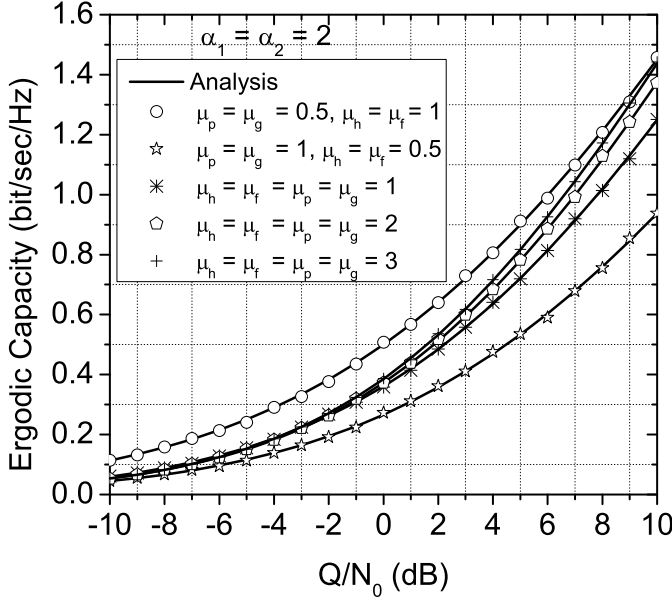


Figure 3: Ergodic capacity per unit bandwidth of an underlay cognitive system employing DF relaying over α - μ fading channels. Nakagami- m and its special cases ($\alpha_1 = \alpha_2 = 2, \mu_p = \mu_g = m$ and $\mu_h = \mu_f = m$, solid line: Analysis; markers: Simulation).

low values of Q/N_0 , the Weibull fading scenario provides better capacity than the other fading scenarios. At higher values of Q/N_0 , the capacity of Weibull channels becomes worse and matches with that of the one-sided Gaussian scenario. This can be explained by the fact that as there is no distinction in severity between communication and interference links, only low values of Q/N_0 show the advantages of severe fading at the interference links. That is, when the PUs can only tolerate very low interference powers, the transmission of the SU network can improve in severe fading conditions. In the case that the PUs can tolerate high values of powers, for instance at higher values of Q/N_0 , there is no benefit of severe fading channels at the interference links for the secondary transmission. Also, it is interesting to observe that, generally, as the μ or α parameter increase, the capacity increases as well.

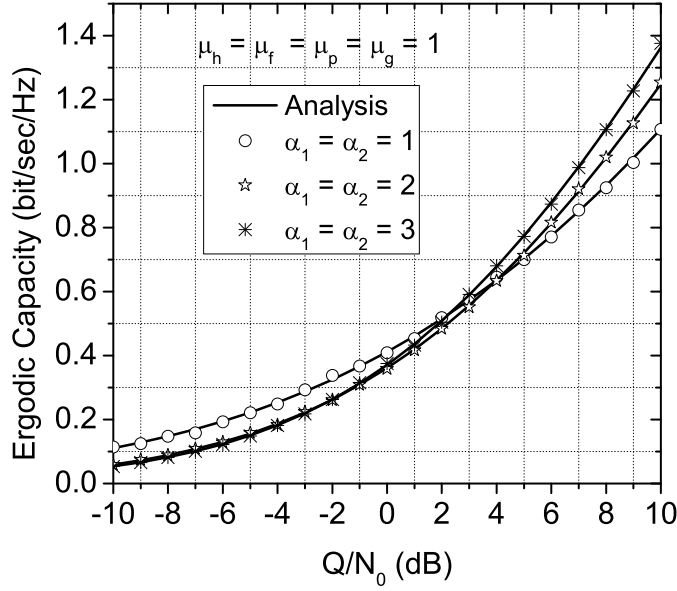


Figure 4: Ergodic capacity per unit bandwidth of an underlay cognitive system employing DF relaying over α - μ fading channels. Weibull and its special cases ($\mu_p = \mu_g = \mu_h = \mu_f = 1$, solid line: Analysis; markers: Simulation).

Fig. 3 shows the ergodic capacity per unit bandwidth for the case where all channels are facing Nakagami- m fading. It can be easily observed that for interference channels with severe fading, $\mu_p = \mu_g = 0.5$, and communication channels under Rayleigh fading $\mu_h = \mu_f = 1$, the system provides the highest capacity over the whole range of Q/N_0 . For the reverse situation ($\mu_p = \mu_g = 1$ for interference channels, and $\mu_h = \mu_f = 0.5$ for communication channels), the capacity is the lowest. In fact, the capacity achieved by the aforementioned scenarios are the upper and lower bound among all the investigated scenarios throughout this study. Clearly, this explains the benefit of severe fading channels at the interference links.

Fig. 4 shows the Weibull scenario ($\mu_p = \mu_g = \mu_h = \mu_f = 1$) from which the exponential fading scenario can be attained with $\alpha_1 = \alpha_2 = 1$. Clearly, when all channels are facing exponential fading, the ergodic capacity is higher

at low values of Q/N_0 and becomes lower for increased values of Q/N_0 . An increase of one unit of the α parameter from $\alpha_1 = \alpha_2 = 2$ to $\alpha_1 = \alpha_2 = 3$ does not have any effect in the low regime of Q/N_0 . In the high regime of Q/N_0 , $\alpha_1 = \alpha_2 = 3$ (less fading severity) provides the highest capacity because there is no distinction in severity between interference and communication channels.

5 Conclusions

In this paper, we have investigated the ergodic capacity of an underlay cognitive relay network under the peak interference power constraint. We have utilized the ratio of independent α - μ random variables to derive the PDF and CDF expressions for the instantaneous SNR at the SU-Rx of a dual-hop DF spectrum sharing system over α - μ fading channels with non-identical μ parameters. Numerical results of the ergodic capacity of the system in fading channels according to Nakagami- m , Weibull as well as some other related distributions have been presented.

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Part II-A

PART II-A

Outage Probability and Ergodic Capacity of a Spectrum Sharing System with Multiuser Diversity

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Outage Probability and Ergodic Capacity of a Spectrum Sharing System with Multiuser Diversity

Charles Kabiri, Hung Tran, and Hans-Jürgen Zepernick

Abstract

In this paper, we derive analytical expressions for outage probability and ergodic capacity of a spectrum sharing system with multiuser diversity under Nakagami- m fading. In particular, multiuser diversity effects in such systems where several secondary users utilize the licensed spectrum of the primary user are considered. The best and worst channel quality scenarios are investigated for transmission from a number of secondary transmitters. Numerical analysis and simulations are shown to reveal the effects of multiusers and fading parameters on the considered system.

1 Introduction

The concept of cognitive radios emerged from the fact that frequency resources have become increasingly exhausted. Measurements conducted by the Federal Communications Commission (FCC) through the Spectrum Policy Task Force (SPTF) showed that frequency bands are utilized inefficiently, indicating many portions of unused or lightly used spectrum for significant periods of time [1]. Since many years, back to the 1920's, the main approach to protect systems from interfering with each other was to establish rigorous limits on the permitted transmit power. This traditional approach for spectrum management started being perceived as inefficient due to its inflexibility.

In recent years, there have been many studies on spectrum sharing systems [1][2] due to their potential of improving spectrum utilization. In particular, spectrum sharing refers to an approach that offers resources to cognitive users, so-called secondary users (SUs), while keeping interference to the licensees of the spectrum, referred to as primary users (PUs), within acceptable limits. This can be achieved by coordinating the multiple access of users and allocating radio resources adaptively to changes of the radio environment. Moreover, the secondary network may benefit through opportunistic user selection which is known to offer multiuser diversity gains in fading environments [3]. There are several works investigating the capacity gains offered by dynamic spectrum sharing systems such as [4], [5], and [6] among others. However, still few of them extended to the case of having multiple users, i.e. multiple PUs and/or multiple SUs.

The authors of [7] allowed multiple PUs, where the capacity of a spectrum sharing system is evaluated in the presence of Rayleigh and Nakagami fading. In [8], capacity gains of opportunistic user selection in a spectrum sharing system subject to Rayleigh fading are examined. First, multiple SUs are considered and then multiple PUs are allowed. It is shown that multiple PUs result in poor performance in terms of interference temperature compared to the case of a single PU. In [9], outage probability and average transmission time in the presence of Nakagami- m fading channels have been derived for a single SU scenario. Given a peak interference power constraint, in [10], queuing aspects for a spectrum sharing system with a single SU in the presence of Nakagami- m have been studied.

Motivated, especially by the advantages of opportunistic transmission in fading environment, we focus here on the outage probability and ergodic capacity of a spectrum sharing system with multiuser diversity in Nakagami- m fading, i.e. considering multiple SUs instead of multiple PUs like in [7].

The remainder of this paper is organized as follows. In Section II, we describe the system model. Section III provides the performance analysis utilizing various statistics to derive expressions for outage probability and ergodic capacity under Nakagami m fading channels of the system. Section IV gives numerical analysis and discussions and finally, Section V concludes our study.

2 System Model

In this paper, we adopt the spectrum sharing system suggested in [8] where N SU transmitters (SU-Tx) utilize the spectrum licensed to the PU as shown in Fig. 1. In this system, the SU-Txs are not allowed to transmit at power levels that would cause unacceptable interference at the PU receiver (PU-Rx).

In other words, the maximum allowed interference power level at the PU is constrained to not exceed a predefined level of Q .

Let us denote the channels gains from the i -th SU-Tx _{i} to the PU-Rx and to the target SU receiver (SU-Rx) by α_i and β_i , respectively. These channel gains are assumed to be independent and identically distributed (i.i.d.) Gamma random variables with unit mean establishing independent Nakagami- m fading channels. Furthermore, we assume that all SU-Txs have perfect information about the interference channel gains α_i . This could be obtained, for example, through direct feedback from the PU-Rx [7] or indirect feedback from a third-party [8]. Given perfect channel state information (CSI) at both SU-Txs and

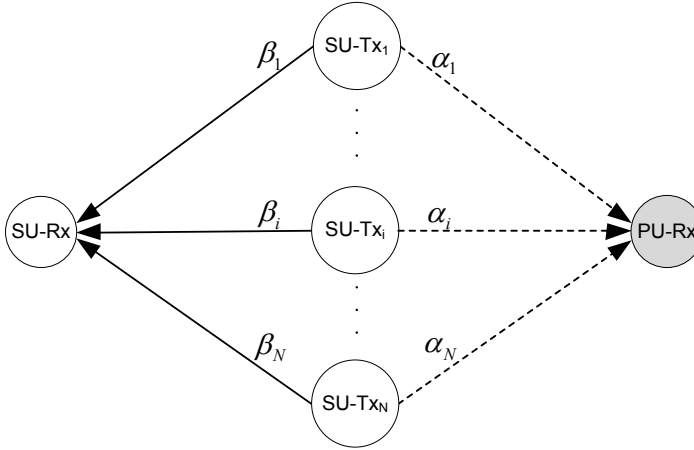


Figure 1: A spectrum sharing model [8].

SU-Rx, the instantaneous power adaptation at each SU-Tx can be formulated as a function of channel gains as $P(\alpha_i, \beta_i)$. Then, each SU-Tx reports its transmit power to the SU-Rx which selects the best SU-Tx in terms of the signal-to-noise ratio (SNR).

Interference from the PU to the secondary network is ignored following the assumption of [8] that the PU-Tx is located sufficiently far from the SU-Rx. In this case, interference from PU-Tx to SU-Rx may be lumped together with the noise term at the SU-Rx following a Gaussian distribution.

3 Performance Analysis

First of all, it is noted that the received SNR at the SU-Rx corresponding to the i -th SU-Tx is given by

$$\gamma_i = \frac{\beta_i P(\alpha_i, \beta_i)}{N_0} \quad i = 1, 2, \dots, N \quad (1)$$

where $P(\alpha_i, \beta_i)$ is the transmit power of the i -th SU-Tx and N_0 is the noise power spectral density. Given the peak interference power constraint of the i -th SU-Tx as

$$\alpha_i P(\alpha_i, \beta_i) \leq Q, \quad i = 1, 2, \dots, N \quad (2)$$

the corresponding maximum transmit power of the i -th SU-Tx is obtained as

$$P(\alpha_i, \beta_i) = \frac{Q}{\alpha_i}, \quad i = 1, 2, \dots, N \quad (3)$$

In the remainder of this paper we assume that all SU-Txs transmit at their maximum allowable transmit power, which translates to the following SNRs at the SU-Rx:

$$\gamma_i = \frac{\beta_i Q}{\alpha_i N_0} \quad i = 1, 2, \dots, N \quad (4)$$

In particular, we consider the two cases of the SU-Rx selecting that SU-Tx among the N SU-Txs which has best channel quality and worst channel quality, i.e. highest and lowest SNR, respectively.

3.1 Probability distributions

In this paper, we consider the general scenario of a spectrum sharing system under Nakagami- m fading, where m denotes the fading severity parameter. The related Nakagami- m distribution [11] can be fitted to a wide range of empirical data by adjusting the fading severity parameter m . For example, Rayleigh fading is obtained for the special case of $m = 1$.

Given Nakagami- m fading in the considered spectrum sharing systems, channel power gains α_i and β_i are distributed according to the following Gamma distribution:

$$f_X(x) = \frac{m^m x^{m-1}}{\Gamma(m)} e^{-mx}, \quad x \geq 0 \quad (5)$$

where $X \in \{\alpha_i, \beta_i\}$ and $\Gamma(\cdot)$ is the Gamma function defined as $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ [12, eq. (8.310.1)]. The fading severity parameters of the SU-Tx _{i} to PU-Rx and SU-Tx _{i} to SU-Rx links shall be denoted as m_0 and m_1 , respectively.

It can be seen from (4) that the distribution of the ratio β_i/α_i is crucial for calculating the capacity and outage probability. In this context, it has been shown in [7] that both metrics depend on the joint channel statistics only through the first-order probability density function (PDF) of β_i/α_i , i.e. $f_{\beta_i/\alpha_i}(\cdot)$.

As the function $f_{\beta_i/\alpha_i}(\cdot)$ has been developed in [7], we can focus on deriving the cumulative distribution function (CDF) of the SNRs γ_i as follows:

$$F_{\gamma_i}(\gamma) = \Pr\left\{\frac{\beta_i}{\alpha_i} \leq \gamma \frac{N_0}{Q}\right\} \quad (6)$$

where α_i and β_i are Gamma distributed random variables with fading severity parameters m_0 and m_1 , respectively. Then, the CDF of $Z = \frac{\beta_i}{\alpha_i}$ can be obtained by using the PDF, $f_{\beta_i/\alpha_i}(\cdot)$, from [7] as follows:

$$\begin{aligned} F_Z(z) &= \left(\frac{m_1}{m_0}\right)^{m_1} \int_0^z \frac{z^{m_1-1}}{\mathcal{B}(m_0, m_1)(1 + z \frac{m_1}{m_0})^{m_0+m_1}} dz \\ &= {}_2F_1\left(m_1, m_0 + m_1; 1 + m_1; -z \frac{m_1}{m_0}\right) \\ &\quad \times \left(\frac{m_1}{m_0}\right)^{m_1} \frac{\Gamma(m_1)z^{m_1}}{\mathcal{B}(m_0, m_1)} \end{aligned} \quad (7)$$

where ${}_2F_1(\cdot, \cdot, \cdot; \cdot)$ stands for the hypergeometric function [12, eq. (3.194.5)] and $\mathcal{B}(a, b)$ is the beta function [12, eq.(8.384.1)] defined as

$$\mathcal{B}(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \quad (8)$$

Therefore, the CDF of γ_i can be found as

$$\begin{aligned} F_{\gamma_i}(\gamma) &= F_Z\left(\gamma \frac{N_0}{Q}\right) \\ &= \left(\frac{m_1}{m_0}\right)^{m_1} \left(\frac{N_0}{Q}\right)^{m_1} \frac{\Gamma(m_1)\gamma^{m_1}}{\mathcal{B}(m_0, m_1)} \\ &\quad \times {}_2F_1\left(m_1, m_0 + m_1; 1 + m_1; -\gamma \frac{m_1 N_0}{m_0 Q}\right) \end{aligned} \quad (9)$$

Furthermore, the PDF of γ_i is found by differentiating (9) with respect to γ yielding

$$f_{\gamma_i}(\gamma) = \left(\frac{N_0 m_1}{Q m_0}\right)^{m_1} \frac{\gamma^{m_1-1}}{\mathcal{B}(m_0, m_1)(1 + \gamma \frac{N_0 m_1}{Q m_0})^{m_0+m_1}} \quad (10)$$

3.2 Outage probability with SU-Tx in best channel condition

In this case, the SU-Rx selects the SU-Tx that offers the best channel quality among the N SU-Txs. Then, the SNR at the SU-Rx with respect to the selected SU-Tx is obtained as

$$\gamma_{max} = \max_{i=1,2,\dots,N} \{\gamma_i\} = \max_{i=1,2,\dots,N} \left\{ \frac{\beta_i}{\alpha_i} \right\} \frac{Q}{N_0} \quad (11)$$

From the order statistics theory, the PDF and CDF of the end-to-end SNR, that is, when an SU-Tx with the best channel quality among N SU-Txs is selected, is given by

$$\begin{aligned} f_{\gamma_{max}}(\gamma) &= N f_{\gamma_i}(\gamma) [F_{\gamma_i}(\gamma)]^{N-1} \\ &= N \left(\frac{N_0}{Q} \frac{m_1}{m_0} \right)^{m_1 N} \frac{\Gamma(m_1)^{N-1}}{\mathcal{B}(m_0, m_1)^N} \\ &\quad \times \left[{}_2F_1 \left(m_1, m_0 + m_1; 1 + m_1; -\gamma \frac{m_1 N_0}{m_0 Q} \right) \right]^{N-1} \\ &\quad \times \frac{\gamma^{m_1 N - 1}}{\left(1 + \gamma \frac{N_0}{Q} \frac{m_1}{m_0} \right)^{m_0 + m_1}} \end{aligned} \quad (12)$$

$$F_{\gamma_{max}}(\gamma) = [F_{\gamma_i}(\gamma)]^N \quad (13)$$

Using (12), the outage probability for the case of selecting the SU-Tx with best channel condition is given by

$$P_{out}^b = \Pr\{\gamma_{max} < 2^{\frac{r_0}{B}} - 1\} = [F_{\gamma_i}(2^{\frac{r_0}{B}} - 1)]^N \quad (14)$$

where r_0 denotes transmission rate and B is the total available bandwidth given in normalized units (nats/Hz).

3.3 Outage probability with SU-Tx in worst channel condition

Similarly, when the SU-Rx selects the SU-Tx with the worst channel quality among N SU-Txs, the received SNR with respect to the selected SU-Tx is given by

$$\gamma_{min} = \min_{i=1,2,\dots,N} \{\gamma_i\} = \min_{i=1,2,\dots,N} \left\{ \frac{\beta_i}{\alpha_i} \right\} \frac{Q}{N_0} \quad (15)$$

The PDF and CDF, respectively, of the minimum SNR is given according to the order statistics theory as follows:

$$f_{\gamma_{min}}(\gamma) = N f_{\gamma_i}(\gamma) [1 - F_{\gamma_i}(\gamma)]^{N-1} \quad (16)$$

$$F_{\gamma_{min}}(\gamma) = 1 - [1 - F_{\gamma_i}(\gamma)]^N \quad (17)$$

from which the outage probability can be easily obtained as

$$\begin{aligned} P_{out}^w &= \Pr\{\gamma_{min} < 2^{\frac{r_0}{B}} - 1\} \\ &= 1 - \prod_{i=1}^N [1 - \Pr\{\gamma_i < 2^{\frac{r_0}{B}} - 1\}] \\ &= 1 - \prod_{i=1}^N [1 - F_{\gamma_i}(2^{\frac{r_0}{B}} - 1)] \end{aligned} \quad (18)$$

3.4 Ergodic capacity for best channel condition and interference power constraint

Let us now consider the capacity of the secondary network subject to the peak interference power constraint imposed by the PU. It should be noted that it is in some cases desirable to impose a peak interference power constraint even though average received power constraint is advantageous for delay insensitive applications [7][13]. Assuming that there are no other limitations on the transmit power of an SU-Tx, then the capacity in the secondary network is maximized by transmitting at maximum instantaneous power allowed, i.e. when $P(\alpha_i, \beta_i) = Q/\alpha_i$, and by making the SU-Rx selecting the SU-Tx with best channel quality among the N SU-Txs. Specifically, the ergodic capacity is defined as

$$\mathbb{E}[B \log_2(1 + \gamma)] = \int_0^\infty B \log_2(1 + \gamma) f_{\gamma_{max}}(\gamma) d\gamma \quad (19)$$

Substituting (12) into (19), we obtain the ergodic capacity of the considered spectrum sharing system with best channel condition as

$$\begin{aligned}
 C_{erg} = & BN \left(\frac{N_0}{Q} \frac{m_1}{m_0} \right)^{m_1 N} \Gamma(m_1)^{N-1} \\
 & \times \frac{1}{\mathcal{B}(m_0, m_1)^N} \int_0^\infty \log_2(1 + \gamma) \\
 & \times \left[{}_2F_1 \left(m_1, m_0 + m_1; 1 + m_1; -\gamma \frac{m_1 N_0}{m_0 Q} \right) \right]^{N-1} \\
 & \times \frac{\gamma^{m_1 N-1}}{\left(1 + \gamma \frac{N_0}{Q} \frac{m_1}{m_0} \right)^{m_0 + m_1}} d\gamma
 \end{aligned} \tag{20}$$

To the best of our knowledge, a closed-form expression for the ergodic capacity given in (20) is not available due to the complexity of the hypergeometric function. However, we can obtain analytical results by using a popular computational software such as Mathematica.

4 Numerical Results

Based on the analytical outage probability and capacity expressions derived from statistics in the previous sections, numerical results are presented for different fading conditions. In particular, the analysis is performed for noise power spectral density of $N_0 = 1$ W/Hz and transmission rate of $r_0 = 1$ bit/sec/Hz.

Figure 2 shows the outage probability for the case when the SU-Rx selects the SU-Tx with best channel conditions resulting in maximum SNR. As can be seen from the figure, the outage probability decreases with the increase of the peak interference power Q . The results also illustrate that it is beneficial to increase the number of SU-Txs. Specifically, a significant performance improvement is obtained by increasing the number of SU-Txs from $N = 3$ to $N = 5$. This behavior is thought to be attributed to the fact that increasing the number of SU-Txs also increases the options for the best channel conditions. As far as changes in the fading severity parameters m_0 and m_1 are concerned, we consider the following three scenarios:

- $(m_0, m_1) = (2, 2)$: SU-Tx to PU-Rx and SU-Tx to SU-Rx links are less severe than Rayleigh fading.
- $(m_0, m_1) = (1, 2)$: SU-Tx to PU-Rx links are subject to Rayleigh fading while SU-Tx to SU-Rx links are less severe than Rayleigh fading.
- $(m_0, m_1) = (2, 1)$: SU-Tx to PU-Rx links are less severe than Rayleigh fading while SU-Tx to SU-Rx links are subject to Rayleigh fading.

For the scenarios when the SU-Tx to PU-Rx links are subject to Rayleigh fading while SU-Tx to SU-Rx links are less severe than Rayleigh fading, the lowest outage probability is achieved for the considered values of the peak interference power Q . Because of the more severe fading in the SU-Tx to PU-Rx links, the interference to the PU-Rx is reduced. On the other hand, if the SU-Tx to PU-Rx links are less severe than Rayleigh fading but the SU-Tx to SU-Rx links suffer from Rayleigh fading, inferior performance in terms of outage probability is observed. Finally, for the scenario where SU-Tx to PU-Rx and SU-Tx to SU-Rx links are all less severe than Rayleigh fading, outage probability is bound within the results obtained for the above two scenarios. In particular, the outage probability aligns closely with the results for $(m_0, m_1) = (2, 1)$ in the low peak interference power regime and converges to the results for $(m_0, m_1) = (1, 2)$ with increasing peak interference power.

Figure 3 shows the outage probability for the case when the SU-Rx selects the SU-Tx having worst channel conditions and hence giving the lowest SNR. Accordingly, the outage probability increases with the increase of the number of SU-Tx, i.e. from $N = 3$ to $N = 5$ for the considered example. This is because the options for worst channel conditions also increase with more SU-Tx used in the spectrum sharing system. Regarding the different constellations of fading severity parameters, the same ordering of outage probability as for the best channel conditions is retained. However, it should be noted that the absolute levels of outage probabilities are significantly higher with selection of worst channel condition compared to best channel condition.

Figure 4 shows the achievable ergodic capacity versus the peak interference power constraint Q for various numbers N of SU-Tx. Here, we have chosen the scenario where the SU-Tx selects the SU-Rx with the best channel condition. Clearly, for a given set of fading severity parameters (m_0, m_1) and fixed peak interference power Q , the ergodic capacity increases with the number N of SU-Tx. Likewise, for a fixed number of SU-Tx, ergodic capacity increases

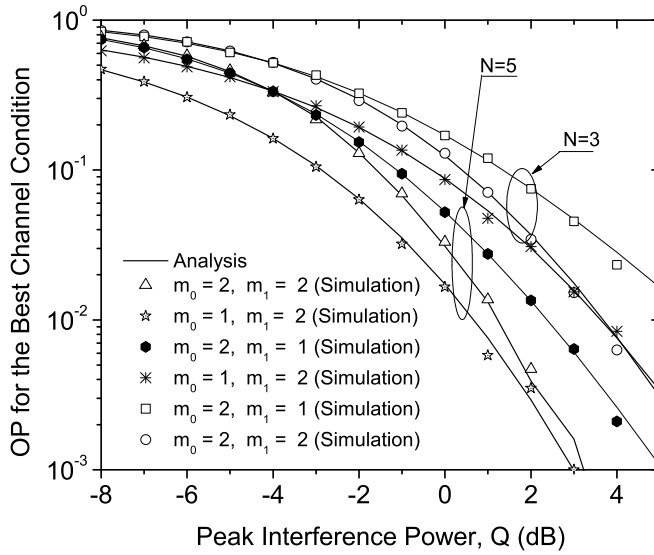


Figure 2: Outage probability for the case when the SU-Rx selects the SU-Tx with the best channel condition versus tolerable peak interference power.

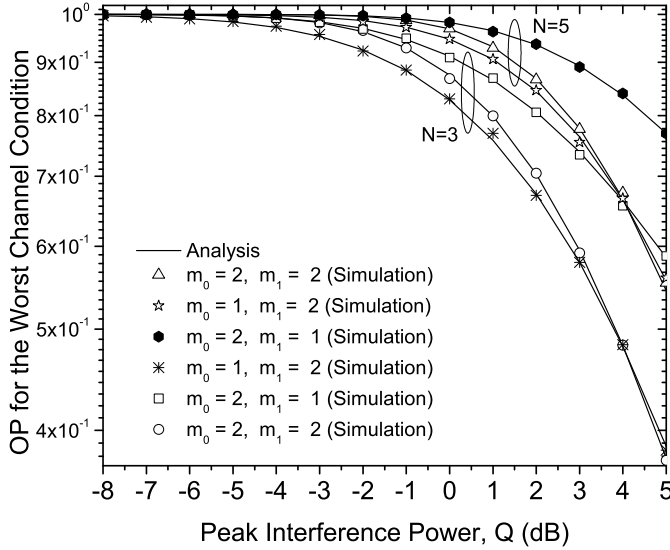


Figure 3: Outage probability for the case when the SU-Rx selects the SU-Tx with the worst channel condition versus tolerable peak interference power.

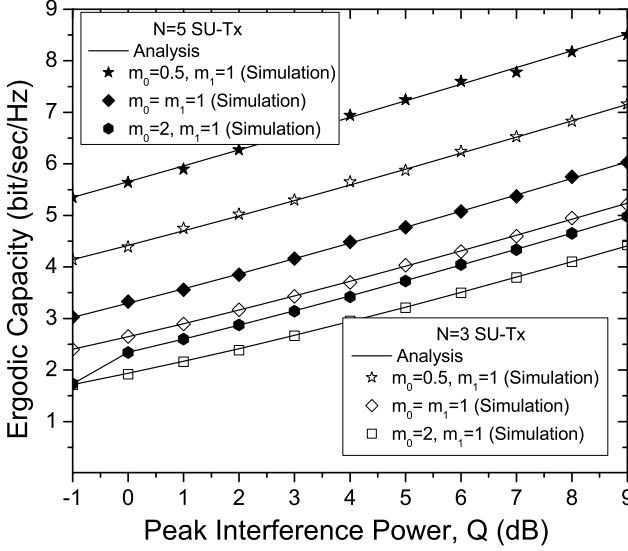


Figure 4: Ergodic capacity for the case when the SU-Rx selects the SU-Tx with the best channel condition versus tolerable peak interference power.

significantly with the increase of tolerable peak interference power Q . It can also be observed from these results that the ergodic capacity substantially increases when the SU-Tx to PU-Rx links suffer more severe fading ($m_0 = 0.5$) compared to the SU-Tx to SU-Rx links ($m_1 = 1$). On the other hand, the ergodic capacity decreases when the SU-Tx to PU-Rx links undergo less severe fading ($m_0 = 2$) compared to the SU-Tx to SU-Rx links ($m_1 = 1$).

5 Conclusions

In this paper, we considered a spectrum sharing system with multiuser diversity in the presence of Nakagami- m fading. In particular, we obtained analytical expression for outage probability and ergodic capacity. For this purpose, PDF and CDF of the end-to-end SNR at the SU-Tx have been derived. These results were then used to derive the outage probabilities for the cases when the SU-Rx selects the SU-Tx out of a given number of SU-Tx having best and worst channel condition. Numerical results were presented to illustrate applications of the derived expressions and to reveal both the impact of the number of SU-Tx as well as the impact of the fading severity parameters on the system performance. It should also be mentioned that numerical results

and simulations closely match for the considered scenarios.

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Part II-B

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Outage Probability and Ergodic Capacity of Underlay Cognitive Radio Systems with Adaptive Power Transmission

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Outage Probability and Ergodic Capacity of Underlay Cognitive Radio Systems with Adaptive Power Transmission

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Abstract

In this paper, we consider a spectrum sharing system in which a primary transmission coexists with a secondary network composed of a secondary base station (SBS) and multiple secondary user receivers (SU-Rxs). The secondary user (SU) can access the spectrum licensed to the primary user (PU) as long as the SU does not cause any harmful interference to the PU. Among all available SU-Rxs, only the best SU-Rx with highest signal-to-interference-plus-noise ratio (SINR) is selected to receive the transmit signals from the SBS. We derive an analytical expression for the outage probability (OP) of the SU-Rx with highest SINR. Moreover, we derive an approximation of the ergodic capacity at the SU-Rx. In order to meet the requirements at the PU receiver (PU-Rx), at the same time, satisfying the quality of the SU link, an adaptive power allocation strategy is developed. The power allocation policy imposes the SU transmitter (SU-Tx) to adapt its transmit power in order to avoid harmful interference at the PU receiver (PU-Rx), with a PU predefined outage constraint.

1 Introduction

Cognitive radios (CRs) allow significant increase of spectrum utilization through intelligent awareness of its surrounding environment [1]. The transmission in cognitive radio networks (CRNs) can be categorized in three modes:

interweave, overlay and underlay modes according to how the transmission does affect the primary (licensed) user, PU. In interweave mode, secondary (unlicensed) users, (SUs) opportunistically exploit unused license band, i.e., spectral holes, to transmit with minimum interference to other communications. The overlay mode, allows SUs to transmit at any power, simultaneously with PUs. In particular, the underlay mode allows the SUs to transmit as long as they do not cause any harmful interference to PUs. That is, a certain prescribed interference threshold is imposed to the SU transmission to not interfere with the PU communication [2], [3]. The capacity of fading channels in underlay CRN has attracted a lot of attention. In [4], the capacity of underlay CRN for additive white Gaussian noise (AWGN) channels is investigated. In [5] and [6], when channels vary due to fading, the capacity gain and power allocation strategy are presented for a basic spectrum sharing scenario composed of a pair of secondary transmitter (SU-Tx) and secondary receiver (SU-Rx) in the presence of a primary receiver (PU-Rx). The capacity of fading channels subject to a received power constraint at the PU-Rx is investigated. The results indicate a significant capacity gain in varying the fading channels with full channel state information (CSI) available at the SU-Tx. In [7], flat Rayleigh fading has been considered to derive the capacity and optimum power allocation scheme for three different capacity notions, namely, ergodic, outage, and minimum-rate. Therein, the averaged and peak interference power constraints at the PU-Rx are utilized. With full CSI available at the SU-Tx, [8] developed the ergodic capacity and the outage capacity under average/peak transmit power constraints and average/peak interference constraints. In [9] and [10], the impact of imperfect channel knowledge between the SU-Tx and the PU-Rx is investigated. A closed-form expression of the mean SU capacity under a peak power constraint is derived. The multi-user communication aspect, naturally present in CRNs, attracted intensive attention to include multi-user scenarios in studies involving optimal transmission strategies [4], [3]. Common in most of the above mentioned works is that the power allocation strategies developed, ignored the interference from the PU-Tx to the SU-Rx. Moreover, the impact of multiple SU-Rxs is not considered in any of the mentioned studies.

In this study, we take into account the presence of a PU-Tx and extend the scenario like that in [2] and [9] to multiple SU-Rxs. On this basis, an adaptive transmit power strategy for the SU-Tx is investigated. Here, the SU-Tx may be a secondary base station (SBS). Using the derived power transmission strategy, we derive the expression for the outage probability (OP) for the SU-Rx, selected to receive, among all available SU-Rxs in the network. That is, the SU-Rx having the highest signal-to-interference-plus-noise ratio (SINR) is selected to receive. Moreover, an approximation of the ergodic capacity of

the SU-Rx is derived.

The remainder of this paper is organized as follows. Section II presents the system model and defines the SU and PU transmissions. In Section III, the adaptive transmit power strategy, outage probability, and ergodic capacity are derived. In Section IV, numerical results are presented. Finally, conclusions are provided in Section V.

Notation: Throughout this paper, the following notations are used. The operator $\mathbb{E}[\cdot]$ expresses the statistical expectation. The logarithm of e with base 2 is given by $\log_2 e$ where e is the base of the natural logarithm. The operator $\max_{i=1,\dots,N} \{\cdot\}$ denotes the maximum among N elements. The symbol $\binom{n}{m}$ denotes the binomial coefficient indexed by n and m . The factorial of i is denoted by $i!$ and $E_i(\cdot)$ denotes the exponential integral function as in [11].

2 System Model

2.1 Channel Model

We consider a scenario in which a pair of PU-Tx and PU-Rx coexists with a secondary network composed of a SBS communicating with N SU-Rxs as shown in Fig. 1. Here, the random variables (RVs) g_i and f_i , $i = 1, \dots, N$, denote the channel power gains of the SBS $\rightarrow$ SU-Rx $_i$ and PU-Tx $\rightarrow$ SU-Rx $_i$ link, respectively. The channel power gain between PU-Tx and PU-Rx is denoted by h and the channel power gain between SBS and PU-Rx is denoted by β . We assume that all channel power gains g_i , f_i , h , and β are independent and exponentially distributed RVs with mean Ω_g , Ω_f , Ω_p , and Ω_β , respectively. We further assume that CSI is available at all SUs. This may be achieved by classical channel training, estimation, and feedback mechanisms [3].

2.2 PU Transmission

We assume that the CSI is not available at the PU-Rx and that the PU-Tx transmits with a constant power P_p . In many cases, the PU transmission may have a non-zero outage probability margin, so that the interference from the SU-Tx can be tolerated at a certain range [2]. Let ϵ be the desired outage probability at the PU-Rx. Considering the effect of the SU transmission on the PU link, the primary outage constraint can be formulated as follows:

$$\Pr \left\{ \frac{P_p h}{P_s \beta + N_0} \leq \gamma_{p_{th}} \right\} \leq \epsilon \quad (1)$$

where $\gamma_{p_{th}}$ is the outage threshold at the PU-Rx, N_0 is the noise power, and P_s is the transmit power of the SBS.

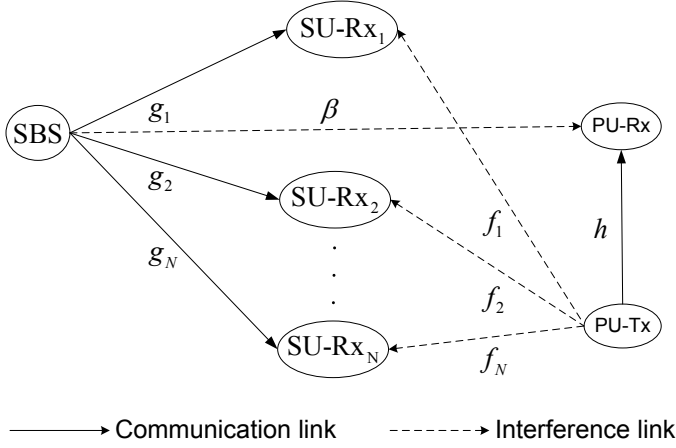


Figure 1: Spectrum sharing system where an SBS communicates with N SU-Rx_s in the presence of a primary communication composed of a PU-Tx and a PU-Rx.

2.3 SU Transmission

The SBS transmit power, P_s , should stay less or equal to a certain limit P_{peak} . In this regards, the SBS should be able to adapt its transmit power to ensure that there is no harmful interference to the PU, at the same time satisfying the primary outage constraint defined in (1). The SU peak transmit power constraint can be formulated as follows:

$$P_s \leq P_{peak} \quad (2)$$

In addition to the PU protection against harmful interference from the SU-Tx, the SU link need to be optimized such that the SU-Rx capacity is maximized and the outage probability is kept to a minimum.

3 Performance Analysis

In this section, we derive the probability density function (PDF) and cumulative distribution function (CDF) of the SINR at a SU-Rx_i. First, the outage

probability at the PU-Rx, denoted by $P_{out,p}$, can be found with the help of (1) as

$$\begin{aligned}
 P_{out,p} &= \Pr \left(\frac{P_p h}{P_s \beta + N_0} \leq \gamma_{p_{th}} \right) \\
 &= \int_0^\infty \Pr [P_p h \leq (P_s x + N_0) \gamma_{p_{th}}] f_\beta(x) dx \\
 &= \int_0^\infty \frac{1}{\Omega_\beta} \left[1 - \exp \left(-\frac{(P_s x + N_0) \gamma_{p_{th}}}{P_p \Omega_p} \right) \right] \\
 &\quad \times \exp \left(-\frac{x}{\Omega_\beta} \right) dx \\
 &= 1 - \frac{P_p \Omega_p}{\gamma_{p_{th}} P_s \Omega_\beta + P_p \Omega_p} \exp \left(-\frac{N_0 \gamma_{p_{th}}}{P_p \Omega_p} \right)
 \end{aligned} \tag{3}$$

where $f_\beta(x) = \frac{1}{\Omega_\beta} e^{-\frac{x}{\Omega_\beta}}$ is the PDF of the channel power gain from SU-Tx to PU-Rx. Substituting (3) into (1) and equating the result to ε , we obtain P_s as

$$P_s = \frac{P_p \Omega_p}{\gamma_{p_{th}} \Omega_\beta (1 - \varepsilon)} \left[\exp \left(-\frac{N_0 \gamma_{p_{th}}}{P_p \Omega_p} \right) - (1 - \varepsilon) \right] \tag{4}$$

Hence, the maximum transmit power allowable for the SBS can be written as

$$P_{s,max} = \min \left(\frac{P_p \Omega_p}{\gamma_{p_{th}} \Omega_\beta (1 - \varepsilon)} \left[\exp \left(-\frac{N_0 \gamma_{p_{th}}}{P_p \Omega_p} \right) - (1 - \varepsilon) \right], P_{peak} \right) \tag{5}$$

Furthermore, the SINR at the SU-Rx_i can be formulated as follows:

$$\gamma_i = \frac{P_s g_i}{P_p f_i + N_0} \tag{6}$$

The CDF of the instantaneous SINR at an SU-Rx_i can, therefore, be

obtained as follows:

$$\begin{aligned}
 F_{\gamma_i}(\gamma) &= \Pr \left(\frac{P_s g_i}{P_p f_i + N_0} < \gamma \right) \\
 &= \int_0^\infty \Pr [P_s g_i < (P_p x + N_0)\gamma] f_{f_i}(x) dx \\
 &= \int_0^\infty \frac{1}{\Omega_f} \left\{ 1 - \exp \left[-\frac{(P_p x + N_0)\gamma}{P_s \Omega_g} \right] \right\} \\
 &\quad \times \exp \left(-\frac{x}{\Omega_f} \right) dx \\
 &= 1 - \frac{P_s \Omega_g}{\gamma P_p \Omega_f + P_s \Omega_g} \exp \left(-\frac{N_0 \gamma}{P_s \Omega_g} \right)
 \end{aligned} \tag{7}$$

where $f_{f_i} = \frac{1}{\Omega_f} e^{-\frac{x}{\Omega_f}}$ is the PDF of the channel power gain from PU-Tx to SU-Rx_i.

The PDF of the instantaneous SINR at an SU-Rx_i can readily be obtained by differentiating (7) with respect to γ , giving

$$\begin{aligned}
 f_{\gamma_i}(\gamma) &= P_s \Omega_g \exp \left(-\frac{N_0 \gamma}{P_s \Omega_g} \right) \\
 &\quad \times \left[\frac{P_s \Omega_f}{(P_p \gamma \Omega_f + P_s \Omega_g)^2} + \frac{N_0}{P_s \Omega_g (P_p \gamma \Omega_f + P_s \Omega_g)} \right]
 \end{aligned} \tag{8}$$

In the sequel, with (7) and (8), various performance metrics of the SU link are analyzed.

3.1 Outage capacity

The outage capacity is defined as the maximum constant rate that can be attained with a prescribed outage probability. Given a transmission rate r_0 in bit/sec, the outage probability at an SU-Rx_i can be expressed as

$$P_{out,s} = \Pr [\log_2(1 + \gamma_i) < r_0] \tag{9}$$

The SINR corresponding to the selected SU-Rx is obtained as

$$\gamma_{\max} = \max_{i=1, \dots, N} \{\gamma_i\} \tag{10}$$

From the order statistics theory, the PDF and CDF of (10) are, respectively, given by

$$f_{\gamma_{\max}}(\gamma) = N f_{\gamma_i}(\gamma) [F_{\gamma_i}(\gamma)]^{N-1} \quad (11)$$

$$F_{\gamma_{\max}}(\gamma) = [F_{\gamma_i}(\gamma)]^N \quad (12)$$

An analytical expression of (11) can be obtained after some algebraic manipulations as follows:

$$\begin{aligned} f_{\gamma_{\max}}(\gamma) = Nba \sum_{i=0}^{N-1} \binom{N-1}{i} \frac{(-1)^i b^i \exp \left[-\frac{\gamma(i+1)N_0}{b} \right]}{(\gamma a + b)^{i+2}} \\ + N_0 N \sum_{i=0}^{N-1} \binom{N-1}{i} \frac{(-1)^i b^i \exp \left[-\frac{\gamma(i+1)N_0}{b} \right]}{(\gamma a + b)^{i+1}} \end{aligned} \quad (13)$$

where $a = P_p \Omega_f$ and $b = P_s \Omega_g$. Similarly, (12) can be readily obtained as

$$F_{\gamma_{\max}}(\gamma) = \left[1 - \frac{b \exp \left(-\frac{\gamma N_0}{b} \right)}{\gamma a + b} \right]^N \quad (14)$$

Clearly, the outage probability of the SU-Rx with maximum SINR can readily be obtained by replacing the threshold $\gamma_{s_{th}} = 2^{r_0} - 1$, obtained from (9) into (14). Therefore, the outage probability is expressed as

$$P_{out,s} = \left[1 - \frac{\exp \left(-\frac{N_0}{b} \gamma_{s_{th}} \right) b}{a \gamma_{s_{th}} + b} \right]^N \quad (15)$$

3.2 Ergodic Capacity

The ergodic capacity is defined as the maximum achievable rate averaged with a long-term delay constraint [12]. It is a suitable metric for delay insensitive services. Here, for the considered system, the ergodic capacity can be expressed as

$$C_{erg} = \int_0^{\infty} \log_2(1 + \gamma) f_{\gamma_{max}}(\gamma) d\gamma \quad (16)$$

As the closed-form expression in (16) is not available, we proceed by approximating the logarithm function with Taylor's series as

$$\begin{aligned} \log_2(1+x) &= \log_2(e) \ln(1+\bar{x}) + \log_2(e) \frac{(x-\bar{x})}{1+\bar{x}} \\ &\quad - \log_2(e) \frac{(x-\bar{x})^2}{2(1+\bar{x})^2} + O[(x-\bar{x})^3] \end{aligned} \quad (17)$$

where $O[x^n]$ is defined as a polynomial function with power equal or greater than n and $\bar{x} = \int_x x f_X(x) dx$. The first moment and second moment of the SINR of the selected SU-Rx are given, respectively, by $\bar{\gamma} = \int_0^\infty \gamma f_{\gamma_{max}}(\gamma) d\gamma$ and $\mathbb{E}[\gamma^2] = \int_0^\infty \gamma^2 f_{\gamma_{max}}(\gamma) d\gamma$. Therefore, the ergodic capacity in (16) can be approximated as [13]

$$C_{erg} \approx \log_2(e) \ln(1+\bar{\gamma}) - \frac{\log_2(e) (\mathbb{E}[\gamma^2] - \bar{\gamma}^2)}{2(1+\bar{\gamma})^2} \quad (18)$$

Accordingly, given the expressions of the first moment and the second moment of (10), along with (13) applied into (18), we can easily obtain an approximation of the ergodic capacity at the SU-Rx.

Theorem 1 *Given constants $K_1 = Nba$ and $K_2 = N_0N$, the first moment and the second moment of the maximum SINR for the best SU-Rx can be written as*

$$\begin{aligned} \bar{\gamma} &= K_1 \sum_{i=0}^{N-1} \binom{N-1}{i} (-1)^i b^i \frac{\exp\left[\frac{(i+1)N_0}{a}\right]}{a^{i+2}} (L_1 - \frac{b}{a} L_3) \\ &\quad + K_2 \sum_{i=0}^{N-1} \binom{N-1}{i} (-1)^i b^i \frac{\exp\left[\frac{(i+1)N_0}{a}\right]}{a^{i+1}} (L_2 - \frac{b}{a} L_1) \end{aligned} \quad (19)$$

$$\begin{aligned} \mathbb{E}[\gamma^2] &= K_1 \sum_{i=0}^{N-1} \binom{N-1}{i} (-1)^i b^i \frac{\exp\left[\frac{(i+1)N_0}{a}\right]}{a^{i+2}} \\ &\quad \times [L_2 - \frac{2b}{a} L_1 + (b/a)^2 L_3] \\ &\quad + K_2 \sum_{i=0}^{N-1} \binom{N-1}{i} (-1)^i b^i \frac{\exp\left[\frac{(i+1)N_0}{a}\right]}{a^{i+1}} \\ &\quad \times [L_4 - \frac{2b}{a} L_2 + (b/a)^2 L_1] \end{aligned} \quad (20)$$

where L_1 , L_2 , L_3 , and L_4 are, respectively, given by

$$L_1 = \exp \left[-\frac{(i+1)N_0}{a} \right] \sum_{k=1}^i \frac{(k-1)! \left[-\frac{(i+1)N_0}{b} \right]^{i-k}}{i! \left(\frac{b}{a} \right)^k} - \frac{\left[-\frac{(i+1)N_0}{b} \right]^i}{i!} E_i \left(-\frac{(i+1)N_0}{a} \right) \quad (21)$$

$$L_2 = \exp \left[-\frac{(i+1)N_0}{a} \right] \sum_{k=1}^{i-1} \frac{(k-1)! \left[-\frac{(i+1)N_0}{b} \right]^{i-k-1}}{(i-1)! \left(\frac{b}{a} \right)^k} - \frac{\left[-\frac{(i+1)N_0}{b} \right]^{i-1}}{(i-1)!} E_i \left(-\frac{(i+1)N_0}{a} \right) \quad (22)$$

$$L_3 = \exp \left[-\frac{(i+1)N_0}{a} \right] \sum_{k=1}^{i+1} \frac{(k-1)! \left[-\frac{(i+1)N_0}{b} \right]^{i-k+1}}{(i+1)! \left(\frac{b}{a} \right)^k} - \frac{\left[-\frac{(i+1)N_0}{b} \right]^{i+1}}{(i+1)!} E_i \left(-\frac{(i+1)N_0}{a} \right) \quad (23)$$

$$L_4 = \exp \left[-\frac{(i+1)N_0}{a} \right] \sum_{k=1}^{i-2} \frac{(k-1)! \left[-\frac{(i+1)N_0}{b} \right]^{i-k-2}}{(i-2)! \left(\frac{b}{a} \right)^k} - \frac{\left[-\frac{(i+1)N_0}{b} \right]^{i-2}}{(i-2)!} E_i \left(-\frac{(i+1)N_0}{a} \right) \quad (24)$$

Proof 1 See Appendix A.

4 Numerical Results

In this section, numerical and simulation results are presented to evaluate the performance of the proposed model. All the channels are assumed to undergo Rayleigh fading and thus, their channel power gains are exponentially distributed RVs. The channel power gains g_i and h are assumed to be with unit mean in all cases but β and f_i are assumed to be with mean 0.5 for certain cases. We set the upper limit for the transmit power of the SBS, $P_{peak} = 10$ dB. Also, the outage threshold at the SU-Rx is set such that $\gamma_{s_{th}} = 6.02$ dB, the target SINR at PU-Rx, $\gamma_{p_{th}} = -20$ dB, and the desired outage probability

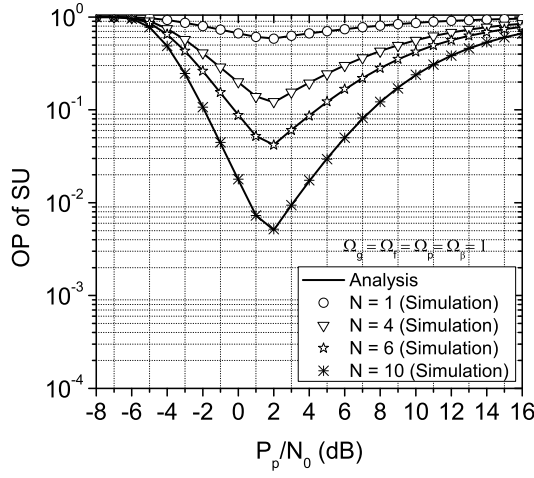


Figure 2: Outage probability in best channel conditions, $P_{peak} = 10$ dB, outage threshold at the SU-Rx, $\gamma_{sth} = 6.02$ dB.

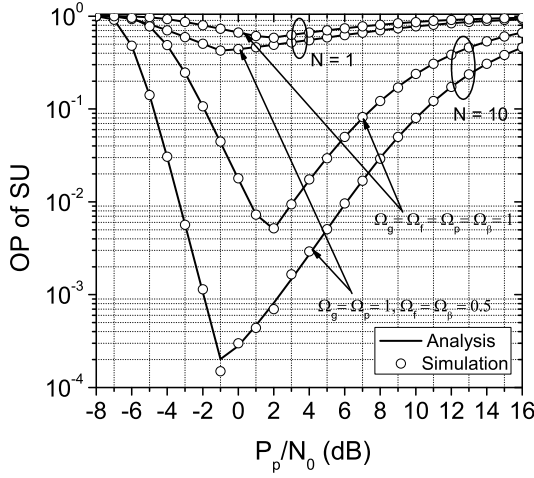


Figure 3: Outage probability in best channel conditions, $P_{peak} = 10$ dB, outage threshold at the SU-Rx, $\gamma_{sth} = 6.02$ dB, with channel characteristics of [2].

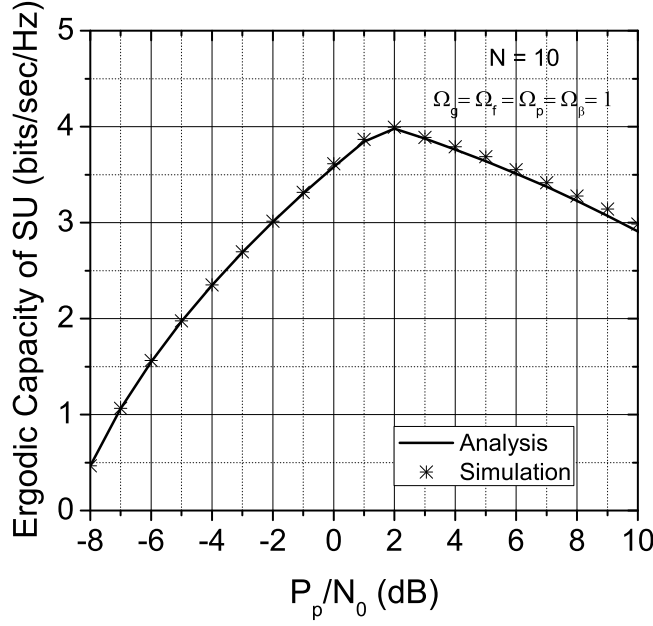


Figure 4: Ergodic Capacity in best channel conditions, $P_{peak} = 10$ dB, outage threshold at the SU-Rx, $\gamma_{sth} = 6.02$ dB.

at PU, $\epsilon = 0.07$. Fig. 2 depicts the multi-user effect of the model on the outage probability for the SU link. The outage probability decreases as the SU-Rx number grows. However, at a certain level of the PU-Tx transmit power, $P_p/N_0 = 2$ dB in our case, the performance starts degrading. This can be explained by carefully observing the function in the power allocation strategy of (5). When the PU-Tx increases its transmit power beyond the optimum transmit power, the PU-Rx becomes a severe source of fading to the SU transmission.

In Fig. 3, we capture two important contributions of this study. First, the multi-user effect on the outage probability of the SU is depicted where it is clearly shown that when $N = 10$, the system offers the best performance than the case $N = 1$. Again, we can easily observe high performance when we adopt the channel characteristics like those in [2]. That is, $g_i = h = 1$ and $\beta = f_i = 0.5$. However, there is a cost of constraining the PU-Tx to reducing its transmit power.

In Fig. 4, the results for an approximation of the ergodic capacity are shown with the number of SU-Rxs, $N = 10$. Clearly, the ergodic capacity increases quickly with the increase of P_p/N_0 , and suddenly, it starts dropping from $P_p/N_0 = 2$ dB. The degradation observed for both the outage probability and the ergodic capacity is most sensitive to P_{peak} , the limit of SBS transmit power.

5 Conclusions

In this paper, we have studied the performance of a CRN where N SU-Rxs communicate with a SBS in the presence of a primary communication. The impact of the PU transmission on the SU link has been evaluated in terms of outage probability and ergodic capacity. In addition, the effect of multiple SU-Rxs to the system performance is investigated for different settings of Rayleigh fading channels. The derived power allocation strategy reveals the severe source of fading that PU-Tx becomes against the SU link for higher values of P_p/N_0 . This happens for P_p/N_0 values beyond the optimal PU transmit power. The derived analytical expressions for the outage probability and the approximated ergodic capacity have been numerically evaluated and simulated.

A Proof of Theorem 1

A.1 First moment

Using (13), the first moment of (10) can be written as

$$\begin{aligned} \bar{\gamma} = & K_1 \sum_{i=0}^{N-1} \binom{N-1}{i} (-1)^i b^i I_1 \\ & + K_2 \sum_{i=0}^{N-1} \binom{N-1}{i} (-1)^i b^i I_2 \end{aligned} \quad (25)$$

where I_1 and I_2 are, respectively, given as

$$I_1 = \int_0^{\infty} \gamma \frac{\exp\left[-\frac{(i+1)N_0}{b}\gamma\right]}{(\gamma a + b)^{i+2}} d\gamma \quad (26)$$

$$I_2 = \int_0^{\infty} \gamma \frac{\exp\left[-\frac{(i+1)N_0}{b}\gamma\right]}{(\gamma a + b)^{i+1}} d\gamma \quad (27)$$

Applying a change of variable, with $\gamma + \frac{b}{a} = y$, to integrals I_1 and I_2 , then, decomposing the two integrals results in the following:

$$I_1 = \frac{e^{\frac{(i+1)N_0}{a}}}{a^{i+2}} (I_{11} - \frac{b}{a} I_{12}) \quad (28)$$

$$I_2 = \frac{e^{\frac{(i+1)N_0}{a}}}{a^{i+1}} (I_{21} - \frac{b}{a} I_{22}) \quad (29)$$

where I_{11} , I_{21} , I_{12} and I_{22} are given as

$$I_{11} = I_{22} = \int_{b/a}^{\infty} \frac{e^{-\frac{(i+1)N_0}{b}y}}{y^{i+1}} dy \quad (30)$$

$$I_{21} = \int_{b/a}^{\infty} \frac{e^{-\frac{(i+1)N_0}{b}y}}{y^i} dy \quad (31)$$

$$I_{12} = \int_{b/a}^{\infty} \frac{e^{-\frac{(i+1)N_0}{b}y}}{y^{i+2}} dy \quad (32)$$

Grouping all the terms together and replacing them into (25), we get

$$\begin{aligned} \bar{\gamma} = & K_1 \sum_{i=0}^{N-1} \binom{N-1}{i} (-1)^i b^i \frac{\exp\left[\frac{(i+1)N_0}{a}\right]}{a^{i+2}} (I_{11} - \frac{b}{a} I_{12}) \\ & + K_2 \sum_{i=0}^{N-1} \binom{N-1}{i} (-1)^i b^i \frac{\exp\left[\frac{(i+1)N_0}{a}\right]}{a^{i+1}} (I_{21} - \frac{b}{a} I_{22}) \end{aligned} \quad (33)$$

A.2 Second moment

Using (13), the second moment of (10) can be obtained as

$$\begin{aligned} \mathbb{E} [\gamma^2] = & K_1 \sum_{i=0}^{N-1} \binom{N-1}{i} (-1)^i b^i I_3 \\ & + K_2 \sum_{i=0}^{N-1} \binom{N-1}{i} (-1)^i b^i I_4 \end{aligned} \quad (34)$$

where I_3 and I_4 are given, respectively, by

$$I_3 = \int_0^\infty \gamma^2 \frac{\exp \left[-\frac{(i+1)N_0}{b} \gamma \right]}{(\gamma a + b)^{i+2}} d\gamma \quad (35)$$

$$I_4 = \int_0^\infty \gamma^2 \frac{\exp \left[-\frac{(i+1)N_0}{b} \gamma \right]}{(\gamma a + b)^{i+1}} d\gamma \quad (36)$$

Applying a change of variables, with $\gamma + \frac{b}{a} = y$, then, decomposing the integrals I_3 and I_4 , we get

$$I_3 = \frac{e^{\frac{(i+1)N_0}{a}}}{a^{i+2}} [I_{31} - \frac{2b}{a} I_{32} + (\frac{b}{a})^2 I_{33}] \quad (37)$$

$$I_4 = \frac{e^{\frac{(i+1)N_0}{a}}}{a^{i+1}} [I_{41} - \frac{2b}{a} I_{42} + (\frac{b}{a})^2 I_{43}] \quad (38)$$

where I_{33} and I_{41} are, respectively, expressed as

$$I_{33} = \int_{b/a}^\infty \frac{e^{-\frac{(i+1)N_0}{b} y}}{y^{i+2}} dy \quad (39)$$

$$I_{41} = \int_{b/a}^\infty \frac{e^{-\frac{(i+1)N_0}{b} y}}{y^{i-1}} dy \quad (40)$$

Hence, the expression in (34) can be rewritten as

$$\begin{aligned} \mathbb{E} [\gamma^2] &= K_1 \sum_{i=0}^{N-1} \binom{N-1}{i} (-1)^i b^i \\ &\quad \times \frac{e^{\frac{(i+1)N_0}{a}}}{a^{i+2}} [I_{31} - \frac{2b}{a} I_{32} + (\frac{b}{a})^2 I_{33}] \\ &\quad + K_2 \sum_{i=0}^{N-1} \binom{N-1}{i} (-1)^i b^i \\ &\quad \times \frac{e^{\frac{(i+1)N_0}{a}}}{a^{i+1}} [I_{41} - \frac{2b}{a} I_{42} + (\frac{b}{a})^2 I_{43}] \end{aligned} \quad (41)$$

Noting that $I_{11} = I_{22} = I_{32} = I_{43}$, $I_{21} = I_{31} = I_{42}$ and $I_{33} = I_{12}$, for convenience, we denote them, respectively, by L_1 , L_2 and L_3 . Finally, we denote L_4 such that $L_4 = I_{41}$.

By using the relation between exponential and rational functions [14, eq. (3.353.1)], the integrals L_1, L_2, L_3 , and L_4 can be expressed as in (21)-(24). Substituting the expressions of L_1, L_2, L_3 and L_4 into (33) and (41), we get the first moment and the second moment of the SINR expressed in (10). This completes the proof.

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Part III

PART III

Outage Probability of a Cognitive Cooperative Relay Network with Multiple Primary Users Under Primary Outage Constraint

Part III is based on:

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Outage Probability of a Cognitive Cooperative Relay Network with Multiple Primary Users Under Primary Outage Constraint

Charles Kabiri, Hans-Jürgen Zepernick, and Hung Tran

Abstract

In this paper, the impact of multiple primary user transmitters and multiple primary user receivers (PU-Rxs) on the outage probability of a cognitive cooperative relay network (CCRN) with decode-and-forward relaying is studied. Specifically, power allocation policies for the secondary user transmitter and secondary relay are formulated, subject to the outage constraint at the PU-Rxs and the peak transmit power limit of the secondary transmitters. Given these power allocation policies, an expression for the outage probability of the CCRN is derived. Numerical examples are provided to illustrate the impact of system parameters on the outage probability of the CCRN.

1 Introduction

Combining cognitive radios (CRs) with cooperative relaying has gained attention as a promising solution to better utilize radio spectrum. This approach allows secondary networks to opportunistically access spectrum, share spectrum with primary networks, and provide performance gains due to the spatial diversity of cooperative relaying transmission. Specifically, in [1], taking into account the interference from the primary user transmitter (PU-Tx) to the secondary user receiver (SU-Rx) and the interference from the secondary user transmitter (SU-Tx) to the primary user receiver (PU-Rx), for an underlay

cognitive relay network, a lower bound on the outage probability has been derived. In [2], different power allocation strategies to minimize the outage probability and transmit power subject to different constraints have been provided for wireless networks in Rayleigh fading. Similarly, CRs mainly distinguish between interference constraints and power constraints. In particular, interference constraints refer to the peak interference level that a primary network can tolerate without sacrificing quality of service. In this context, interference constraints can be expressed in terms of peak interference power limit or average interference power limit at the PU-Rx. In order to address the limitations on transmit power in practical devices, transmit power constraints such as peak transmit power limit or average transmit power limit at the SU-Tx have to be considered in system design and performance analysis of CRs. Furthermore, outage probability constraints with respect to the primary network have recently attracted the attention of the research community. In [3], different types of interference constraints and power allocation strategies to maximize the secondary network capacity have been studied. In [4], using an interference power constraint to protect the primary network, but without accounting for a predefined outage constraint at the PU-Rx, the impact of multiple transceivers on the performance of the secondary network has been analyzed. Protecting the primary network from interfering signals of the secondary network using an outage constraint at the PU-Rx has revealed benefits over the use of the interference power constraint [5]. In particular, protecting the primary network using an interference power constraint requires full channel state information of the interference channels at the secondary transmitters for controlling their transmit powers accordingly. However, this information is not easy to obtain and may be costly in terms of feedback. To overcome the problems with the fast variation of the interferences channels, the outage probability constraint, which is based on statistical information such as distance and path loss, is used in this paper. In [6], the outage performance of a cognitive cooperative relay network (CCRN) under the outage constraint of the primary network and peak transmit power limit of the SU-Tx has been analyzed with only a single PU-Rx interfering with the secondary network.

In view of the above discussion, in this paper, an analytical expression of the outage probability for the SU-Rx, assisted by a secondary relay (SR), is derived subject to a predefined outage probability at multiple PU-Rxs. The peak transmit power limit of the secondary transmitters is taken into consideration as an additional constraint in the formulation of a suitable power allocation policy for the secondary network.

The remainder of this paper is organized as follows. Section 2 introduces the system model of the considered CCRN in terms of topology, communi-

cation links, interference links, and transmission channels. In Section 3, an analytical expression for the outage probability of the secondary network is derived along with power allocation policies for the SU-Tx and SR. Numerical results are provided in Section 4 in order to illustrate the impact of system parameters such as the number of PU-Txs, peak transmit power of the SU-Tx and SR, and PU-Tx transmit power on the outage performance of the secondary network. Finally, Section 5 concludes the paper.

2 System model

Let us consider a CCRN as shown in Fig. 1, where the decode-and-forward (DF) protocol and half-duplex mode is used at the SR. We assume that there is no direct link from the SU-Tx to the SU-Rx due to severe shadowing. In the first transmission phase, the SU-Tx transmits its signal to the SR with transmit power P_s . In the second transmission phase, the SR decodes and forwards the received message to the SU-Rx with transmit power P_r . The transmit power of the SU-Tx and SR must comply with a predefined outage constraint, say $\varepsilon_k = \varepsilon$, at any PU-Rx $_k$, $k = 1, 2, \dots, N$. On the other hand, the received signals at the SR and SU-Rx are affected by the interference caused by the PU-Tx $_k$, $k = 1, 2, \dots, N$, transmit power P_p . It is assumed that the interference power from the PU-Txs to the secondary network is dominant compared to the noise, hence, the noise can be neglected as in [7]. Moreover, for the primary network, it is assumed that each PU-Tx $_k$ transmits to its correspondent PU-Rx $_k$. In this paper, the PU outage constraint is utilized instead of the interference power constraint [4], as the latter requires full channel state information of the interference links.

The channels between PU-Tx $_k \rightarrow$ PU-Rx $_k$, SU-Tx $\rightarrow$ SR, SR $\rightarrow$ SU-Rx, SU-Tx $\rightarrow$ PU-Rx $_k$, SR $\rightarrow$ PU-Rx $_k$, PU-Tx $_k \rightarrow$ SR and PU-Tx $_k \rightarrow$ SU-Rx, respectively, are assumed to be subject to Rayleigh fading with channel mean power Ω_a and related channel power gain $a \in \{h_k, g_1, g_2, \alpha_{1k}, \alpha_{2k}, \beta_{1k}, \beta_{2k}\}$. Here, the symbol a represents an exponentially distributed random variable with probability density function (PDF) given as

$$f_a(x) = \frac{1}{\Omega_a} e^{-x/\Omega_a} u(x) \quad (1)$$

where $u(x)$ is the Heaviside step function, i.e. $u(x) = 1$ for $x \geq 0$, and $u(x) = 0$ for $x < 0$.

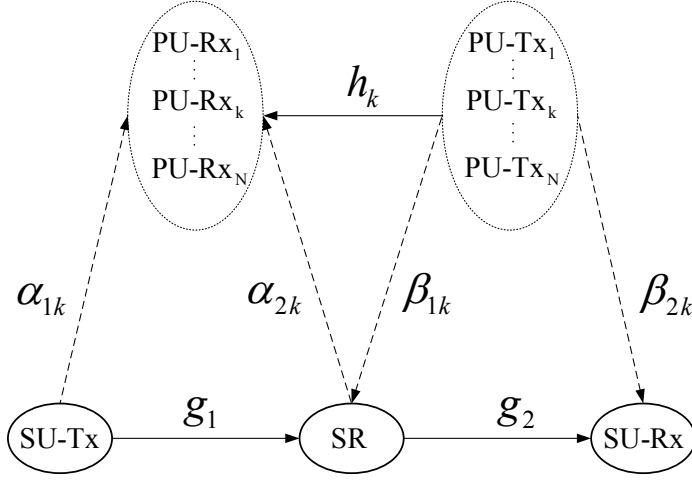


Figure 1: System model of a CCRN with co-linear topology (Solid lines: Communication links; Dashed lines: Interference links).

3 Outage probability

The instantaneous signal-to-interference ratios (SIRs) at the SR and the SU-Rx during the two transmission phases can, respectively, be expressed as

$$\gamma_1 = \frac{g_1 P_s}{\sum_{k=1}^N \beta_{1k} P_p} \quad (2)$$

$$\gamma_2 = \frac{g_2 P_r}{\sum_{k=1}^N \beta_{2k} P_p} \quad (3)$$

On the primary links, accounting for the interference from the secondary transmission, the outage constraints in the two transmission phases are given by

$$P_{out}^{p_{1k}} = \Pr \left\{ \frac{h_k P_p}{\alpha_{1k} P_s} < \gamma_{p_{th}} \right\} \leq \varepsilon \quad (4)$$

$$P_{out}^{p_{2k}} = \Pr \left\{ \frac{h_k P_p}{\alpha_{2k} P_r} < \gamma_{p_{th}} \right\} \leq \varepsilon \quad (5)$$

where $\gamma_{p_{th}}$ is the SIR threshold at the PU-Rx_k and ϵ is the outage constraint of the primary network. The instantaneous transmit powers P_s and P_r at the SU-Tx and SR, respectively, must be less than or equal to the peak transmit power P_{peak} , i.e.

$$P_s \leq P_{peak} \quad (6)$$

$$P_r \leq P_{peak} \quad (7)$$

This is due to the fact that practical transmitters apparently can operate only with finite power, a constraint that has not been considered in several works such as [7]. In addition, the SU-Tx and the SR have to adapt their transmit powers to ensure that there is no harmful interference caused to the primary network. As such, the transmit powers at the SU-Tx and SR can be readily obtained once expressions for the outage probabilities in the two transmission phases of the secondary network are available which shall therefore be derived in the sequel.

In the first transmission phase, the outage probability of the PU-Rx_k, with all primary communications being protected from harmful interference of the secondary network, can be expressed with the help of the order statistics theory as

$$\begin{aligned} P_{out}^{p_{1k}} &= \Pr \left\{ \min_{k=1,2,\dots,N} \left\{ \frac{P_p h_k}{\alpha_{1k} P_s} \right\} < \gamma_{p_{th}} \right\} \\ &= 1 - \prod_{k=1}^N \left(1 - \Pr \left\{ \frac{P_p h_k}{\alpha_{1k} P_s} < \gamma_{p_{th}} \right\} \right) \\ &= 1 - \prod_{k=1}^N \left(1 - \int_0^\infty \Pr \{ P_p h_k < x P_s \gamma_{p_{th}} \} f_{\alpha_{1k}}(x) dx \right) \end{aligned} \quad (8)$$

It is assumed that all channel power gains α_{1k} and h_k are identically distributed random variables, i.e., $\forall k: \Omega_{\alpha_{1k}} = \Omega_{\alpha_1}$ and $\Omega_{h_k} = \Omega_h$. Then, $\forall k: P_{out}^{p_{1k}} = P_{out}^{p_1}$, the outage probability related to the first transmission phase can be obtained from (8) as

$$P_{out}^{p_1} = 1 - \left(\frac{P_p \Omega_h}{\gamma_{p_{th}} P_s \Omega_{\alpha_1} + P_p \Omega_h} \right)^N \quad (9)$$

In the second transmission phase, the outage probability at PU-Rx_k is found in a similar way as (9) by substituting $\alpha_1 \rightarrow \forall k: \alpha_{2k} = \alpha_2$ and $P_s \rightarrow P_r$, thus, giving

$$P_{out}^{p_2} = 1 - \left(\frac{P_p \Omega_h}{\gamma_{p_{th}} P_r \Omega_{\alpha_2} + P_p \Omega_h} \right)^N \quad (10)$$

The transmit powers for the SU-Tx and SR, in the presence of N PU-Rxs, can then be deduced from (9) and (10), respectively, as

$$P_s = \frac{P_p \Omega_h}{\gamma_{p_{th}} \Omega_{\alpha_1}} \left(\frac{1}{\sqrt[N]{1-\varepsilon}} - 1 \right) \quad (11)$$

$$P_r = \frac{P_p \Omega_h}{\gamma_{p_{th}} \Omega_{\alpha_2}} \left(\frac{1}{\sqrt[N]{1-\varepsilon}} - 1 \right) \quad (12)$$

Given the peak transmit power P_{peak} of the SU-Tx, the transmit power policies for the SU-Tx and SR, respectively, can then be formulated as

$$P_s^a = \min \left\{ \frac{P_p \Omega_h}{\gamma_{p_{th}} \Omega_{\alpha_1}} \rho(N, \epsilon), P_{peak} \right\} \quad (13)$$

$$P_r^a = \min \left\{ \frac{P_p \Omega_h}{\gamma_{p_{th}} \Omega_{\alpha_2}} \rho(N, \epsilon), P_{peak} \right\} \quad (14)$$

where the number N of PU-Txs and the outage constraint ϵ of the corresponding N PU-Rxs are accounted for by the term

$$\rho(N, \epsilon) = \frac{1}{\sqrt[N]{1-\varepsilon}} - 1 \quad (15)$$

Using the order statistics theory, along with the expressions (2) and (3) of the SIRs γ_1 and γ_2 at SR and SU-Rx, respectively, the outage probability of the secondary network, i.e., the probability that the end-to-end SIR, $\min\{\gamma_1, \gamma_2\}$, falls below a certain threshold γ_{th} , is obtained as

$$P_{out} = 1 - [1 - F_{\gamma_1}(\gamma_{th})][1 - F_{\gamma_2}(\gamma_{th})] \quad (16)$$

where $F_{\gamma_i}(\gamma)$ is the cumulative distribution functions (CDF) of γ_i , $i = 1, 2$.

Let us first find the CDF of the SIR γ_1 in the first transmission phase at the SR as follows. The SIR γ_1 can be rewritten as $\gamma_1 = (P_s g_1)/Y$, where $Y = P_p S_N$ and $S_N = \sum_{k=1}^N \beta_{1k}$. Then, the CDF of γ_1 , conditioned on Y , is given by

$$F_{\gamma_1|Y}(\gamma) = \Pr(\gamma_1 < \gamma | Y) = \Pr\left(g_1 < \frac{Y\gamma}{P_s} | Y\right) = 1 - \exp\left(-\frac{Y\gamma}{P_s \Omega_{g_1}}\right) \quad (17)$$

The unconditional CDF of γ_1 can be derived by taking the expectation of $F_{\gamma_1|Y}(\gamma)$ with respect to Y , i.e. $F_{\gamma_1}(\gamma) = \mathbb{E}_Y \{F_{\gamma_1|Y}(\gamma)\}$. Given independent and identically distributed (i.i.d.) random variables, β_{1k} , $k = 1, 2, \dots, N$, the PDF of the random variable Y for a real-valued argument y is given by

$$f_Y(y) = \frac{\lambda_1^N (y/P_p)^{N-1}}{P_p (N-1)!} \exp\left(-\frac{\lambda_1}{P_p} y\right) \quad (18)$$

where $\forall k: \lambda_{1k} = \lambda_1$ assuming that all channel mean powers are equal, i.e., $\forall k: \Omega_{\beta_{1k}} = \Omega_{\beta_1} = 1/\lambda_1$. Thus, $F_{\gamma_1}(\gamma)$ can be obtained as

$$\begin{aligned} F_{\gamma_1}(\gamma) &= \int_0^\infty \left[1 - \exp\left(-\frac{Y\gamma}{P_s\Omega_{g_1}}\right) \right] f_Y(y) dy \\ &= 1 - \left(1 + \gamma \frac{P_p\Omega_{\beta_1}}{P_s\Omega_{g_1}} \right)^{-N} \end{aligned} \quad (19)$$

Similarly, the CDF of the SIR γ_2 for the second transmission phase can be obtained by substituting $\Omega_{\beta_{1k}} \rightarrow \Omega_{\beta_2}$, $\Omega_{g_1} \rightarrow \Omega_{g_2}$, and $P_s \rightarrow P_r$, thus, giving

$$F_{\gamma_2}(\gamma) = 1 - \left(1 + \gamma \frac{P_p\Omega_{\beta_2}}{P_r\Omega_{g_2}} \right)^{-N} \quad (20)$$

From (16), (19), (20), and taking into account the peak transmitter power through the transmitter power policies P_s^a and P_r^a given in (13) and (14), respectively, the outage probability of the secondary network with respect to a given SIR threshold γ_{th} is easily obtained as

$$P_{out} = 1 - \left[\left(1 + \gamma_{th} \frac{P_p\Omega_{\beta_1}}{P_s^a\Omega_{g_1}} \right) \left(1 + \gamma_{th} \frac{P_p\Omega_{\beta_2}}{P_r^a\Omega_{g_2}} \right) \right]^{-N} \quad (21)$$

4 Numerical results

To gain insights into the performance of the CCRN, the outage probability of the secondary network is computed using the derived expression and is verified by Monte Carlo simulations. The channel mean powers are set according to the exponential-decay path loss model. Let us denote the distance between transmitting and receiving nodes as $d_{u,v}$ with $u \in \{\text{SU-Tx, SR, PU-Tx}_k\}$ and $v \in \{\text{PU-Rx}_k, \text{SR, SU-Rx}\}$. Thus, the corresponding channel mean power is $\Omega_a \sim d_{u,v}^{-\mu}$, where μ is the path loss exponent and $a \in \{h_k, g_1, g_2, \alpha_{1k}, \alpha_{2k}, \beta_{1k}, \beta_{2k}\}$. For the secondary network, we assume a co-linear topology where SU-Tx, SR, and SU-Rx are placed along the x -axis. Further, the SR is placed half-way between the SU-Tx and the SU-Rx. Assuming a non-line-of-sight propagation model, we set $\mu = 4$ which represents a shadowed urban cellular radio scenario. Other parameters are set to $\gamma_{p_{th}} = -15$ dB, $\gamma_{th} = -3$ dB, $\epsilon = 0.01$, and the number of PU-Txs and PU-Rxs is varied as $N = 1, 2, 3$. Assuming normalized distances, the channel mean powers are selected as $\Omega_{g_1} = \Omega_{g_2} = 16$, $\Omega_{\beta_1} = \Omega_{\beta_2} = 0.5$, $\Omega_{\alpha_1} = \Omega_{\alpha_2} = 0.5$, and $\Omega_h = 16$.

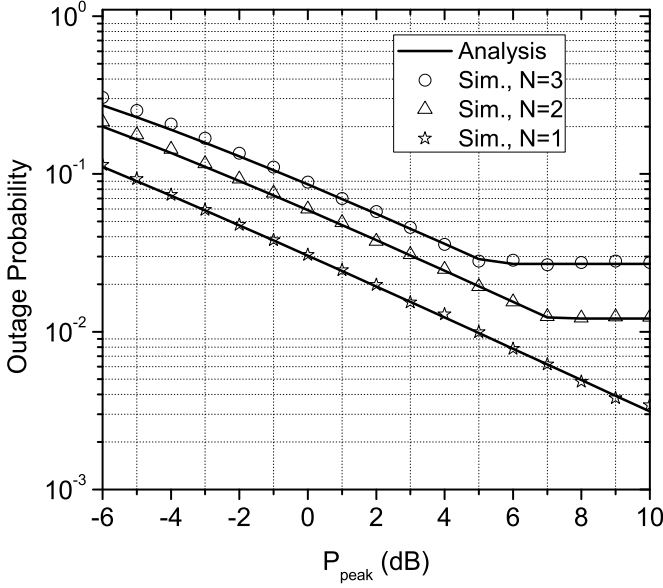


Figure 2: Outage probability of the SU-Rx versus peak transmit power P_{peak} of the secondary transmitters for different numbers N of PU-Txs and PU-Tx transmit power set to $P_p = 0$ dB.

Fig. 2 shows the outage probability at the SU-Rx versus the peak transmit power P_{peak} of the secondary transmitters. Here, the PU-Tx transmit power is set to $P_p = 0$ dB. As can be observed from this figure, the outage performance degrades as the number N of PU-Txs increases. However, this degradation does not increase in the same proportion as the increase of N . For example, the degradation in outage probability when the number of primary transceivers increases from $N = 1$ to 2 is larger than for the increase from $N = 2$ to 3. It is also interesting to note that at about $P_{peak} = 5$ dB, the outage probability for $N = 3$ reaches a floor while it stays at a floor from about $P_{peak} = 7$ dB for $N = 2$. This floor effect can be explained by the transmit power policies P_s^a and P_r^a as follows. As long as the terms $\frac{P_p \Omega_h}{\gamma_{pth} \Omega_{\alpha_1}} \rho(N, \epsilon)$ and $\frac{P_p \Omega_h}{\gamma_{pth} \Omega_{\alpha_2}} \rho(N, \epsilon)$ in the transmit power policies of the SU-Tx and SR, respectively, stay above the peak transmit power, the $\min\{\cdot, \cdot\}$ operator of the respective transmit power policy releases the term P_{peak} . As such, the outage

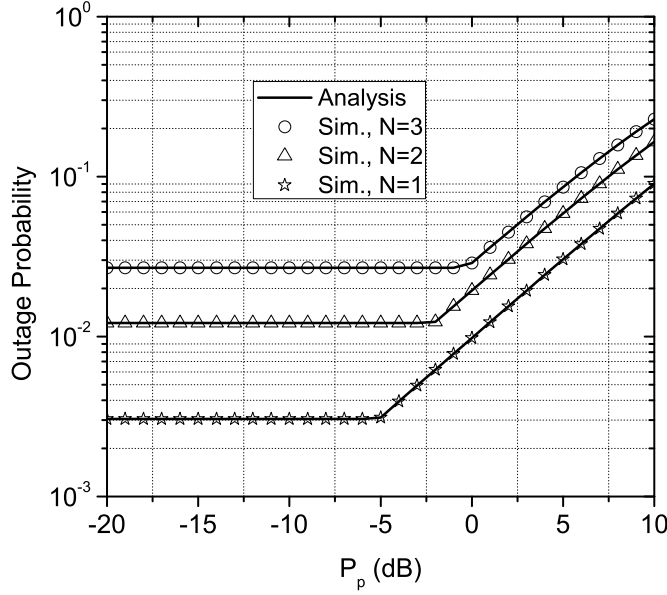


Figure 3: Outage probability of the SU-Rx versus PU-Tx transmit power P_p for different numbers N of PU-Txs and peak transmit power set to $P_{peak} = 5$ dB.

probability given by (21) decreases with the increase of P_{peak} . Once the peak transmit power is sufficiently large, the transmit power policies release the terms $\frac{P_p \Omega_h}{\gamma_{p_{th}} \Omega_{\alpha_1}} \rho(N, \epsilon)$ and $\frac{P_p \Omega_h}{\gamma_{p_{th}} \Omega_{\alpha_2}} \rho(N, \epsilon)$. Substituting these terms into (21) and after some algebraic manipulations, an expression for the outage probability is obtained that does not depend on the transmitter power P_p of the PU-Txs and the outage probability stays at a constant value when further increasing P_{peak} .

Fig. 3 plots the SU outage probability versus the PU transmit power P_p . The peak transmit power of SU-Tx and SR is set to $P_{peak} = 5$ dB. Clearly, depending on the number N of PU-Txs, the outage probability stays constant until a certain value of the PU-Tx transmit power P_p is reached. In particular, an increase of the number N of PU-Rxs significantly degrades the outage performance of the CCRN. This progression for a given N can again be explained by the $\min\{\cdot, \cdot\}$ operation in the transmit power policies P_s^a and

P_r^a and their impact on the calculation of the outage probability.

Fig. 4 illustrates the relationship between the transmit power P_s^a of the SU-Tx and the PU-Tx transmit power P_p (see also (13)). Note that the same progression is obtained for the relationship between the transmit power P_r^a of the SR and the PU transmit power P_p for the case of the selected channel mean powers. It can be seen from the figure that the SU-Tx transmit power P_s^a follows the increase of the PU-Tx transmit power P_p as long as the term $\frac{P_p \Omega_h}{\gamma_{pth} \Omega_{\alpha_1}} \rho(N, \epsilon)$ in (13) stays below the respective peak transmit power P_{peak} of the SU-Tx. Once the PU-Tx transmit power P_p reaches a certain value, the $\min\{\cdot, \cdot\}$ operator releases P_{peak} and hence the SU-Tx transmit power is kept constant at this limit. It can also be observed that the SU-Tx transmit power P_s^a is higher for the case of a single PU-Tx/PU-Rx ($N = 1$) until it reaches the peak transmit power compared to the case of two PU-Txs/PU-Rxs ($N = 2$). In the latter case, the SU-Tx has to control its transmit power such that the outage constraint of the two PU-Rxs is not violated.

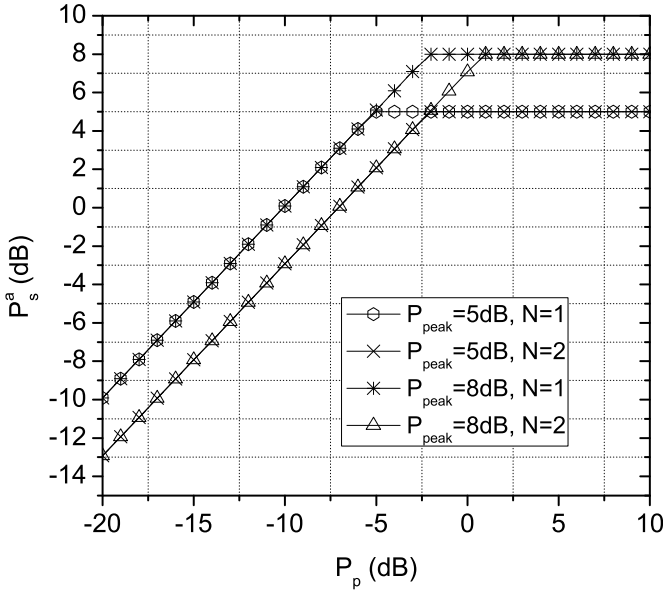


Figure 4: SU-Tx transmit power P_s^a versus PU-Tx transmit power P_p for different numbers N of PU-Txs and different peak transmit powers P_{peak} .

5 Conclusion

This paper has investigated the effect of multiple PU-Txs and PU-Rxs on the outage performance of a CCRN. Taking into account the outage constraint at the PU-Rxs and the peak transmit power P_{peak} of the secondary transmitters, power allocation policies for the secondary transmitters, and an analytical expression for the outage probability of the CCRN have been derived. Numerical results have been provided to illustrate the impact of system parameters such as the number N of PU-Txs, peak transmit power P_{peak} of the SU-Tx and SR, and PU-Tx transmit power P_p on the outage probability of the CCRN for some example scenarios.

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Part IV-A

PART IV-A

On the Performance of Cognitive Radio Networks with DF Relay Assistance Under Primary Outage Constraint Using SC and MRC

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On the Performance of Cognitive Radio Networks with DF Relay Assistance Under Primary Outage Constraint Using SC and MRC

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Abstract

In this paper, we analyze the performance of a cognitive radio network (CRN) that is assisted by a single relay. In particular, the secondary user (SU) transmitter (SU-Tx) and the secondary relay (SR) utilize the licensed frequency band of the primary user (PU). To protect the PU from harmful interference, the SU-Tx and SR must regulate their transmit power to satisfy the outage probability constraint of the PU. System performance in terms of outage probability is analyzed for selection combining (SC) and maximal ratio combining (MRC). Specifically, a power allocation policy and analytical expressions for the outage probability with SC and MRC are derived. Our results show that the upper bound of the outage probability corresponding to MRC is equal to the exact expression for the outage probability for SC.

1 Introduction

Cognitive radio networks (CRNs) are considered as a promising solution to alleviate the problem of spectrum scarcity and inefficient spectrum utilization. In particular, opportunistic spectrum access and spectrum sharing are the two main paradigms in CRNs. The first paradigm allows a secondary user (SU) to opportunistically access the frequency band allocated to a primary user (PU) when the PU is inactive. The second paradigm allows the SU to transmit

simultaneously with the PU over the same frequency band as long as the PU performance does not fall below a certain level. Moreover, in spectrum sharing access, an interference power constraint to the SU transmitter (SU-Tx) is imposed to protect the PU from harmful interference. Due to such a constraint and the fading nature of wireless channels, the quality of service of the SU may be degraded considerably.

On the other hand, it is well known that the spatial diversity of cooperative relaying transmission can offer large performance gains. Therefore, the combination of CRNs with cooperative relaying has attracted much interest [1–4]. In [5], for a typical spectrum sharing scenario, the author studied the channel capacities and the corresponding power control policies where the SU applies different interference power constraints. A lower bound on the outage performance for an underlay cognitive relay network considering the interference from the PU transmitter (PU-Tx) to the SU receiver (SU-Rx) and the interference from the SU-Tx to the PU receiver (PU-Rx) is derived in [6]. Other types of interference constraints and power allocation strategies to maximize the secondary network capacity have been studied in [7–9]. In [10], different power allocation strategies to minimize the outage probability and transmit power with different constraints are provided for wireless networks in Rayleigh fading. None of the aforementioned works evaluated the effect of the PU communication on the outage performance of the SU in a relay-based spectrum sharing system with PU outage probability constraint taken into consideration.

In this paper, an analytical expression of the outage probability for the SU, assisted by a secondary relay (SR), is derived under the constraint of satisfying the PU outage probability. In practice, further, the transmit power of the SU-Tx cannot be infinite or very large. Therefore, we take into account the maximum transmit power of the SU-Tx as an additional constraint. Accordingly, a power allocation strategy is proposed for the SU to adaptively regulate its transmit power under such constraints.

The rest of this paper is organized as follows. Section II describes the system and channel model. In Section III, we derive the power allocation policy and statistics corresponding to the instantaneous signal-to-interference plus noise ratio (SINR) of the first-hop and second-hop to provide an outage probability expression for the SU. Numerical results are presented in Section IV. Concluding remarks are given in Section V.

2 System Model

Consider a cognitive cooperative relay network (CCRN) as shown in Fig. 1, where the decode-and-forward (DF) protocol at the SR and half-duplex com-

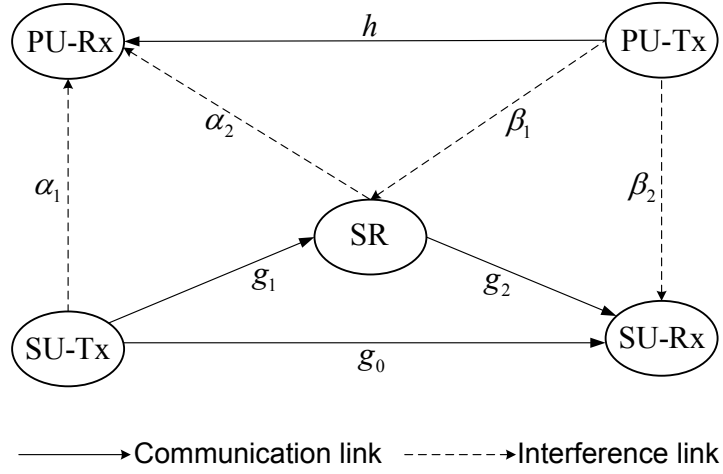


Figure 1: System model of a CCRN.

munication are used. In the first phase, the SU-Tx broadcasts its signal to the SU-Rx and the SR with transmit power P_s . In the second phase, the SR decodes and forwards the received message to the SU-Rx with transmit power P_r . The transmit power of SU-Tx and SR should guarantee a predefined outage constraint of the PU-Rx. The channels between PU-Tx→PU-Rx, SU-Tx→SU-Rx, SU-Tx→SR, SR→SU-Rx, SU-Tx→PU-Rx, SR→PU-Rx, PU-Tx→SR and PU-Tx→SU-Rx, respectively, are assumed to be subject to independent flat Rayleigh fading with channel mean powers Ω_a , $a \in \{h, g_0, g_1, g_2, \alpha_1, \alpha_2, \beta_1, \beta_2\}$, where $\{\cdot\}$ denotes the set of channel gains. Here, a is exponentially distributed with probability density function (PDF) $f_a(x) = 1/\Omega_a e^{-x/\Omega_a} u(x)$, where $u(x)$ is the Heaviside step function, $u(x) = 1$ for $x \geq 0$, and $u(x) = 0$ for $x < 0$.

3 Outage Performance Analysis

The instantaneous signal-to-interference-plus-noise ratio (SINR) of the links SU-Tx→SU-Rx, SU-Tx→SR and SR→SU-Rx can, respectively, be written as

$$\gamma_0 = \frac{g_0 P_s}{\beta_2 P_p + N_0} \quad (1)$$

$$\gamma_1 = \frac{g_1 P_s}{\beta_1 P_p + N_0} \quad (2)$$

$$\gamma_2 = \frac{g_2 P_r}{\beta_2 P_p + N_0} \quad (3)$$

where P_p is the transmit power of the PU-Tx and N_0 is the noise power.

Let ε denote the desired outage probability at the PU-Rx. Considering the effect of the secondary transmission on the primary network, the primary outage constraint can be formulated in the first and second transmission phase as

$$P_{out}^{p1} = \Pr \left\{ \frac{h P_p}{\alpha_1 P_s + N_0} < \gamma_{p_{th}} \right\} \leq \varepsilon \quad (4)$$

$$P_{out}^{p2} = \Pr \left\{ \frac{h P_p}{\alpha_2 P_r + N_0} < \gamma_{p_{th}} \right\} \leq \varepsilon \quad (5)$$

where $\Pr \{\cdot\}$ denotes probability, $\gamma_{p_{th}} = 2^{r_p/B} - 1$ is the SINR outage threshold of the PU-Rx and r_p is the transmission rate of the PU-Tx. The system bandwidth, B , is normalized to unity. The instantaneous transmit powers P_s and P_r at the SU-Tx and SR, respectively, should stay less or equal to the allowed maximum power P_{max} . As such, SU-Tx and SR must be able to adapt their transmit powers to ensure that there is no harmful interference to the primary network, subject to the primary outage constraint as defined in (4) and (5). Hence, the outage probabilities in the two phases are needed for deriving an adaptive transmit power policy. For this purpose, the following lemma is utilized.

Lemma 1 *The primary outage probabilities for the two transmission phases are given by*

$$P_{out}^{p1} = 1 - D_{p1} \exp \left(\frac{-N_0 \gamma_{p_{th}}}{P_p \Omega_h} \right) \quad (6)$$

$$P_{out}^{p2} = 1 - D_{p2} \exp \left(\frac{-N_0 \gamma_{p_{th}}}{P_p \Omega_h} \right) \quad (7)$$

where

$$D_{p1} = \frac{P_p \Omega_h}{\gamma_{p_{th}} P_s \Omega_{\alpha_1} + P_p \Omega_h} \quad (8)$$

$$D_{p2} = \frac{P_p \Omega_h}{\gamma_{p_{th}} P_r \Omega_{\alpha_2} + P_p \Omega_h} \quad (9)$$

Proof 1 See *Appendix A*

Note that if in the first transmission phase, the SU-Tx keeps silent or if the SR, in the second phase, cannot decode the message from SU-Tx, then, $D_{p_1} = D_{p_2} = 1$.

From (4), (5), and Lemma 1, along with some algebraic manipulations, the transmit power for the SU-Tx and SR can be written as

$$P_s = \frac{P_p \Omega_h}{\gamma_{p_{th}} \Omega_{\alpha_1}} \left[\frac{1}{(1 - \varepsilon)} \exp \left(\frac{-N_0 \gamma_{p_{th}}}{P_p \Omega_h} \right) - 1 \right] \quad (10)$$

$$P_r = \frac{P_p \Omega_h}{\gamma_{p_{th}} \Omega_{\alpha_2}} \left[\frac{1}{(1 - \varepsilon)} \exp \left(\frac{-N_0 \gamma_{p_{th}}}{P_p \Omega_h} \right) - 1 \right] \quad (11)$$

Given P_{max} , the adaptive transmit power policies at the SU-Tx and SR, can, respectively, be formulated as

$$\mathcal{P}_s = \min \left(\frac{P_p \Omega_h}{\gamma_{p_{th}} \Omega_{\alpha_1}} \rho^+, P_{max} \right) \quad (12)$$

$$\mathcal{P}_r = \min \left(\frac{P_p \Omega_h}{\gamma_{p_{th}} \Omega_{\alpha_2}} \rho^+, P_{max} \right) \quad (13)$$

where $\rho^+ = \max(\rho, 0)$ with

$$\rho = \frac{1}{(1 - \varepsilon)} \exp \left(\frac{-N_0 \gamma_{p_{th}}}{P_p \Omega_h} \right) - 1 \quad (14)$$

3.1 Outage Probability for Selection Combining

Selection combining (SC) at the SU-Rx allows selecting the link with the highest SINR (direct or relay link). For SC, the instantaneous SINR at the SU-Rx is given by

$$\gamma_{SC} = \max \left[\gamma_0, \min(\gamma_1, \gamma_2) \right] \quad (15)$$

Therefore, the outage probability of the secondary network with SC can be derived as

$$\begin{aligned}
 P_{out}^{SC} &= \Pr \left\{ \max \left[\gamma_0, \min \left(\gamma_1, \gamma_2 \right) \right] < \gamma_{th} \right\} \\
 &= \underbrace{\int_0^\infty \Pr \left\{ \frac{g_0 P_s}{x P_p + N_0} < \gamma_{th} \right\}}_{I_1} \\
 &\quad \times \underbrace{\Pr \left\{ \min \left(\frac{g_1 P_s}{\beta_1 P_p + N_0}, \frac{g_2 P_r}{x P_p + N_0} \right) < \gamma_{th} \right\}}_{I_2} \\
 &\quad \times f_{\beta_2}(x) dx
 \end{aligned} \tag{16}$$

where $\gamma_{th} = 2^{2r_s/B} - 1$ is the SINR outage threshold of the SU-Rx and r_s is the transmission rate of the SU-Tx. From (16), I_1 can be obtained following the cumulative distribution function (CDF) of an exponential random variable as

$$I_1 = \Pr \left\{ \frac{g_0 P_s}{x P_p + N_0} < \gamma_{th} \right\} = 1 - \exp \left(\frac{-(x P_p + N_0) \gamma_{th}}{\Omega_{g_0} P_s} \right) \tag{17}$$

The integral I_2 in (16) can be found by the order statistics theory as shown in Appendix B assuming that

$$F_1(\gamma_{th}) = \Pr \left\{ \frac{g_1 P_s}{\beta_1 P_p + N_0} < \gamma_{th} \right\} \tag{18}$$

$$F_2(\gamma_{th}) = \Pr \left\{ \frac{g_2 P_r}{x P_p + N_0} < \gamma_{th} \right\} \tag{19}$$

It should be noted that $F_2(\gamma_{th})$ can be calculated in a similar way as in (17) while $F_1(\gamma_{th})$ is provided by the following lemma.

Lemma 2 *Given the independent exponentially distributed random variables g_1 and β_1 with means Ω_{g_1} and Ω_{β_1} , respectively, the CDF $F_1(\gamma_{th})$ is defined as*

$$F_1(\gamma_{th}) = 1 - D_{s_1} \exp \left(\frac{-N_0 \gamma_{th}}{P_s \Omega_{g_1}} \right) \tag{20}$$

where N_0 , P_s , P_r , and P_p are positive constants and

$$D_{s_1} = \frac{P_s \Omega_{g_1}}{\gamma_{th} P_p \Omega_{\beta_1} + P_s \Omega_{g_1}} \tag{21}$$

Proof 2 See Appendix B.

Given Lemma 2 and (16), we can state the following theorem for the outage probability in case of SC.

Theorem 1 Given are the following constants:

$$K_1 = \frac{1}{P_s \Omega_{g_1}} + \frac{1}{P_r \Omega_{g_2}} \quad (22)$$

$$K_2 = \frac{1/\Omega_{g_0} + 1/\Omega_{g_1}}{P_s} + \frac{1}{P_r \Omega_{g_2}} \quad (23)$$

Then, the outage probability for the case of SC is given by

$$\begin{aligned} P_{out}^{SC} = & 1 - D_{s_0} e^{-\frac{N_0 \gamma_{th}}{P_s \Omega_{g_0}}} - D_{s_1} D_{s_2} e^{-N_0 \gamma_{th} K_1} \\ & + \frac{D_{s_1} D_{s_0} e^{-N_0 \gamma_{th} K_2}}{1 + D_{s_0} \gamma_{th} \frac{P_r \Omega_{\beta_2}}{P_r \Omega_{g_2}}} \end{aligned} \quad (24)$$

where D_{s_1} is defined in Lemma 2, and D_{s_0} and D_{s_2} are, respectively, defined as

$$D_{s_0} = \frac{P_s \Omega_{g_0}}{\gamma_{th} P_r \Omega_{\beta_2} + P_s \Omega_{g_0}} \quad (25)$$

$$D_{s_2} = \frac{P_r \Omega_{g_2}}{\gamma_{th} P_r \Omega_{\beta_2} + P_r \Omega_{g_2}} \quad (26)$$

Proof 3 See Appendix C

3.2 Outage Probability for Maximum Ratio Combining: Upper Bound

Although SC is widely used due to its simplicity and low implementation cost, maximum ratio combining (MRC) achieves higher performance at the expense of complexity. Using MRC, the SU-Rx combines the received data from the SR and the SU-Tx for joint decoding. Hence, the instantaneous SINR at the SU-Rx is given as [11]

$$\gamma_{MRC} = \gamma_0 + \min \{\gamma_1, \gamma_2\} \quad (27)$$

From (27), the outage probability of the secondary link using MRC can

be written as

$$\begin{aligned}
 P_{out}^{MRC} &= \Pr \{ \gamma_0 + \min(\gamma_1, \gamma_2) < \gamma_{th} \} \\
 &= \int_0^\infty \Pr \left\{ \frac{g_0 P_s}{x P_p + N_0} + \min \left(\frac{g_1 P_s}{\beta_1 P_p + N_0}, \frac{g_2 P_r}{x P_p + N_0} \right) \right. \\
 &\quad \left. < \gamma_{th} \mid \beta_2 = x \right\} f_{\beta_2}(x) dx
 \end{aligned} \tag{28}$$

Then, an upper bound on the outage probability, i.e. $P_{out}^{MRC^{ub}} \geq P_{out}^{MRC}$, can be formulated as

$$\begin{aligned}
 P_{out}^{MRC^{ub}} &= \int_0^\infty \underbrace{\Pr \left\{ \frac{g_0 P_s}{x P_p + N_0} < \gamma_{th} \right\}}_{I_3} \\
 &\quad \times \underbrace{\Pr \left\{ \min \left(\frac{g_1 P_s}{\beta_1 P_p + N_0}, \frac{g_2 P_r}{x P_p + N_0} \right) < \gamma_{th} \right\}}_{I_4} \\
 &\quad \times f_{\beta_2}(x) dx
 \end{aligned} \tag{29}$$

where $I_3 = I_1$ and $I_4 = I_2$. Therefore, the upper bound on the outage probability for MRC is given by the same analytical expression as (24).

3.3 Outage Probability for MRC: Lower Bound

The lower bound of the outage probability can be determined as [12]

$$\begin{aligned}
 P_{out}^{MRC} &= \Pr \{ \gamma_0 + \min(\gamma_1, \gamma_2) < \gamma_{th} \} \\
 &\geq \Pr \{ 2 \max[\gamma_0, \min(\gamma_1, \gamma_2)] < \gamma_{th} \}
 \end{aligned} \tag{30}$$

Then, the lower bound of the outage probability for MRC is then given, according to (30) and (16), by

$$\begin{aligned}
 P_{out}^{MRC^{tb}} &= \int_0^\infty \underbrace{\Pr \left\{ \frac{g_0 P_s}{x P_p + N_0} < \gamma_{th}/2 \right\}}_{I_5} \\
 &\quad \times \underbrace{\Pr \left\{ \min \left(\frac{g_1 P_s}{\beta_1 P_p + N_0}, \frac{g_2 P_r}{x P_p + N_0} \right) < \gamma_{th}/2 \right\}}_{I_6} \\
 &\quad \times f_{\beta_2}(x) dx
 \end{aligned} \tag{31}$$

where $I_5 \neq I_3 = I_1$ and $I_6 \neq I_4 = I_2$. This is due to the SINR threshold, γ_{th} , at SU-Rx being now divided by 2.

4 Numerical Results

In this section, numerical results are provided to give some insight into the performance of the secondary network using SC and MRC subject to the primary outage constraint. Specifically, the outage constraint at the PU is set as $\varepsilon = 0.01$, the transmission rates at the SU-Tx and the PU-Tx are set, respectively, to $r_s = 0.1$ bits/s/Hz, and $r_p = 0.4$ bits/s/Hz. These rates may suit the cell edge spectral efficiency requirements of 3GPP LTE which are specified to be $> 0.02 - 0.03$ bits/s/Hz/user [13]. The channel mean powers are set according to the exponential-decay path loss model. In particular, we assume that the distances between transmitting and receiving nodes are $d_{u,v}$, $u \in \{\text{SU-Tx}, \text{SR}, \text{PU-Tx}\}$, and $v \in \{\text{PU-Rx}, \text{SR}, \text{SU-Rx}\}$. Then, we obtain the impact of distance on the channel mean power $\Omega_a \sim d_{u,v}^{-\nu}$, where ν is the path loss exponent and $a \in \{h, g_0, g_1, g_2, \alpha_1, \alpha_2, \beta_1, \beta_2\}$. In the sequel, we assume non-line-of-sight propagation with $\nu = 4$. In addition, the SR shall be placed half-way between SU-Tx and SU-Rx, i.e., $d_{\text{SU-Tx}, \text{SR}} = d_{\text{SR}, \text{SU-Rx}} = 1/2$.

Fig. 2 depicts the SU outage probability for the case of SC. We fix a reference scenario with settings $\Omega_h = 16$, $\Omega_{\alpha_1} = \Omega_{\alpha_2} = 0.5$, $\Omega_{\beta_1} = \Omega_{\beta_2} = 0.5$, $\Omega_{g_0} = 1$, and $\Omega_{g_1} = \Omega_{g_2} = 16$. The reference scenario is compared with three other scenarios resulting from decreasing/increasing some of the channel mean powers. First, decreasing Ω_h to 2 significantly degrades the outage performance of the SU-Rx. At large primary SNR of approximately $P_p/N_0 = 20$ dB, performance recovers to that of the reference case. Second, the increase of channel mean powers Ω_{β_1} and Ω_{β_2} does not cause much degradation but outage probability is still higher than in the reference case. Third, increasing the channel mean power of the interference links from the SU to the PU network, i.e., Ω_{α_1} and Ω_{α_2} , from 0.5 to 1, provides approximately the same

performance as in the second scenario (increasing Ω_{β_1} and Ω_{β_2}) within the range $P_p/N_0 = 0$ dB to 13 dB. In all three cases, an increase of P_p/N_0 beyond a certain level causes severe impairments to the secondary network.

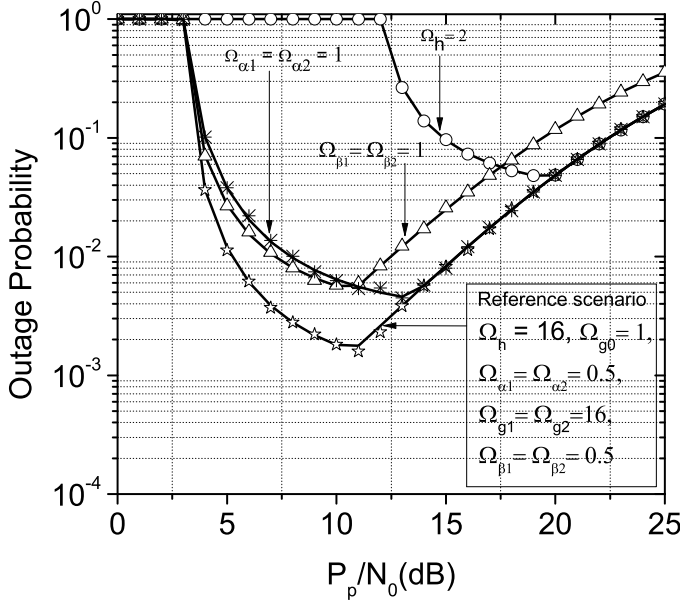


Figure 2: Outage probability of the CCRN using SC (Solid line: Analysis; Markers: Simulation).

In Fig. 3, the performance using SC and MRC with parameter setting of the reference scenario are compared. As expected, MRC outperforms SC. It should be noted that the mismatch between analysis and simulation for MRC is due to the fact that the upper and lower bounds of (28) and (30), respectively, were used.

Fig. 4 shows the outage probability of the PU and the SU where the effect of the interference from the SU to the PU and the PU to the SU network, respectively, is examined. Given the parameters of the reference scenario and without predefined PU outage threshold, we observe that the PU would remain in outage due to the SU interference when the primary SNR is in the range $P_p/N_0 = 0$ dB to 7.5 dB. As such, the shown performance curves reveal

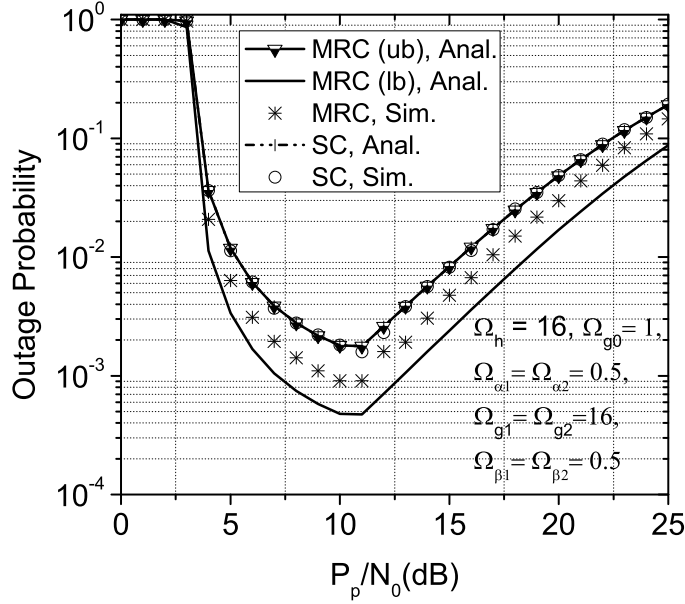


Figure 3: Comparison of the outage probability of SC and MRC for the CCRN.

the primary SNR range under which the SU can coexist with the PU without violating the outage constraint given by the PU. Similarly, the primary SNR range under which the SU has satisfactory outage performance can also be deduced from these results.

Fig. 5 depicts the effect of the SU maximum allowed transmit power, P_{max} , on the outage probability of the SU-Rx. The results show that the increase of P_{max} from 4 dB to 10 dB translates to an improvement of outage performance with respect to the minima of the curves. This implies that an increase of the secondary SNR results in a lower outage probability at the SU-Rx and also allows an increase of the primary SNR. Specifically, the primary SNR can be increased by about 5 dB when P_{max} of the SU-Tx is increased from 4 dB to 10 dB. This interrelationship between secondary and primary network can also be observed in the power allocation policy formulated in (12) and (13).

Fig. 6 plots the results for the reference scenario and the following three other cases with different rates and channel parameters:

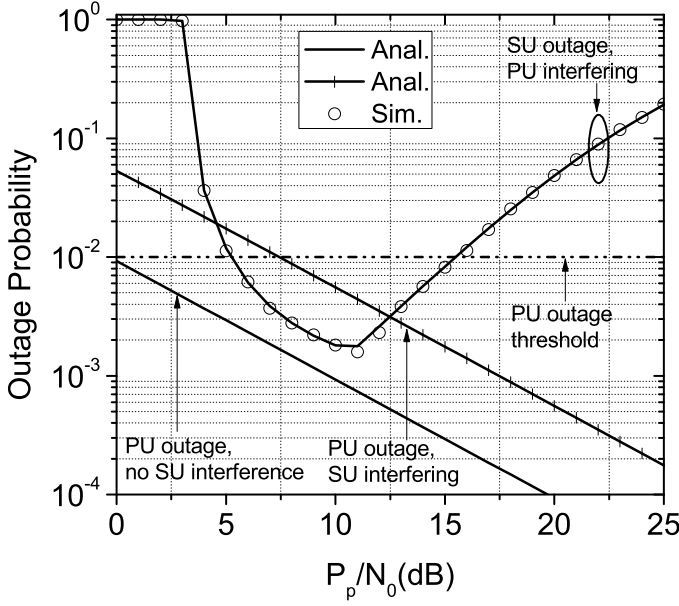


Figure 4: SU and PU outage probability with the interference effect between the two networks for the case of SC.

Case 1: $r_s = 0.02$ bits/s/Hz/user, $r_p = 0.03$ bits/s/Hz/user; Channel mean powers as with reference scenario

Case 2: $r_s = 0.1$ bits/s/Hz/user, $r_p = 0.4$ bits/s/Hz/user; $\Omega_h = 1$, and $\Omega_{g_1} = \Omega_{g_2} = 1$

Case 3: $r_s = 0.02$ bits/s/Hz/user, $r_p = 0.03$ bits/s/Hz/user; $\Omega_h = 1$, and $\Omega_{g_1} = \Omega_{g_2} = 1$

In Case 1, SU and PU use the lowest considered rates but the same channel mean powers as in the reference scenario. In Case 2, SU and PU rates remain as in the reference scenario but channel mean power Ω_h of the primary communication link and channel mean powers Ω_{g_1} and Ω_{g_2} of the secondary

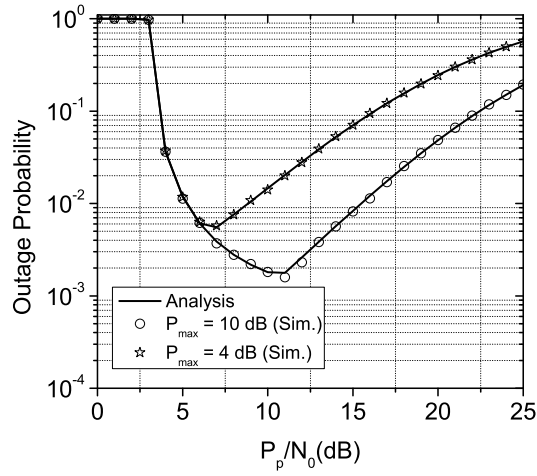


Figure 5: Effect of SU maximum allowed transmit power, P_{max} , on the SU outage probability for the case of SC.

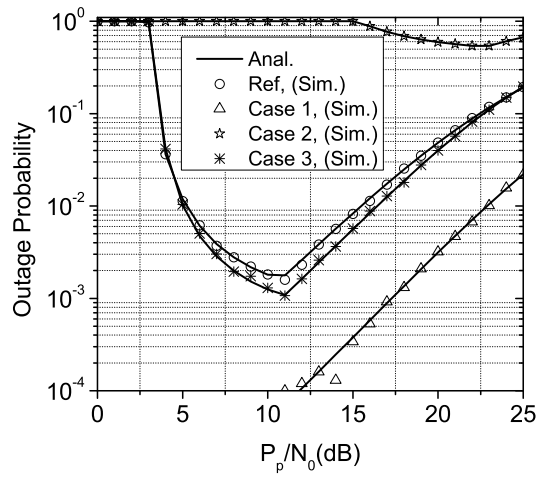


Figure 6: Outage probability with different rates and channel parameters: A comparison of three different cases with the reference scenario and SC.

communication links are decreased. In Case 3, the transmission rates are reduced compared to Case 2. Clearly, decreasing the transmission rate (Case 1) compared to the reference scenario improves the outage probability of the secondary network while a reduction of the channel mean powers of the communication links (Case 2) causes severe performance degradation. In order to compensate for such degradation, the transmission rates may be reduced as in Case 3.

5 Conclusions

In this paper, a power allocation policy for a CCRN has been considered to meet the outage constraint of the PU-Rx. At the destination, two combining techniques, SC and MRC, have been used and compared in order to evaluate the performance offered by the CCRN network. In particular, an analytical expression for the outage probability in case of SC has been derived while upper and lower bounds of the outage probability have been obtained for the case of MRC. Numerical examples have been provided to illustrate the effect of channel mean powers, transmission rates, and maximum allowed transmission power of the SU on outage probability of the secondary network in view of the PU transmission.

Appendix

A. Proof of Lemma 1

Given the outage constraints (4) and (5), the primary outage probability in the first and second phase can be derived as

$$\begin{aligned}
 P_{out}^{p1} &= \Pr \left\{ \frac{P_p h}{\alpha_1 P_s + N_0} < \gamma_{p_{th}} \right\} \\
 &= \int_0^\infty \Pr \{ P_p h < (x P_s + N_0) \gamma_{p_{th}} \} f_{\alpha_1}(x) dx \\
 &= \int_0^\infty \frac{1}{\Omega_{\alpha_1}} \exp\left(\frac{-x}{\Omega_{\alpha_1}}\right) \left[1 - \exp\left(\frac{-(x P_s + N_0) \gamma_{p_{th}}}{P_p \Omega_h}\right) \right] dx \\
 &= 1 - \frac{P_p \Omega_h}{\gamma_{p_{th}} P_s \Omega_{\alpha_1} + P_p \Omega_h} \exp\left(\frac{-N_0 \gamma_{p_{th}}}{P_p \Omega_h}\right)
 \end{aligned} \tag{32}$$

$$\begin{aligned}
 P_{out}^{p2} &= \Pr \left\{ \frac{P_p h}{\alpha_2 P_r + N_0} < \gamma_{p_{th}} \right\} \\
 &= \int_0^\infty \Pr \{ P_p h < (x P_r + N_0) \gamma_{p_{th}} \} f_{\alpha_2}(x) dx \\
 &= \int_0^\infty \frac{1}{\Omega_{\alpha_2}} \exp\left(\frac{-x}{\Omega_{\alpha_2}}\right) \left[1 - \exp\left(\frac{-(x P_r + N_0) \gamma_{p_{th}}}{P_p \Omega_h}\right) \right] dx \\
 &= 1 - \frac{P_p \Omega_h}{\gamma_{p_{th}} P_r \Omega_{\alpha_2} + P_p \Omega_h} \exp\left(\frac{-N_0 \gamma_{p_{th}}}{P_p \Omega_h}\right)
 \end{aligned} \tag{33}$$

B. Proof of Lemma 2

From the order statistics theory, I_2 in (16) can be easily determined as long as the probabilities, $F_1(\gamma_{th}) = \Pr \left\{ \frac{g_1 P_s}{\beta_1 P_p + N_0} < \gamma_{th} \right\}$ and $F_2(\gamma_{th}) = \Pr \left\{ \frac{g_2 P_r}{x P_p + N_0} < \gamma_{th} \right\}$ are available.

Similar to (32) and (33), $F_1(\gamma_{th})$ can be derived as

$$\begin{aligned}
 F_1(\gamma_{th}) &= \Pr \left\{ \frac{P_s g_1}{\beta_1 P_p + N_0} < \gamma_{th} \right\} \\
 &= \int_0^\infty \Pr \{ P_s g_1 < (x P_p + N_0) \gamma_{th} \} f_{\beta_1}(x) dx \\
 &= \int_0^\infty \frac{1}{\Omega_{\beta_1}} \exp\left(\frac{-x}{\Omega_{\beta_1}}\right) \left[1 - \exp\left(\frac{-(x P_p + N_0) \gamma_{th}}{P_s \Omega_{g_1}}\right) \right] dx \\
 &= 1 - \frac{P_s \Omega_{g_1}}{\gamma_{th} P_p \Omega_{\beta_1} + P_s \Omega_{g_1}} \exp\left(\frac{-N_0 \gamma_{th}}{P_s \Omega_{g_1}}\right)
 \end{aligned} \tag{34}$$

In addition, from the order statistics theory, we obtain

$$\begin{aligned}
 I_2 &= 1 - [1 - F_1(\gamma_{th})] [1 - F_2(\gamma_{th})] \\
 &= 1 - \frac{P_s \Omega_{g_1}}{\gamma_{th} P_p \Omega_{\beta_1} + P_s \Omega_{g_1}} \exp\left(\frac{-N_0 \gamma_{th}}{P_s \Omega_{g_1}}\right) \\
 &\quad \times \exp\left(\frac{-(x P_p + N_0) \gamma_{th}}{P_r \Omega_{g_2}}\right)
 \end{aligned} \tag{35}$$

where $F_2(\gamma_{th})$ is calculated in a similar way as in (17) as

$$\begin{aligned} F_2(\gamma_{th}) &= \Pr \left\{ \frac{P_r g_2}{x P_p + N_0} < \gamma_{th} \right\} \\ &= 1 - \exp \left(\frac{-(x P_p + N_0) \gamma_{th}}{P_r \Omega_{g_2}} \right) \end{aligned} \quad (36)$$

C. Proof of Theorem 1

The integral of (16), i.e.,

$$I = \int_0^\infty I_1 I_2 f_{\beta_2}(x) dx \quad (37)$$

can be solved by substituting (17) for I_1 , and using Lemma 2 for I_2 . Hence, with some algebraic manipulations, the outage probability for SC is obtained as in (24).

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Part IV-B

PART IV-B

Symbol Error Probability of Cognitive Cooperative Relay Networks Under Primary Outage and Secondary Peak Transmit Power Constraints

Part IV-B is based on:

C. Kabiri, H.-J. Zepernick, L. Sibomana, H. Tran, “Symbol Error Probability of Cognitive Cooperative Relay Networks Under Primary Outage and Secondary Peak Transmit Power Constraints,” *Wireless Personal Communications*, Mar. 2015, submitted.

Symbol Error Probability of Cognitive Cooperative Relay Networks Under Primary Outage and Secondary Peak Transmit Power Constraints

C. Kabiri, H.-J. Zepernick, L. Sibomana, and H. Tran

Abstract

In this article, we analyze the performance of cognitive radio networks (CRNs) in terms of symbol error probability (SEP) that utilizes decode-and-forward relaying. The communication in the secondary network is subject to the peak transmit power constraint of the secondary user transmitter and the outage constraint given by the primary user receiver. Given these constraints, an adaptive transmit power policy is formulated and the SEP of the secondary network is derived for different cognitive cooperative relay network (CCRN) topologies. In particular, a single relay CCRN with and without direct link as well as a multiple relay CCRN are considered. Numerical results of the SEP for CCRNs with different modulation schemes and different number of relays are provided. These results illustrate that the availability of a direct link gives small performance improvements while an increase of the number of relays improves performance more significantly for the scenarios considered.

1 Introduction

Cognitive radio networks (CRNs) have been introduced to deal with the problems of radio spectrum scarcity and spectrum under-utilization. Specifically,

spectrum sharing access allows secondary users (SUs) to concurrently transmit with primary users (PUs) over the same licensed radio spectrum provided that the interference caused to the PUs is kept below a certain level. In spectrum sharing access, it is therefore common to impose an interference power constraint to the SU transmitter (SU-Tx) to protect the PUs. In addition, the peak transmit power of the SU-Tx is also constrained to be finite depending on the actual technology used. Given these two constraints along with the severe impairments caused by wireless channels such as signal fading, the quality of service (QoS) and range of the secondary network may be degraded considerably. In order to improve QoS and coverage of the secondary network, advanced radio transmission technologies such as cooperative communications can be utilized. In particular, spatial diversity gains are provided by cooperative relaying transmission schemes which have gained large interest as an approach to be combined with CRNs establishing cognitive cooperative relay networks (CCRN) [1–4].

Regarding performance assessment, channel capacities and power control policies of a spectrum sharing scenario are studied in [5] where the PU is protected from harmful interference by applying different interference power constraints. Considering the interference from the primary network to the secondary network and vice versa, a lower bound of the outage performance for underlay cognitive relay networks is derived in [6]. In order to maximize secondary network capacity, other types of interference constraints and power allocation strategies have been studied in [7–9]. For the more general case of wireless networks under Rayleigh fading, the work reported in [10] studies different power allocation problems to minimize outage probability and transmit power with different constraints.

In view of the above discussion, in this article, we examine the performance of a CCRN with the aforementioned two constraints in terms of symbol error probability (SEP) for decode-and-forward (DF) relaying and selection combining (SC) at the receiver. Furthermore, we extend the analysis from a single relay to a multiple relay system and assess their SEP performances. Specifically, analytical expressions for the SEP of the secondary network are derived subject to the outage constraint of the primary network and the peak transmit power constraint of the secondary network. Although neglected in some of the research works, it should be mentioned that the latter constraint is motivated by the fact that practical devices apparently cannot transmit with infinite or very high transmit power. Accordingly, in this article, a power allocation policy is adopted for the SUs to regulate their transmit powers under such constraints. Furthermore, the interference from the primary network to the secondary network and vice versa are taken into account.

The remainder of this article is organized as follows. Section 2 describes

the system model of the considered CCRN. In Section 3, we derive the power allocation policy based on the outage constraint of the primary network and the peak transmit power constraint of the secondary network. Then, in Section 4, the cumulative distribution functions (CDFs) of the instantaneous signal-to-interference-plus-noise ratios (SINRs) in the secondary network are derived. On this basis, the analysis of the SEP for the different CCRN topologies is provided in Section 5. Numerical results are given in Section 6. Finally, concluding remarks are provided in Section 7.

2 System Model

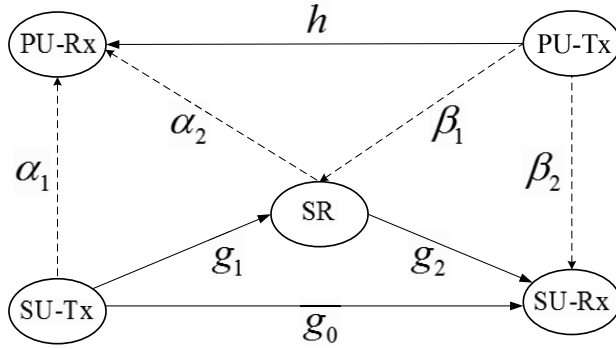
The topology of the considered CCRN is shown in Fig. 1. The transmission from SU-Tx to SU receiver (SU-Rx) is assisted by a secondary relay (SR) that uses the DF protocol and operates in half-duplex mode. In the first transmission phase, given a direct link, the SU-Tx broadcasts its signal to both the SR and SU-Rx with transmit power P_s . In the second transmission phase, the SR decodes, re-encodes, and then forwards the received message to the SU-Rx with transmit power P_r . The transmit power of the SU-Tx and SR must be chosen such that a predefined outage constraint, ε , of the primary network is not violated. The channels between PU transmitter (PU-Tx) $\rightarrow$ PU receiver (PU-Rx), SU-Tx $\rightarrow$ SU-Rx, SU-Tx $\rightarrow$ SR, SR $\rightarrow$ SU-Rx, SU-Tx $\rightarrow$ PU-Rx, SR $\rightarrow$ PU-Rx, PU-Tx $\rightarrow$ SR, and PU-Tx $\rightarrow$ SU-Rx are assumed to be subject to independent Rayleigh fading with channel mean power Ω_a and corresponding channel power gain $a \in \{h, g_0, g_1, g_2, \alpha_1, \alpha_2, \beta_1, \beta_2\}$.

Thus, the channel power gain a is exponentially distributed with corresponding probability density function (PDF) given by $f_a(x) = 1/\Omega_a e^{-x/\Omega_a} u(x)$ where $u(x)$ is the Heaviside step function, $u(x) = 1$ for $x \geq 0$ and $u(x) = 0$ for $x < 0$.

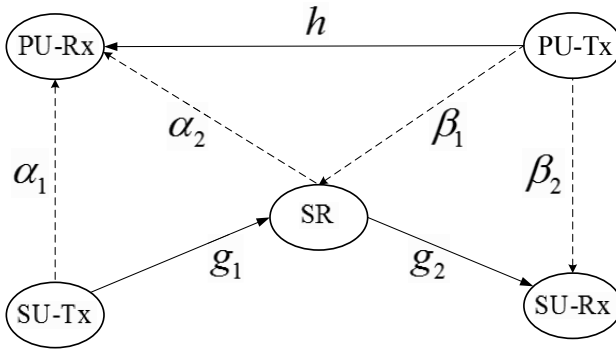
3 Power Allocation Policies of the Secondary Network

The power allocation policies of the secondary network are adopted from our work on the outage probability in CCRNs reported in [11] and shall be summarized in the sequel for convenience.

Accounting for the interference from the secondary network to the primary network, the primary outage constraint for the first and second transmission



(a)



(b)

Figure 1: Topology of the considered cognitive cooperative relay network: a) with direct link between SU-Tx and SU-Rx, b) without direct link between SU-Tx and SU-Rx (Solid lines: Communication links; Dashed lines: Interference links).

phase can be expressed as

$$P_{out}^{p_1} = \Pr \left\{ \frac{hP_p}{\alpha_1 P_s + N_0} < \gamma_{p_{th}} \right\} \leq \varepsilon \quad (1)$$

$$P_{out}^{p_2} = \Pr \left\{ \frac{hP_p}{\alpha_2 P_r + N_0} < \gamma_{p_{th}} \right\} \leq \varepsilon \quad (2)$$

where ε is the maximum tolerable outage probability of the primary network, P_p is the transmit power of the PU-Tx, P_s is the transmit power of the SU-Tx, P_r is the transmit power of the SR, and N_0 denotes the noise power. Further, $\Pr\{A\}$ denotes the probability of an event A , $\gamma_{p_{th}} = 2^{r_p} - 1$ is the SINR outage threshold, r_p is the transmission rate at the PU network, and system bandwidth is normalized to unity. The instantaneous transmit powers P_s and P_r at the SU-Tx and SR, respectively, must stay less or equal to the allowed peak transmit power P_{peak} and must respect the primary outage constraint defined in (1) and (2). In view of these constraints, the outage probabilities of the primary network for the two transmission phases are given in Lemma 1 [11].

Lemma 1 *The primary outage probabilities $P_{out}^{p_1}$ and $P_{out}^{p_2}$ associated with the first and second transmission phase in the secondary network, respectively, are given by*

$$P_{out}^{p_1} = 1 - D_{p_1} \exp \left(-\frac{N_0 \gamma_{p_{th}}}{P_p \Omega_h} \right) \quad (3)$$

$$P_{out}^{p_2} = 1 - D_{p_2} \exp \left(-\frac{N_0 \gamma_{p_{th}}}{P_p \Omega_h} \right) \quad (4)$$

where

$$D_{p_1} = \frac{P_p \Omega_h}{\gamma_{p_{th}} P_s \Omega_{\alpha_1} + P_p \Omega_h} \quad (5)$$

$$D_{p_2} = \frac{P_p \Omega_h}{\gamma_{p_{th}} P_r \Omega_{\alpha_2} + P_p \Omega_h} \quad (6)$$

Given (3) and (4), the transmit powers for SU-Tx and SR can be written as

$$P_s = \frac{P_p \Omega_h}{\gamma_{p_{th}} \Omega_{\alpha_1}} \left[\frac{1}{(1 - \varepsilon)} \exp \left(-\frac{N_0 \gamma_{p_{th}}}{P_p \Omega_h} \right) - 1 \right] \quad (7)$$

$$P_r = \frac{P_p \Omega_h}{\gamma_{p_{th}} \Omega_{\alpha_2}} \left[\frac{1}{(1 - \varepsilon)} \exp \left(-\frac{N_0 \gamma_{p_{th}}}{P_p \Omega_h} \right) - 1 \right] \quad (8)$$

Then, taking the peak transmit power constraint of the secondary nodes into consideration, i.e. $P_s \leq P_{peak}$ and $P_r \leq P_{peak}$, the adaptive transmit power policies of the SU-Tx and SR, respectively, can be formulated as

$$\mathcal{P}_s = \min\left(\frac{P_p\Omega_h}{\gamma_{p_{th}}\Omega_{\alpha_1}}\rho^+, P_{peak}\right) \quad (9)$$

$$\mathcal{P}_r = \min\left(\frac{P_p\Omega_h}{\gamma_{p_{th}}\Omega_{\alpha_2}}\rho^+, P_{peak}\right) \quad (10)$$

where $\rho^+ = \max(\rho, 0)$ and

$$\rho = \frac{1}{(1-\varepsilon)} \exp\left(-\frac{N_0\gamma_{p_{th}}}{P_p\Omega_h}\right) - 1 \quad (11)$$

4 CDFs of the SINR in the Secondary Network

The derivation of an analytical SEP expression of the secondary network can take advantage of the following result of [12]:

$$P_e = \frac{\eta\sqrt{\theta}}{2\sqrt{\pi}} \int_0^\infty F_{\gamma_s}(\gamma) \frac{e^{-\theta\gamma}}{\sqrt{\gamma}} d\gamma \quad (12)$$

where η and θ are constants that depend on the specific modulation scheme used [13]. For m -ary phase shift keying (m -PSK), these parameters are given as $\eta = 2$ and $\theta = \sin^2(\pi/m)$. Note that subscript γ_s on the CDF $F_{\gamma_s}(\gamma)$ in the integral expression (12) denotes the instantaneous SINR at SU-Rx. Apparently, the availability of the CDF of the instantaneous SINR is key in the calculation of the SEP of the secondary network. In the rest of this section, we therefore derive the CDFs for different secondary network topologies which can then be used in Section 5 to derive the related analytical SEP expressions.

4.1 CDF of the SINR for Single Relay CCRN with Direct Link

In order to derive the SEP of the secondary network, let us introduce the SINR of the links SU-Tx→SU-Rx, SU-Tx→SR, and SR→SU-Rx as

$$\gamma_0 = \frac{g_0 P_s}{\beta_2 P_p + N_0} \quad (13)$$

$$\gamma_1 = \frac{g_1 P_s}{\beta_1 P_p + N_0} \quad (14)$$

$$\gamma_2 = \frac{g_2 P_r}{\beta_2 P_p + N_0} \quad (15)$$

Let us recall that the SC technique is based on selecting the transmission link with the higher SINR among the direct and relay link. As such, SC is simple to implement with relatively low implementation costs. In the context of SC in a single relay CCRN with direct link (see Fig. 1a), the instantaneous SINR at the SU-Rx is given by

$$\gamma_{SC} = \max[\gamma_0, \min(\gamma_1, \gamma_2)] \quad (16)$$

The CDF of the SINR γ_{SC} for the case of having a single relay CCRN with direct link has been derived in our work on the outage performance of CCRNs comparing SC and maximal radio combining reported in [11] and hence shall be briefly revisited in the sequel for completeness. In particular, and in view of (16), the CDF of the SINR γ_{SC} for this case can be formulated as

$$\begin{aligned} F_{\gamma_{SC}}(\gamma) &= \Pr\{\gamma_{SC} < \gamma\} \\ &= \int_0^\infty \underbrace{\Pr\left\{\frac{g_0 P_s}{x P_p + N_0} < \gamma\right\}}_{I_1} \\ &\quad \times \underbrace{\Pr\left\{\min\left(\frac{g_1 P_s}{\beta_1 P_p + N_0}, \frac{g_2 P_r}{x P_p + N_0}\right) < \gamma\right\}}_{I_2} f_{\beta_2}(x) dx \quad (17) \end{aligned}$$

The first term I_1 under the integral in (17) is basically the CDF of an exponentially distributed random variable and can be obtained as

$$I_1 = 1 - \exp\left[-\frac{(x P_p + N_0)\gamma}{\Omega_{g_0} P_s}\right] \quad (18)$$

The second term I_2 under the integral in (17) can be solved by using the order statistics theory which enables us to reformulate this term as

$$I_2 = 1 - [1 - F_1(\gamma)][1 - F_2(\gamma)] \quad (19)$$

As such, the problem reduces to finding the following CDFs:

$$F_1(\gamma) = \Pr\left\{\frac{g_1 P_s}{\beta_1 P_p + N_0} < \gamma\right\} \quad (20)$$

$$F_2(\gamma) = \Pr\left\{\frac{g_2 P_r}{x P_p + N_0} < \gamma\right\} \quad (21)$$

which can be straightforwardly obtained after some elementary manipulations as

$$F_1(\gamma) = 1 - \frac{P_s \Omega_{g_1}}{\gamma P_p \Omega_{\beta_1} + P_s \Omega_{g_1}} \exp\left(-\frac{N_0 \gamma}{P_s \Omega_{g_1}}\right) \quad (22)$$

$$F_2(\gamma) = 1 - \exp\left[-\frac{(x P_p + N_0) \gamma}{P_r \Omega_{g_2}}\right] \quad (23)$$

Note that the expressions given in (22) and (23) differ due to the fact that (20) comprises two random variables g_1 and β_1 while only one random variable g_2 is involved in (21). Then, substituting (22) and (23) into (19) gives

$$I_2 = 1 - \frac{P_s \Omega_{g_1}}{\gamma P_p \Omega_{\beta_1} + P_s \Omega_{g_1}} \exp\left(-\frac{N_0 \gamma}{P_s \Omega_{g_1}}\right) \exp\left[-\frac{(x P_p + N_0) \gamma}{P_r \Omega_{g_2}}\right] \quad (24)$$

Having solved all the individual terms, i.e. I_1 and I_2 , the CDF $F_{\gamma_{SC}}(\gamma)$ of γ_{SC} can be obtained by substituting (18) and (24) into (17) as

$$F_{\gamma_{SC}}(\gamma) = 1 - D_{s_0} e^{-\frac{N_0 \gamma}{P_s \Omega_{g_0}}} - D_{s_1} D_{s_2} e^{-N_0 \gamma K_1} + D_{s_1} D_{s_0} \frac{e^{-N_0 \gamma K_2}}{1 + D_{s_0} \gamma \frac{P_p \Omega_{\beta_2}}{P_r \Omega_{g_2}}} \quad (25)$$

where

$$D_{s_0} = \frac{P_s \Omega_{g_0}}{\gamma P_p \Omega_{\beta_2} + P_s \Omega_{g_0}} \quad (26)$$

$$D_{s_1} = \frac{P_s \Omega_{g_1}}{\gamma P_p \Omega_{\beta_1} + P_s \Omega_{g_1}} \quad (27)$$

$$D_{s_2} = \frac{P_r \Omega_{g_2}}{\gamma P_p \Omega_{\beta_2} + P_r \Omega_{g_2}} \quad (28)$$

and

$$K_1 = \frac{1}{P_s \Omega_{g_1}} + \frac{1}{P_r \Omega_{g_2}} \quad (29)$$

$$K_2 = \frac{1/\Omega_{g_0} + 1/\Omega_{g_1}}{P_s} + \frac{1}{P_r \Omega_{g_2}} \quad (30)$$

4.2 CDF of the SINR for Single Relay CCRN without Direct Link

For the case of a single relay and no direct link, the topology of the CCRN reduces to the scenario shown in Fig. 1b which also accounts for the interference from the primary network to the secondary network and vice versa. Then, the CDF $F_{\gamma_{NDL}}(\gamma)$ of $\gamma_{NDL} = \min(\gamma_1, \gamma_2)$ is formulated as

$$\begin{aligned} F_{\gamma_{NDL}}(\gamma) &= \Pr\{\gamma_{NDL} < \gamma\} \\ &= \Pr\left\{\min(\gamma_1, \gamma_2) < \gamma\right\} \\ &= \int_0^\infty \Pr\left\{\min\left(\frac{g_1 P_s}{\beta_1 P_p + N_0}, \frac{g_2 P_r}{x P_p + N_0}\right) < \gamma \mid \beta_2 = x\right\} f_{\beta_2}(x) dx \\ &= \int_0^\infty \underbrace{\Pr\left\{\min\left(\frac{g_1 P_s}{\beta_1 P_p + N_0}, \frac{g_2 P_r}{x P_p + N_0}\right) < \gamma\right\}}_{I_2} f_{\beta_2}(x) dx \quad (31) \end{aligned}$$

As the term I_2 has already been derived as (24), the CDF $F_{\gamma_{NDL}}(\gamma)$ of γ_{NDL} can be obtained after some algebraic manipulations as

$$F_{\gamma_{NDL}}(\gamma) = 1 - D_{s_1} D_{s_2} e^{-N_0 \gamma K_1} \quad (32)$$

where the terms D_{s_1} , D_{s_2} , and K_1 are given in (27), (28), and (29), respectively.

4.3 CDF of the SINR for CCRN with Multiple Relays

Let us now extend the single relay topology with no direct link to a multiple relay topology where M relays give a more complex topology (see Fig. 2). Note that the derivation of the CDF relates to our work on outage performance analysis of cognitive relay networks for best relay selection and max-max relay

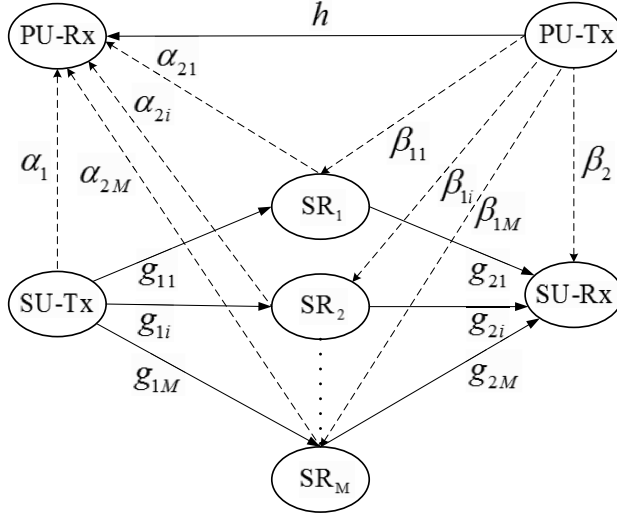


Figure 2: Topology of the considered cognitive cooperative relay network with multiple relays but without direct link between SU-Tx and SU-Rx (Solid lines: Communication links; Dashed lines: Interference links).

selection reported in [14]. In this scenario, the instantaneous SINRs for the two communication hops are given by

$$\gamma_{1i} = \frac{g_{1i}P_s}{\beta_{1i}P_p + N_0} \quad (33)$$

$$\gamma_{2i} = \frac{g_{2i}P_r}{\beta_{2i}P_p + N_0} \quad (34)$$

where g_{1i} , g_{2i} , β_{1i} ; $i = 1, \dots, M$, and β_2 ; are the channel power gains with corresponding channel mean powers $\Omega_{g_{1i}}$, $\Omega_{g_{2i}}$, $\Omega_{\beta_{1i}}$, and Ω_{β_2} , respectively.

Then, the CDF of the SINR at the SU-Rx can be expressed as

$$F_{\gamma_{SC}}(\gamma) = \Pr \left\{ \max_i \left[\min \left(\frac{g_{1i}P_s}{\beta_{1i}P_p + N_0}, \frac{g_{2i}P_r}{\beta_{2i}P_p + N_0} \right) \right] < \gamma \right\} \quad (35)$$

which can be rewritten as

$$F_{\gamma_{SC}}(\gamma) = \int_0^\infty \left(\int_0^\infty \Pr \left\{ \max_i \left[\min \left(\frac{g_{1i}P_s}{xP_p + N_0}, \frac{g_{2i}P_r}{yP_p + N_0} \right) \right] < \gamma \right\} \times f_{\beta_{1i}}(x)dx \right) f_{\beta_2}(y)dy \quad (36)$$

where

$$f_{\beta_{1i}}(x) = \frac{1}{\Omega_{\beta_{1i}}} \exp\left(-\frac{x}{\Omega_{\beta_{1i}}}\right) \quad (37)$$

$$f_{\beta_2}(y) = \frac{1}{\Omega_{\beta_2}} \exp\left(-\frac{y}{\Omega_{\beta_2}}\right) \quad (38)$$

Then, using the order statistic theory, the CDF $F_{\gamma_{SC}}(\gamma)$ of γ_{SC} for this multiple relay scenario can be further simplified as

$$F_{\gamma_{SC}}(\gamma) = \int_0^\infty \left(\int_0^\infty \prod_{i=1}^M [I_{1i}(x, y)] f_{\beta_{1i}}(x) dx \right) f_{\beta_2}(y) dy \quad (39)$$

where

$$I_{1i}(x, y) = \Pr \left\{ \min \left(\frac{g_{1i}P_s}{xP_p + N_0}, \frac{g_{2i}P_r}{yP_p + N_0} \right) < \gamma \right\} \quad (40)$$

In the following, we assume independent and identically distributed Rayleigh fading channels with $g_{1i} = g_1$, $g_{2i} = g_2$, $\beta_{1i} = \beta_1$, and $\alpha_{2i} = \alpha_2$ such that the power allocation proposed in Section 3 for the single relay topology is also valid for the multiple relay case. Then, (39) can be rewritten as

$$F_{\gamma_{SC}}(\gamma) = \int_0^\infty \left(\int_0^\infty [I_{1i}(x, y)]^M f_{\beta_1}(x) dx \right) f_{\beta_2}(y) dy \quad (41)$$

Applying the order statistics theory, expression (40) can be calculated as

$$\forall i: I_{1i}(x, y) = I_1(x, y) = 1 - [1 - F_1(\gamma)] [1 - F_2(\gamma)] \quad (42)$$

where

$$F_1(\gamma) = \Pr \left\{ \frac{g_1 P_s}{x P_p + N_0} < \gamma \right\} = \Pr \left\{ g_1 < \frac{\gamma(x P_p + N_0)}{P_s} \right\} \quad (43)$$

$$F_2(\gamma) = \Pr \left\{ \frac{g_2 P_r}{y P_p + N_0} < \gamma \right\} = \Pr \left\{ g_2 < \frac{\gamma(y P_p + N_0)}{P_r} \right\} \quad (44)$$

In view of the considered independent and identically distributed Rayleigh fading channels, substituting the related CDFs of Rayleigh fading into (42), and performing some algebraic manipulations gives

$$I_1(x, y) = 1 - \exp \left[-\frac{\gamma(P_p x + N_0)}{P_s \Omega_{g_1}} \right] \exp \left[-\frac{\gamma(P_p y + N_0)}{P_r \Omega_{g_2}} \right] \quad (45)$$

Given (45), using the binomial identity, and in view of the independent and identically distributed Rayleigh fading channels, the M -th power of $I_{1i}(x, y) = I_1(x, y)$ in expression (41) can be written as

$$\left[I_1(x, y) \right]^M = \sum_{k=0}^M \binom{M}{k} (-1)^k \exp \left[-\frac{k\gamma(P_p x + N_0)}{P_s \Omega_{g_1}} \right] \exp \left[-\frac{k\gamma(P_p y + N_0)}{P_r \Omega_{g_2}} \right] \quad (46)$$

Substituting $f_{\beta_1}(x) = \frac{1}{\Omega_{\beta_1}} \exp \left(-\frac{x}{\Omega_{\beta_1}} \right)$ and $f_{\beta_2}(y) = \frac{1}{\Omega_{\beta_2}} \exp \left(-\frac{y}{\Omega_{\beta_2}} \right)$ into (41) along with (46), yields

$$F_{\gamma_{SC}}(\gamma) = \sum_{k=0}^M \binom{M}{k} (-1)^k \frac{\exp \left(-\frac{k\gamma N_0}{P_s \Omega_{g_1}} \right) \exp \left(-\frac{k\gamma N_0}{P_r \Omega_{g_2}} \right)}{\left(\frac{k\gamma P_p \Omega_{\beta_1}}{P_s \Omega_{g_1}} + 1 \right) \left(\frac{k\gamma P_p \Omega_{\beta_2}}{P_r \Omega_{g_2}} + 1 \right)} \quad (47)$$

which can be further simplified to finally obtain

$$F_{\gamma_{SC}}(\gamma) = \sum_{k=0}^M \binom{M}{k} (-1)^k D_{s_{1k}} D_{s_{2k}} \exp(-\gamma N_0 k K_1) \quad (48)$$

where

$$D_{s_{1k}} = \frac{P_s \Omega_{g_1}}{\gamma k P_p \Omega_{\beta_1} + P_s \Omega_{g_1}} \quad (49)$$

$$D_{s_{2k}} = \frac{P_r \Omega_{g_2}}{\gamma k P_p \Omega_{\beta_2} + P_r \Omega_{g_2}} \quad (50)$$

$$K_1 = \frac{1}{P_s \Omega_{g_1}} + \frac{1}{P_r \Omega_{g_2}} \quad (51)$$

5 Symbol Error Probability

Having obtained the CDFs for the different CCRN topologies, i.e. single relay CCRN with and without direct link, and multiple relay CCRN without direct link, we can now return to deriving analytical SEP expression for each of these topologies in the following sections.

5.1 SEP of Single Relay CCRN

Substituting (25) into (12) and performing some algebraic manipulations, the SEP of the CCRN with a single relay and availability of a direct link between

SU-Tx and SU-Rx can be derived as

$$P_e^{DL} = \frac{\eta\sqrt{\theta}}{2\pi} \left(I_0 - P_s\Omega_{g_0}I_1 - P_s\Omega_{g_1}P_r\Omega_{g_2}I_2 + P_s\Omega_{g_0}P_s\Omega_{g_1}P_r\Omega_{g_2}I_3 \right) \quad (52)$$

Similar in the case where a direct link is not available due to severe shadowing, an SEP expression for the single relay CCRN can be obtained as

$$P_e^{NDL} = \frac{\eta\sqrt{\theta}}{2\pi} \left(I_0 - P_s\Omega_{g_1}P_r\Omega_{g_2}I_2 \right) \quad (53)$$

The integrals I_0 , I_1 , I_2 and I_3 in (52) and (53) are, respectively, given by

$$I_0 = \int_0^\infty \frac{e^{-\theta\gamma}}{\sqrt{\gamma}} d\gamma \quad (54)$$

$$I_1 = \int_0^\infty \frac{e^{-(\theta + \frac{N_0}{P_s\Omega_{g_0}})\gamma}}{(\gamma P_p\Omega_{\beta_2} + P_s\Omega_{g_0})\sqrt{\gamma}} d\gamma \quad (55)$$

$$I_2 = \int_0^\infty \frac{e^{-(\theta + N_0K_1)\gamma}}{(\gamma P_p\Omega_{\beta_2} + P_r\Omega_{g_2})(\gamma P_p\Omega_{\beta_1} + P_s\Omega_{g_1})\sqrt{\gamma}} d\gamma \quad (56)$$

$$I_3 = \int_0^\infty \frac{e^{-(\theta + N_0K_2)\gamma}}{(\gamma P_p\Omega_{\beta_2} + P_s\Omega_{g_0})(\gamma A_3 + P_s\Omega_{g_0}P_r\Omega_{g_2})\sqrt{\gamma}} d\gamma \quad (57)$$

where $A_3 = P_p\Omega_{\beta_2}P_r\Omega_{g_2} + P_s\Omega_{g_0}P_p\Omega_{\beta_2}$. For brevity, we denote $A_1 = P_p\Omega_{\beta_1}$, $A_2 = P_p\Omega_{\beta_2}$, $B_0 = P_s\Omega_{g_0}$, $B_1 = P_s\Omega_{g_1}$, $B_2 = P_r\Omega_{g_2}$, and $B_3 = P_s\Omega_{g_0}P_r\Omega_{g_2}$. Then, the integrals I_0 , I_1 , I_2 , and I_3 can be rewritten as

$$I_0 = \int_0^\infty \frac{e^{-\theta\gamma}}{\sqrt{\gamma}} d\gamma \quad (58)$$

$$I_1 = \int_0^\infty \frac{e^{-(\theta + \frac{N_0}{B_0})\gamma}}{(\gamma A_2 + B_0)\sqrt{\gamma}} d\gamma \quad (59)$$

$$I_2 = \int_0^\infty \frac{e^{-(\theta + N_0K_1)\gamma}}{(\gamma A_1 + B_1)(\gamma A_2 + B_2)\sqrt{\gamma}} d\gamma \quad (60)$$

$$I_3 = \int_0^\infty \frac{e^{-(\theta + N_0K_2)\gamma}}{(\gamma A_2 + B_0)(\gamma A_3 + B_3)\sqrt{\gamma}} d\gamma \quad (61)$$

Analytical expressions for the SEPs P_e^{DL} and P_e^{NDL} can readily be found by solving the integrals I_0 , I_1 , I_2 , and I_3 using the following general integral form:

$$\mathcal{I}(\alpha, a, b, c, d) = \int_0^{\infty} \frac{e^{-\alpha x}}{(ax+b)(cx+d)\sqrt{x}} dx \quad (62)$$

5.2 Calculation of Integral I_0

Comparing the arguments of (62) with those in the expression under the integral (58) and with the help of [15, eq. 3.361.2], the integral I_0 can be solved as

$$I_0 = \sqrt{\frac{\pi}{\alpha}} \quad (63)$$

5.3 Calculation of Integral I_1

In this case, we have $a = 0$ and $c > 0$ such that the general form (62) reduces to

$$\mathcal{I}(\alpha, a, b, c, d) = \frac{1}{bc} \int_0^{\infty} \frac{e^{-\alpha x}}{(x + \frac{d}{c})\sqrt{x}} dx \quad (64)$$

Then, performing a change of variable $t = x + \frac{d}{c}$ in (64) and using [15, eq. (3.363.1)], the expression (64) can be rewritten as

$$\mathcal{I}(\alpha, a, b, c, d) = \frac{e^{\frac{\alpha d}{c}}}{bc} \int_{\frac{d}{c}}^{\infty} \frac{e^{-\alpha t}}{t\sqrt{t - \frac{d}{c}}} dt = \frac{e^{\frac{\alpha d}{c}} \pi}{b\sqrt{dc}} \left[1 - \mathcal{Q} \left(\sqrt{\frac{\alpha d}{c}} \right) \right] \quad (65)$$

and a comparison of arguments finally gives

$$I_1 = \frac{e^{\frac{(\theta + \frac{N_0}{B_0})B_0}{A_2}} \pi}{\sqrt{A_2 B_0}} \left[1 - \mathcal{Q} \left(\sqrt{\frac{(\theta + \frac{N_0}{B_0})B_0}{A_2}} \right) \right] \quad (66)$$

where $\mathcal{Q}(t)$ denotes the error function which is defined as

$$\mathcal{Q}(t) = \frac{2}{\sqrt{\pi}} \int_t^{\infty} e^{-x^2} dx \quad (67)$$

5.4 Calculation of Integral I_2

In order to solve (60), we observe by comparison with (62) that $a > 0$ and $c > 0$. Furthermore, we need to consider the following three cases:

- Case 1: $a = bc/d$

For this relationship between arguments a , b , c , and d , we have

$$\mathcal{I}(\alpha, a, b, c, d) = \frac{d}{bc^2} \int_0^\infty \frac{e^{-\alpha x}}{\sqrt{x} \left(x + \frac{d}{c}\right)^2} dx \quad (68)$$

which has been calculated in [16, eq. (41)] as

$$\mathcal{I}(\alpha, a, b, c, d) = \frac{\sqrt{\alpha\pi}}{bc} + \left[1 - \mathcal{Q}\left(\frac{\alpha d}{c}\right)\right] \frac{e^{\frac{\alpha d}{c}} d\pi(c - 2\alpha d)}{2b(cd)^{3/2}} \quad (69)$$

Therefore, an expression for (56) can be obtained straightforwardly as

$$\begin{aligned} I_2 &= \frac{\sqrt{(\theta + N_0 K_1)\pi}}{B_1 A_2} + \left[1 - \mathcal{Q}\left(\frac{(\theta + N_0 K_1)B_2}{A_2}\right)\right] \\ &\quad \times \frac{e^{\frac{(\theta + N_0 K_1)B_2}{A_2}} B_2 \pi (A_2 - 2(\theta + N_0 K_1)B_2)}{2B_1 (A_2 B_2)^{3/2}} \end{aligned} \quad (70)$$

- Case 2: $c = ad/b$

In this case, the general integral form can be expressed as

$$\mathcal{I}(\alpha, a, b, c, d) = \frac{b}{da^2} \int_0^\infty \frac{e^{-\alpha x}}{\sqrt{x} \left(x + \frac{b}{a}\right)^2} dx \quad (71)$$

As with the preceding case, (71) can be easily solved in a similar way as that leading to (68), which then results in

$$\mathcal{I}(\alpha, a, b, c, d) = \frac{\sqrt{\alpha\pi}}{da} + \left[1 - \mathcal{Q}\left(\frac{\alpha b}{a}\right)\right] \frac{e^{\frac{\alpha b}{a}} b\pi(a - 2\alpha b)}{2d(ab)^{3/2}} \quad (72)$$

Then, the solution of (56) is obtained as

$$\begin{aligned} I_2 &= \frac{\sqrt{(\theta + N_0 K_1)\pi}}{B_2 A_1} + \left[1 - \mathcal{Q}\left(\frac{(\theta + N_0 K_1)B_1}{A_1}\right)\right] \\ &\quad \times \frac{e^{\frac{(\theta + N_0 K_1)B_1}{A_1}} B_1 \pi (A_1 - 2(\theta + N_0 K_1)B_1)}{2B_2 (A_1 B_1)^{3/2}} \end{aligned} \quad (73)$$

- Case 3: $bc \neq ad$

In this case, we have

$$\mathcal{I}(\alpha, a, b, c, d) = \int_0^{\infty} \frac{e^{-\alpha x}}{\sqrt{x} (ax + b) (cx + d)} dx \quad (74)$$

Using the result of [16, eq. (46)], the expression (74) becomes

$$\begin{aligned} \mathcal{I}(\alpha, a, b, c, d) = & \frac{ac}{ad - cb} \left\{ \pi e^{\frac{\alpha b}{a}} \sqrt{\frac{a}{b}} \left[1 - \mathcal{Q} \left(\sqrt{\frac{\alpha b}{a}} \right) \right] \right. \\ & \left. - \pi e^{\frac{\alpha d}{c}} \sqrt{\frac{c}{d}} \left[1 - \mathcal{Q} \left(\sqrt{\frac{\alpha d}{c}} \right) \right] \right\} \end{aligned} \quad (75)$$

and integral I_2 is solved for this case as

$$\begin{aligned} I_2 = & \frac{A_1 A_2}{A_1 B_2 - A_2 B_2} \left\{ \pi e^{\frac{(\theta + N_0 K_1) B_1}{A_1}} \sqrt{\frac{A_1}{B_1}} \left[1 - \mathcal{Q} \left(\sqrt{\frac{(\theta + N_0 K_1) B_1}{A_1}} \right) \right] \right. \\ & \left. - \pi e^{\frac{(\theta + N_0 K_1) B_2}{A_2}} \sqrt{\frac{A_2}{B_2}} \left[1 - \mathcal{Q} \left(\sqrt{\frac{(\theta + N_0 K_1) B_2}{A_2}} \right) \right] \right\} \end{aligned} \quad (76)$$

5.5 Calculation of Integral I_3

Following the same approach and steps as for the calculation of integral I_2 , expressions for (57) can be found for the three different relationships between the arguments a , b , c , and d of (62) as follows:

- Case 1: $a = bc/d$

$$\begin{aligned} I_3 = & \frac{\sqrt{(\theta + N_0 K_2) \pi}}{B_0 A_3} + \left[1 - \mathcal{Q} \left(\frac{(\theta + N_0 K_2) B_3}{A_3} \right) \right] \\ & \times \frac{e^{\frac{(\theta + N_0 K_2) B_3}{A_3}} B_3 \pi (A_3 - 2(\theta + N_0 K_2) B_3)}{2B_0 (A_3 B_3)^{3/2}} \end{aligned} \quad (77)$$

- Case 2: $c = ad/b$

$$I_3 = \frac{\sqrt{(\theta + N_0 K_2)\pi}}{B_3 A_2} + \left[1 - \mathcal{Q}\left(\frac{(\theta + N_0 K_2)B_0}{A_2}\right) \right] \times \frac{e^{\frac{(\theta + N_0 K_2)B_0}{A_2}} B_0 \pi (A_2 - 2(\theta + N_0 K_2)B_0)}{2B_3(A_2 B_0)^{3/2}} \quad (78)$$

- Case 3: $bc \neq ad$

$$I_3 = \frac{A_2 A_3}{A_2 B_3 - A_3 B_3} \left\{ \pi e^{\frac{(\theta + N_0 K_2)B_0}{A_2}} \sqrt{\frac{A_2}{B_0}} \left[1 - \mathcal{Q}\left(\sqrt{\frac{(\theta + N_0 K_2)B_0}{A_2}}\right) \right] - \pi e^{\frac{(\theta + N_0 K_2)B_3}{A_3}} \sqrt{\frac{A_3}{B_3}} \left[1 - \mathcal{Q}\left(\sqrt{\frac{(\theta + N_0 K_2)B_3}{A_3}}\right) \right] \right\} \quad (79)$$

5.6 SEP of Multiple Relay CCRN

Similar as for the single relay scenario, using the CDF of the SINR derived for the multiple relay topology, the related SEP is obtained as

$$P_e^{NDL} = \frac{\eta\sqrt{\theta}}{2\pi} \sum_{k=0}^M \binom{M}{k} (-1)^k P_s \Omega_{g_1} P_r \Omega_{g_2} I'_2 \quad (80)$$

where

$$I'_2 = \int_0^\infty \frac{e^{-(\theta + k N_0 K_1)\gamma}}{(\gamma k P_p \Omega_{\beta_2} + P_r \Omega_{g_2}) (\gamma k P_p \Omega_{\beta_1} + P_s \Omega_{g_1}) \sqrt{\gamma}} d\gamma \quad (81)$$

Recalling (56), the integral I'_2 in (81) can be solved following similar steps as those used for solving I_2 . For this purpose, we substitute A_1, A_2, B_1, B_2 by A'_1, A'_2, B'_1, B'_2 , respectively, where $A'_1 = k P_p \Omega_{\beta_1}$, $A'_2 = k P_p \Omega_{\beta_2}$, $B'_1 = B_1 = P_s \Omega_{g_1}$, $B'_2 = B_2 = P_r \Omega_{g_2}$. The three cases to be distinguished here, then have the following solutions:

- Case 1: $a = bc/d$

$$I'_2 = \frac{\sqrt{(\theta + N_0 K_1)\pi}}{B_1 A'_2} + \left[1 - \mathcal{Q}\left(\frac{(\theta + N_0 K_1)B_2}{A'_2}\right) \right] \times \frac{e^{\frac{(\theta + N_0 K_1)B_2}{A'_2}} B_2 \pi (A'_2 - 2(\theta + N_0 K_1)B_2)}{2B_1(A'_2 B_2)^{3/2}} \quad (82)$$

- Case 2: $c = ad/b$

$$I'_2 = \frac{\sqrt{(\theta + N_0 K_1)\pi}}{B_2 A'_1} + \left[1 - \mathcal{Q} \left(\frac{(\theta + N_0 K_1) B_1}{A'_1} \right) \right] \times \frac{e^{\frac{(\theta + N_0 K_1) B_1}{A'_1}} B_1 \pi (A'_1 - 2(\theta + N_0 K_1) B_1)}{2 B_2 (A'_1 B_1)^{3/2}} \quad (83)$$

- Case 3: $bc \neq ad$

$$I'_2 = \frac{A'_1 A'_2}{A'_1 B_2 - A'_2 B_2} \left(\pi e^{\frac{(\theta + N_0 K_1) B_1}{A'_1}} \sqrt{\frac{A'_1}{B_1}} \left[1 - \mathcal{Q} \left(\sqrt{\frac{(\theta + N_0 K_1) B_1}{A_1}} \right) \right] - \pi e^{\frac{(\theta + N_0 K_1) B_2}{A'_2}} \sqrt{\frac{A'_2}{B_2}} \left[1 - \mathcal{Q} \left(\sqrt{\frac{(\theta + N_0 K_1) B_2}{A'_2}} \right) \right] \right) \quad (84)$$

6 Numerical results

This section provides numerical results for selected CCRN scenarios to illustrate the impact of the PU transmission on the SEP of the SU network with single relay, multiple relays, and different modulation schemes. The channel mean powers are set according to the exponential-decay path loss model. Specifically, we denote the distance between transmitting and receiving nodes as $d_{u,v}$ and formulate the corresponding channel mean power $\Omega_k \sim d_{u,v}^{-\mu}$ where μ is the path loss exponent, $k \in \{h, g_0, g_1, g_2, \alpha_1, \alpha_2, \alpha_{2i}, \beta_1, \beta_{1i}, \beta_2\}$, $u \in \{\text{SU-Tx}, \text{SR}, \text{PU-Tx}\}$, and $v \in \{\text{PU-Rx}, \text{SR}, \text{SU-Rx}\}$. We assume that the SR is placed half-way between the SU-Tx and the SU-Rx, i.e. $d_{\text{SU-Tx}, \text{SR}} = d_{\text{SR}, \text{SU-Rx}} = 1/2$. Assuming a shadowed urban cellular radio propagation environment, we set $\mu = 4$. Other system parameters are set as follows: PU outage constraint $\varepsilon = 0.01$, PU rate $r_p = 0.4$ bits/s/Hz, channel mean powers $\Omega_h = 16$, $\Omega_{g_0} = 1$, $\Omega_{g_1} = 16$, $\Omega_{g_2} = 16$, $\Omega_{\beta_1} = 0.5$, $\Omega_{\beta_2} = 0.5$, $\Omega_{\alpha_1} = 0.5$, $\Omega_{\alpha_2} = 0.5$, and SU peak transmit power $P_{\text{peak}} = 10$ dB.

Fig. 3 shows the SEP performance of a single relay CCRN without direct link as a function of the PU transmit SNR, P_p/N_0 , and for different modulation schemes. As expected, binary phase shift keying (BPSK) modulation outperforms all considered higher order modulation schemes, i.e. quadrature phase shift keying (QPSK), 8-PSK, and 16-PSK. In particular, for 8-PSK and 16-PSK, the SEP of the secondary network is beyond a level that could be coped with by using a reasonable amount of error control coding in terms of complexity. Another interesting observation is the progression of the SEP curves, i.e. initially decreasing SEP with increasing PU transmit SNR until

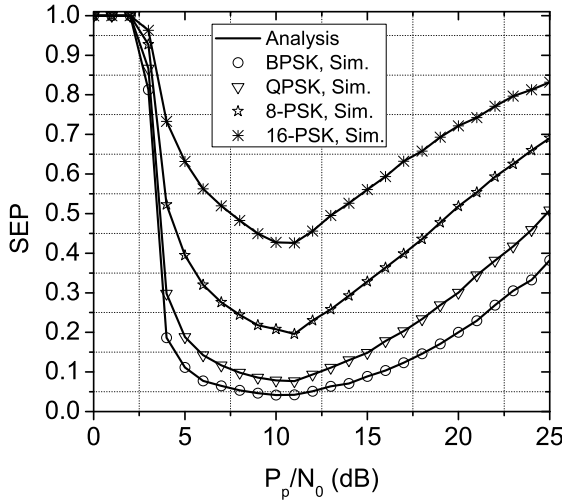


Figure 3: Symbol error probability of a single relay CCRN without direct link for different modulation schemes.

a minimum value is reached and then increasing SEP with further increasing the PU transmit SNR. This behavior is attributed to the power allocation policies of the SU-Tx and SR as introduced in Section 3. Basically, the SU-Tx transmit power can follow the increase of PU transmit power until the peak transmit power limit of the SU-Tx is reached. Once this limit has been reached, the PU transmission becomes a severe source of interference to the secondary network and degrades its SEP performance.

Given the results above, Fig. 4 now shows the SEP performance for the same single relay CCRN but with direct link as a function of the PU transmit SNR, P_p/N_0 , for different modulation schemes. In this case, selection combining can be applied as signals from two links are available at the SU-Rx, i.e. relaying link and direct link. Although selection combining improves the SEP performance for all modulation schemes considered, the gain is rather minor. Further improvement may be obtained by placing the SR at a more beneficial location in an attempt to improve the channel mean power of the relaying link. Another option is to add additional secondary relays to form a multiple relay CCRN which is considered in the results that are subsequently reported.

Figs. 5, 6, 7, and 8, respectively, show the SEP performance for the case

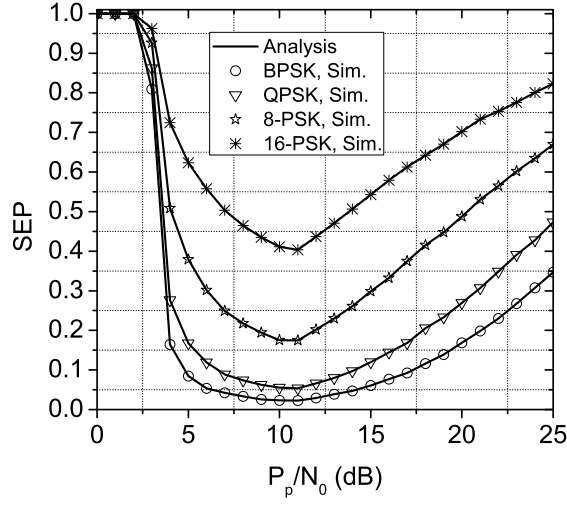


Figure 4: Symbol error probability of a single relay CCRN with direct link for different modulation schemes.

that the secondary network employs multiple relays, $M = 1, 2, 3$ and for BPSK, QPSK, 8-PSK, and 16-PSK modulation. It is assumed that a direct link is not present as the preceding results have indicated that only minor performance improvements are obtained through the availability of a direct link. As can be seen from these figures, increasing the number M of secondary relays indeed significantly improves the SEP performance of the CCRN for the different modulation schemes. In particular, for the mid-range of PU transmit SNR, the CCRNs with three SRs offer an SEP performance for BPSK, QPSK, and to some extent also for 8-PSK modulation that can be coped with by applying error control coding. However, it is observed that the range of PU transmit SNR for which the SEP curves progress rather flat, becomes more narrow for the higher order modulation schemes, i.e. ranges between around 5 dB to 15 dB for BPSK and QPSK, 7.5 dB to 12.5 dB for 8-PSK, and 10 dB to 11 dB for 16-PSK. It should be noted that further performance improvements may be obtained by replacing selection combining by maximal ratio combining which uses a suitable weighting to accumulate the information obtained over each relaying path at the SU-Tx rather than selecting the strongest path. However, this option is out of the scope of this article and will be studied in our future research.

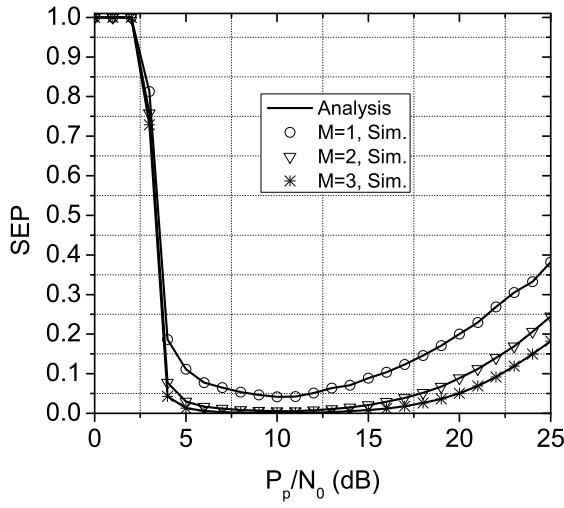


Figure 5: Symbol error probability for multiple relay CCRNs without direct link for BPSK.

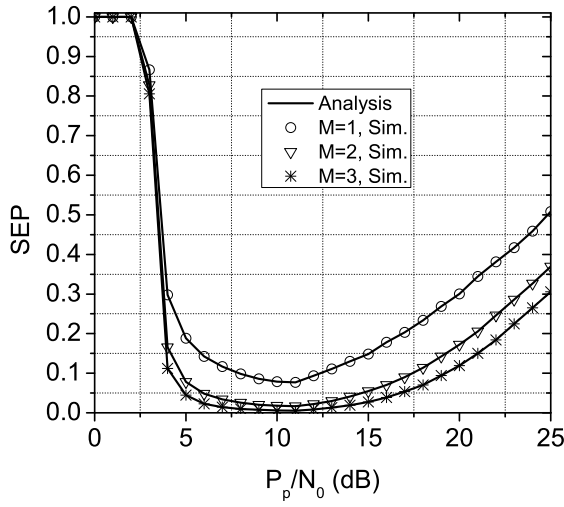


Figure 6: Symbol error probability for multiple relay CCRNs without direct link for QPSK.

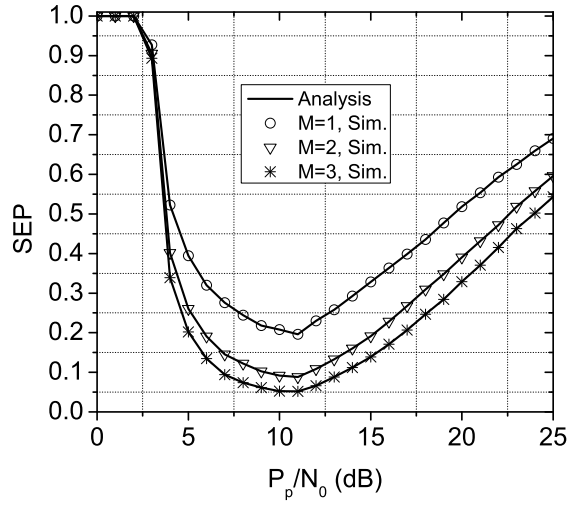


Figure 7: Symbol error probability for multiple relay CCRNs without direct link for 8-PSK.

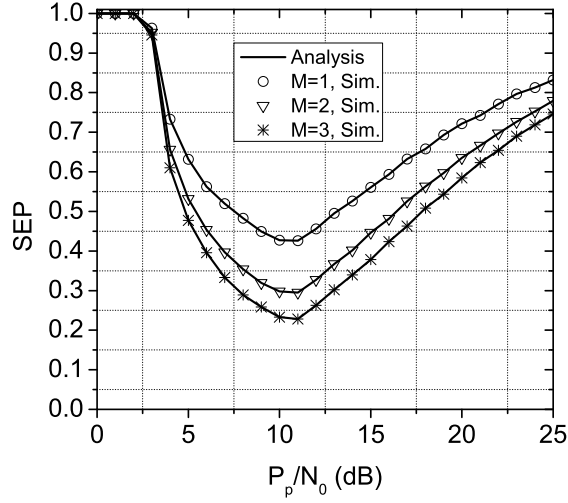


Figure 8: Symbol error probability for multiple relay CCRNs without direct link for 16-PSK.

7 Conclusions

In this article, we have analyzed the symbol error probability of secondary networks assisted by DF relaying subject to the outage constraint of the PU and the peak transmit power constraint of the SUs. Analytical expressions for the symbol error probability of CCRNs with single relay with direct link, single relay without direct link, and multiple relays without direct link have been derived. For this purpose, the power allocation policies of the SUs have been provided considering the PU outage constraint and the SU peak transmit power constraint. Furthermore, the CDFs of the SINRs related to the different topologies have been derived as key functions that are needed to derive the SEP expressions. Numerical examples have been provided to illustrate the impact of the PU transmission on the performance of the secondary network. The results show that having a direct link provides only minor improvements in SEP performance while increasing the number of secondary relays in combination with selection combining is much more powerful in improving SEP performance for the CCRNs considered and modulation schemes used. In order to further improve SEP performance, our future research will consider replacing selection combining with maximal ratio combining to take advantage of the information of all signals arriving from each relaying path at the secondary receiver.

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Part V

PART V

On Power Consumption of Wireless Sensor Nodes with Min(N,T) Policy in Spectrum Sharing Systems

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On Power Consumption of Wireless Sensor Nodes with $\min(N, T)$ Policy in Spectrum Sharing Systems

Charles Kabiri, Hans-Jürgen Zepernick, and Hung Tran

Abstract

In this paper, we analyze the power consumption of wireless sensor nodes with $\min(N, T)$ policy and M/G/1 queue in the presence of Nakagami- m fading. In particular, this system setting is applied to a wireless sensor node operating in a cognitive radio system as secondary user in the presence of a primary user. As such, not only the queue policy influences the power consumption but also the interference power constraint imposed on the wireless sensor node by the primary user. Thus, a queued sleep/wake-up strategy is analyzed in order to mitigate the average power consumption of a sensor node using $\min(N, T)$ policy in the context of an M/G/1 queue and a spectrum sharing environment in the presence of signal fading. Numerical examples are presented to illustrate the impact of queuing parameters and fading channel on the power consumption of a wireless sensor node.

1 Introduction

New applications in the area of wireless sensor networks such as healthcare, environmental monitoring, disaster management, target surveillance and industrial control, are characterized by high data rates and low latency. Some of these applications will require communication over shared spectrum. This is due to the high density of deployed wireless sensor networks (WSNs) and the fact that it is often not economical to allocate a dedicated spectrum band to WSNs. However, sharing a spectrum band with a primary (licensed) system

compromises the reliability and performance of both systems due to interference among the systems. As such, limited bandwidth, computing capacity, data delivery delay, severe energy constraints and severe fading conditions can make the design of WSNs more challenging [1].

Apparently, the operational lifetime of sensor nodes is an important issue in WSNs. As the lifetime of a sensor node is related to its energy consumption, sleep/wake-up schemes and energy efficient routing protocols are two prominent techniques used in WSNs to enhance lifetime [1]. In [2], sleep/wake-up strategies in cognitive radio-based wireless sensor networks are evoked. As most of the energy in sensor nodes is consumed during packet transmission, queuing theory can be an attractive tool to analyze and design efficient power allocation schemes. Furthermore, queuing performance measures in spectrum sharing systems have attracted considerable attention in recent work. In spectrum sharing scenarios, the authors of [3] investigated the queueing behavior of a secondary user (SU) communicating over the channel licensed to the primary user (PU).

In queued sleep/wake-up schemes, to wake up a node from an idle state to the busy state means turning on the radio server and is dealt with by duty cycle schemes [4]. In the context of medium access control (MAC) protocols, the respective sleep/wake-up strategy plays a key role in the level of power savings in sensor nodes. Some MAC protocols such as IEEE 802.11, sensor MAC (S-MAC), sparse topology and energy management (STEM), or Berkeley (B-MAC) use the concept of queued wake-up where a threshold is used to control the average time of turning on the data radio and the latency for the buffered data packets. In this regards, the sensor node triggers the data radio only when the queue holds N packets. The queued packets are transmitted thereafter, in a burst, as soon as the access to the transmission medium is established. It is an interesting question to find the optimal value of N in order to minimize the power consumption. This can be attained from the so called N -policy [5].

The results reported in [4] provide a feasible power-saving technique using the N policy along with an M/M/1 queue, i.e. a single server where arrivals follow a Poisson process and service time is exponentially distributed. A threshold, N , is specified and used to control the average time of turning on the radio transmitter for the buffered data packets. In other words, when the queue holds N packets, the sensor triggers the radio transmitter to start the transmission for the queued packets in a burst. Although the N -policy provides energy savings, a long waiting time may be caused as packets are buffered until the threshold is reached. To prevent such delays, the T -policy has been proposed to deal with the case that the number of queued packets does not reach the threshold N . The T -policy consists of triggering the

radio as soon as the waiting time after the last busy period has reached a given threshold T . A combination of these two policies can be formulated as $\min(N, T)$ operation [6], [7].

Other works related to energy-efficiency in sensor networks are presented in [8], [9] and the references therein. In [9], a discrete-time Markov model is used that allows to study the trade-off between energy consumption and transfer delay in clustered WSNs. Queue aware threshold control policies for the scenarios with infinite and finite buffer have been proposed in [10].

In this paper, to analyze system performance, we consider a spectrum sharing system utilizing an M/G/1 queueing model, i.e. single server with Poisson arrival, service time following a general distribution, and $\min(N, T)$ policy. The contributions of this paper are as follows: 1) We derive an analytical model of a sensor node that uses the $\min(N, T)$ policy and operates in an underlay spectrum sharing environment. 2) Modeling the sensor node as an M/G/1 queue, we assess the power consumption in the idle and busy state. 3) Different to [11], where an error free wireless channel is assumed, we include the effect of channel fading into the analysis.

The rest of the paper is organized as follows. Section 2 describes the system model along with the queuing policy. Section 3 provides the performance analysis by deriving the first and second moment of service time of data packets and a queuing model with $\min(N, T)$ policy used for the power consumption function. In Section 4, numerical results of the power consumption function are presented. Finally, Section 5 concludes the paper.

2 System Model

Let us consider a spectrum sharing scenario as shown in Fig. 1 where a WSN operates in the presence of the primary receiver (PU-Rx) of the primary network. The secondary network comprises of wireless sensor nodes, i.e. a secondary transmitter (SU-Tx) and a secondary receiver (SU-Rx). The data packets arriving and queued in the SU-Tx, which acts as single server, are served in first-in first-out (FIFO) order. We assume that the arrival rate λ at the SU-Tx follows a Poisson process and that service time has a general distribution. In order to cater for a wide range of fading conditions, we assume that packet transmission is subject to Nakagami- m fading where m denotes the fading severity parameter.

2.1 Peak interference power constraint

In underlay cognitive radio networks, the secondary transmitter must adjust its power to avoid harmful interference to the primary receiver. Then, the

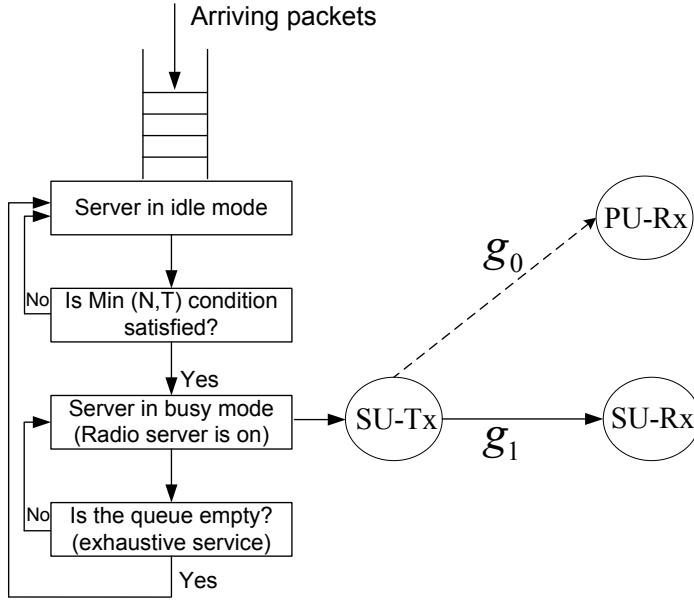


Figure 1: System model of a wireless sensor network with an M/G/1 queue and $\min(N, T)$ policy at the transmitting sensor node that operates in a spectrum sharing environment with a primary receiver.

packet transmission time between SU-Tx and SU-Rx in the secondary network can be determined by

$$T_L = \frac{1}{b \log_2(1 + \gamma_s)} \quad (1)$$

where $b = B/L$, B is the system bandwidth, L is the number of bits per packet, and γ_s is the signal-to-noise ratio (SNR) at the SU-Rx. Given the maximum allowed interference power at the PU-Rx as Q , then the SNR in (1) can be expressed as

$$\gamma_s = \frac{g_1 Q}{g_0 N_0} \quad (2)$$

g_0 and g_1 are the channel power gains of the SU-Tx→PU-Rx and SU-Tx→SU-Rx links, respectively, following the Gamma distribution with unit mean. Further, N_0 denotes the variance of the additive white Gaussian noise at the SU-Rx.

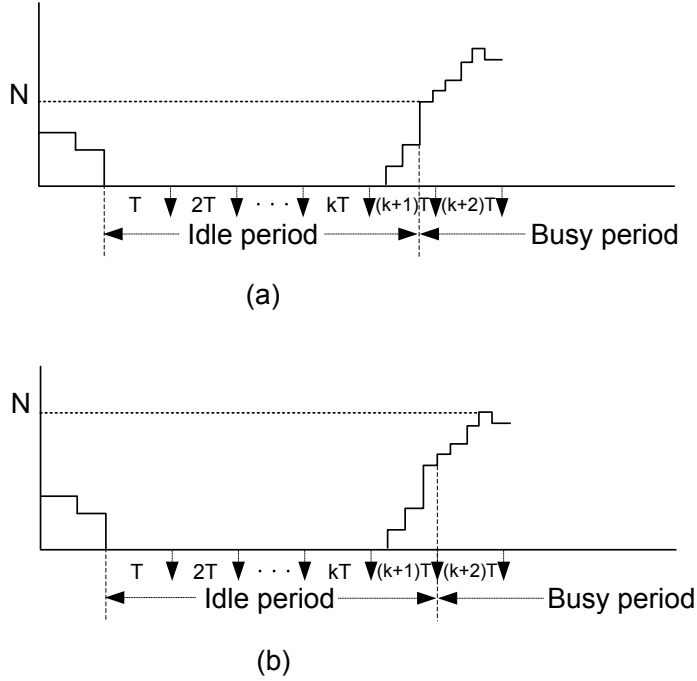


Figure 2: Scenarios of transmitting service started by: (a) N -policy condition, (b) T -policy condition.

2.2 Delay constraint

For simplicity and to reduce further scheduling, we consider a simple stop-and-wait automatic repeat request protocol in the secondary network. As such, if a packet has been successfully received by the SU-Rx, an acknowledgement (ACK) is sent to the SU-Tx. Otherwise, a negative ACK (NACK) is sent which indicates a retransmission request to the SU-Tx. Here, we assume that the ACK/NACK are transmitted over an error-free feedback channel with negligible delay. In order to limit the number of retransmissions, a packet is considered as being timed out and hence being dropped, if it has not been successfully received within a given period or service time T_L . As such, a packet is considered as being received successfully, if the following delay constraint is fulfilled:

$$T_L < t_{out} \quad (3)$$

2.3 $\min(N, T)$ policy

In this work, we allow the sensor node to be in idle state (monitoring) or busy state (transmitting) as illustrated in Fig. 2 [12]. In the idle state, the radio transmitter functionality is switched off to conserve energy while it is switched on in the busy state. In general, the N and T policy switches off the radio transmitter of the server (idle) when accumulating N data packets in the queue and until reaching T time units. Once N packets are in the queue or the time T is reached, the radio transmitter of the server is switched on (busy) and the packets in the queue are transmitted in a burst [7][12].

3 Performance Analysis

3.1 First and second moment of service time T_L

In order to derive the moments of T_L , we first need to obtain the cumulative distribution function (CDF) of the SNR γ_s at the SU-Rx which can be formulated as

$$F_{\gamma_s}(\gamma) = \Pr \left\{ \frac{g_1 Q}{g_0 N_0} < \gamma \right\} \quad (4)$$

According to [13], the CDF of γ_s can be derived as

$$F_{\gamma_s}(\gamma) = \left(\frac{m_1}{m_0} \right)^{m_1} \left(\frac{N_0}{Q} \right)^{m_1} \frac{\Gamma(m_1) \gamma^{m_1}}{\mathcal{B}(m_0, m_1)} \times {}_2F_1 \left(m_1, m_0 + m_1; 1 + m_1; -\gamma \frac{m_1 N_0}{m_0 Q} \right) \quad (5)$$

where m_0 and m_1 are the fading severity parameters of links SU-Tx→PU-Rx and SU-Tx→SU-Rx, respectively. Furthermore, $\Gamma(\cdot)$ is the gamma function defined as $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ [14, eq. (8.310.1)], ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ stands for the hypergeometric function [14, eq. (3.194.5)] and $\mathcal{B}(a, b)$ is the beta function [14, eq.(8.384.1)] defined as

$$\mathcal{B}(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \quad (6)$$

On the other hand, the CDF of T_L can be written as

$$F_{T_L}(t) = \Pr(T_L < t) = 1 - \Pr(T_L \geq t) \quad (7)$$

Substituting (1) into (7) along with some algebraic manipulation, the CDF of T_L can be rewritten as

$$F_{T_L}(t) = 1 - F_{\gamma_s} \left(2^{\frac{1}{bt}} - 1 \right) \quad (8)$$

where $F_{\gamma_s}(\gamma)$ is the CDF determined in (5). Then differentiating (8) with respect to t , the probability density function (PDF) of T_L can be determined as

$$\begin{aligned} f_{T_L}(t) &= \frac{dF_{T_L}(t)}{dt} \\ &= 1/[bm_0t^2\mathcal{B}(m_0, m_1)] \\ &\times 2^{\frac{1}{bt}} m_1 \left[\frac{\left(2^{\frac{1}{bt}} - 1\right) m_1}{m_0} \right]^{-1+m_1} \ln 2 \\ &\times \left(\frac{N_0}{Q}\right)^{m_1} \left[\frac{\left(2^{\frac{1}{bt}} - 1\right) m_1 N_0 + m_0 Q}{m_0 Q} \right]^{-m_0-m_1} \end{aligned} \quad (9)$$

Let us recall that timeout occurs when packet transmission time is greater or equal to a given threshold t_{out} . In this case, a packet is dropped and the related timeout probability is formulated as

$$\begin{aligned} P_{out} &= \Pr(T_L \geq t_{out}) = 1 - F_{T_L}(t_{out}) \\ &= F_{\gamma_s}\left(2^{\frac{1}{bt_{out}}} - 1\right) \end{aligned} \quad (10)$$

On the other hand, the PDF representing the case of packets not being timed out can be written as[15]

$$f_{T_{L,suc}}(t) = \begin{cases} \frac{f_{T_L}(t)}{1-P_{out}} & \text{for } 0 \leq t < t_{out} \\ 0 & \text{for } t \geq t_{out} \end{cases} \quad (11)$$

Therefore, the first and second moment of the transmission time for packets that are not dropped, i.e., $T_{L,suc} = \{T_L | T_L < t_{out}\}$ can be written as

$$\begin{aligned} E\{T_{L,suc}\} &= \int_0^{t_{out}} t f_{T_{L,suc}}(t) dt \\ &= \frac{1}{1-P_{out}} \int_0^{t_{out}} t f_{T_L}(t) dt \end{aligned} \quad (12)$$

$$\begin{aligned} E\{T_{L,suc}^2\} &= \int_0^{t_{out}} t^2 f_{T_{L,suc}}(t) dt \\ &= \frac{1}{1-P_{out}} \int_0^{t_{out}} t^2 f_{T_L}(t) dt \end{aligned} \quad (13)$$

After performing some derivations, it can be shown that the first moment of $T_{L,suc}$ is given by

$$E\{T_{L,suc}\} = \left(\frac{N_0 m_1}{Q}\right)^{m_1} \frac{m_0^{m_0} Q^{m_0+m_1}}{(1 - P_{out}) \mathcal{B}(m_0, m_1)} \Phi_1 \quad (14)$$

where Φ_1 is defined as

$$\Phi_1 = \int_{\alpha}^{\infty} \frac{(u-1)^{m_1-1} u \ln u}{[m_0 Q + m_1 N_0 (u-1)]^{m_0+m_1}} du \quad (15)$$

and $\alpha = 2^{1/bt_{out}}$. Similarly, the second moment of $T_{L,suc}$ can be obtained as

$$E\{T_{L,suc}^2\} = \left(\frac{N_0 m_1}{Q}\right)^{m_1} \frac{m_0^{m_0} Q^{m_0+m_1} \ln 2}{b(1 - P_{out}) \mathcal{B}(m_0, m_1)} \Phi_2 \quad (16)$$

where Φ_2 is given by

$$\Phi_2 = \int_{\alpha}^{\infty} \frac{(u-1)^{m_1-1} u}{[m_0 Q + m_1 N_0 (u-1)]^{m_0+m_1}} du \quad (17)$$

Let us return to service time T_L , which is the time needed by the server to process one packet. Without considering timeout, the mean service time can be calculated as in (14). The second moment of service time without considering timeout, can be calculated from (16). However, if dropped packets are included, mean service time of a packet is given by [15]

$$E\{T_L\} = (1 - P_{out})E\{T_{L,suc}\} + t_{out}P_{out} \quad (18)$$

and the second moment of the service time including dropped packets is given by

$$E\{T_L^2\} = (1 - P_{out})E\{T_{L,suc}^2\} + t_{out}^2 P_{out} \quad (19)$$

3.2 $\min(N, T)$ policy applied to the queuing system

With reference to Fig. 2, let the random variable K_i , $i = 1, 2, \dots$, be the inter-arrival times of packets which are independent, identically, and exponentially distributed with mean arrival rate λ . Further, let the random variable A_n , $n = 1, 2, \dots$, denote the arrival times, i.e. the epoch when the n th packet arrives:

$$A_n = \sum_{i=1}^n K_i \quad (20)$$

which has the CDF [4]

$$F_n(t) = \Pr \{A_n \leq t\} = \int_{x=0}^t \frac{\lambda(\lambda x)^{n-1}}{(n-1)!} e^{-\lambda t} = \Pr \{N(t) \geq n\} \quad (21)$$

where $N(t)$ is the number of packets arriving at the system during $[0, t]$. Then, according to [7, eq. (6)], the expected number of packets L_{NT} for the min(N, T) policy and M/G/1 queue is given by

$$\begin{aligned} L_{NT} &= L_0 + \frac{E\{X(X-1)\}}{2E\{X\}} \\ &= \rho + \frac{\lambda^2 E\{T_L^2\}}{2(1-\rho)} + \frac{\sum_{n=1}^N (n-1)F_n(T)}{\sum_{n=1}^N F_n(T)} \end{aligned} \quad (22)$$

where L_0 is the expected number of packets present in an M/G/1 queue without min(N, T) policy, X is a random variable denoting the number of packets waiting in the queue buffer when the busy period begins, and $E\{X\} = \frac{\sum_{n=1}^N F_n(T)}{F_1(T)}$. The parameter $\rho = \lambda E\{T_L\}$ represents the system utilization.

3.3 Performance measures for min(N, T) policy with M/G/1

In the sequel, several performance metrics are provided that are then utilized to formulate the power consumption function.

3.3.1 Expected length of busy period

The expected length of the busy period, B_{NT} , can be obtained according to [6] as

$$E\{B_{NT}\} = \frac{\sum_{n=1}^N F_n(T)}{1 - e^{-\lambda T}} E\{B_0\} = \frac{E\{X\}E\{T_L\}}{1 - \rho} \quad (23)$$

where $E\{B_0\}$ represents the expected length of the busy period of an M/G/1 queue without min(N, T) policy and $E\{B_0\} = E\{T_L\}/(1 - \rho)$.

3.3.2 Expected length of idle period

According to [6], the expected length of the idle period, I_{NT} , is given by

$$E\{I_{NT}\} = \frac{\sum_{n=1}^N F_n(T)}{1 - e^{-\lambda T}} E\{I_0\} = \frac{E\{X\}}{\lambda} \quad (24)$$

where $E\{I_0\}$ is the expected idle period of an M/G/1 queue without $\min(N, T)$ policy. As the durations between two successive arriving packets are independent, identically, and exponentially distributed with mean $1/\lambda$, we have $E\{I_0\} = 1/\lambda$.

3.3.3 Expected length of the busy cycle

The expected length of the busy cycle, Ω_{NT} , can be obtained as [11, eq. (5)]

$$E\{\Omega_{NT}\} = E\{I_{NT}\} + E\{B_{NT}\} = \frac{E\{X\}E\{T_L\}}{1 - \rho} + \frac{E\{X\}}{\lambda} \quad (25)$$

3.3.4 Probability that the radio server is busy

This probability is equivalent to the proportion of time the radio server is busy and therefore given by

$$P_B = \frac{E\{B_{NT}\}}{E\{\Omega_{NT}\}} = \frac{E\{B_{NT}\}}{E\{B_{NT}\} + E\{I_{NT}\}} = \lambda E\{T_L\} = \rho \quad (26)$$

Note that (26) is independent of N and T and therefore the same as for an M/G/1 queue without $\min(N, T)$ policy.

3.4 Power consumption function

Let C_s denote the setup energy consumption factor. We assume that there is a fixed energy consumption incurred per busy cycle by switching from idle mode to busy mode and vice versa. Further, let C_h denote the holding power for each packet present in the system, C_b the power consumption while the radio server is in busy state, and C_i the power consumption while the radio server is in idle state. Then, the power consumption function, $P_C(N, T)$, can be given as [11]

$$P_C(N, T) = C_h L_{NT} + \frac{C_s}{E\{\Omega_{NT}\}} + C_b \frac{E\{B_{NT}\}}{E\{\Omega_{NT}\}} + C_i \frac{E\{I_{NT}\}}{E\{\Omega_{NT}\}} \quad (27)$$

Then, substituting (22), (23), (24), and (25) into (27) yields

$$\begin{aligned} P_C(N, T) = C_h \left[\rho + \frac{\lambda^2 E\{T_L^2\}}{2(1 - \rho)} + \frac{\sum_{n=1}^N (n - 1) F_n(T)}{\sum_{n=1}^N F_n(T)} \right] \\ + C_s \frac{\lambda(1 - \rho)}{E\{X\}} + C_b \rho + C_i (1 - \rho) \end{aligned} \quad (28)$$

4 Numerical results

In this section, we illustrate the progression of the power consumption function $P_C(N, T)$ when keeping T constant and varying N as well as when keeping N constant and varying T . The system parameters used in the analysis are provided in the legend of the figures. It is noted that the fading severity parameter is set to $m_0 = m_1 = 2$ for all links.

Fig. 3 shows that for a given threshold of waiting time T in the queue, the power consumption decreases steeply with the number of packets N increasing from 1 to 6 and then reaches a constant floor. As such, a maximum of $N = 6$ packets in the queue appears to be sufficient for reducing the power consumption of the wireless sensor node in this considered scenario.

Fig. 4 shows the progression of the power consumption function for fixed N and varying T . Clearly, if only a maximum of a single packet is queued, i.e. $N = 1$, then threshold T has no influence on the power consumption and the largest consumption is observed among the considered scenarios. However, choosing a favorable number of packets N to be accumulated in the queue

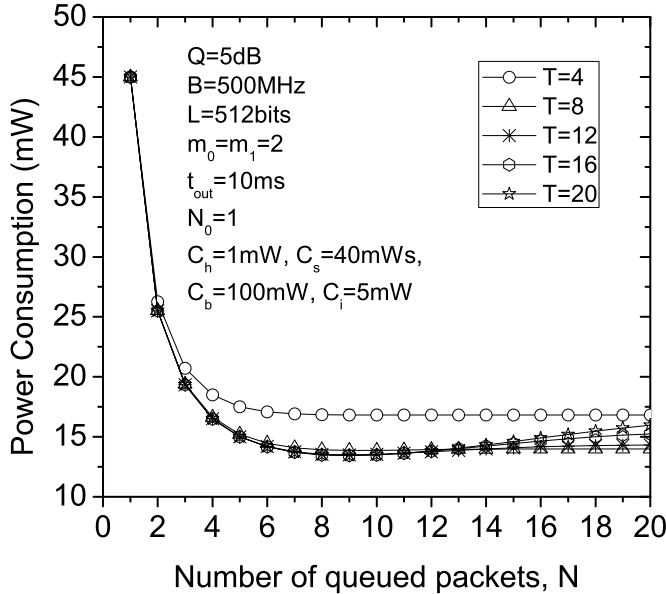


Figure 3: Power consumption function, T constant.

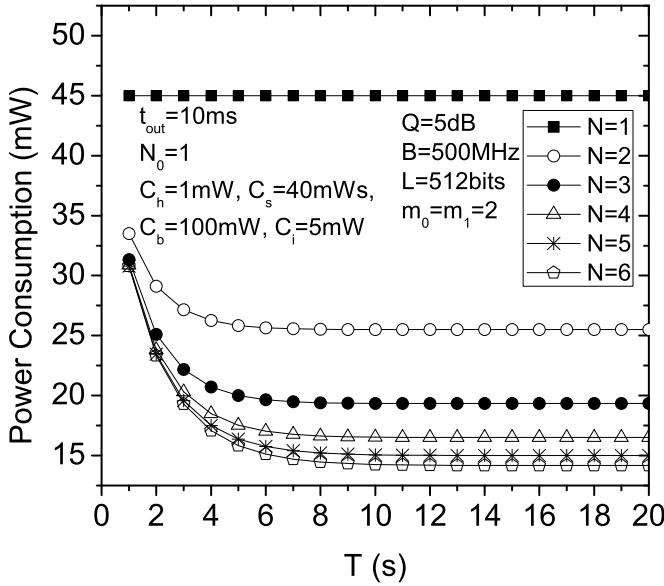


Figure 4: Power consumption function, N constant.

prior transmission, e.g. $N = 6$ or 7 , an increase of the threshold T to around 6 s will further decrease the power consumption while larger thresholds of T offer no significant power savings.

As a conjecture of the results shown in Fig. 3 and Fig. 4, the $\min(6,6)$ policy may be applied to this particular scenario in Nakagami- m fading with fading severity parameters set to $m_0 = m_1 = 2$ for all channels involved.

5 Conclusions

In this paper, we have analyzed the power consumption of wireless sensor nodes with $\min(N, T)$ policy and M/G/1 queue. In particular, a queued sleep/wake-up strategy is analyzed in order to mitigate the average power consumption of a wireless sensor node for use in a spectrum sharing environment subject to signal fading. The power consumption function that we obtained depends upon the first two moments of the service time of the data packets, queueing parameters, and inherently accounts for the Nakagami- m fading channel. The numerical examples illustrate how the parameters N and

T impact on the power consumption for a given fading scenario.

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ABSTRACT

Efficiently allocating the scarce and expensive radio resources is a key challenge for advanced radio communication systems. To this end, cognitive radio (CR) has emerged as a promising solution which can offer considerable improvements in spectrum utilization. Furthermore, cooperative communication is a concept proposed to obtain spatial diversity gains through relays without requiring multiple antennas. To benefit from both CR and cooperative communications, a combination of CR networks (CRNs) with cooperative relaying referred to as cognitive cooperative relay networks (CCRN) has recently been proposed. CCRNs can better utilize the radio spectrum by allowing the secondary users (SUs) to opportunistically access spectrum, share spectrum with primary users (PUs), and provide performance gains offered by cooperative relaying. In this thesis, a performance analysis of underlay CRNs and CCRNs in different fading channels is provided based on analytical expressions, numerical results, and simulations. To allocate power in the CCRNs, power allocation policies are proposed which consider the peak transmit power limit of the SUs and the outage probability constraint of the primary network. Thus, the impact of multiuser diversity, peak transmit power, fading parameters, and modulation schemes on the performance of the CRNs and CCRNs can be analyzed.

The thesis is divided into an introduction and five research parts based on peer-reviewed conference papers and journal articles. The introduction provides fundamental background on spectrum sharing systems, fading channels, and performance metrics. In the first part, a basic underlay CRN is analyzed where the outage probability and the ergodic capacity of the network over general fading channels are derived. In the second part, the outage probability and the ergodic capacity of an underlay CRN are assessed capturing the effect of multiuser diversity on the network subject to Nakagami- m fading. Considering the presence of a PU transmitter (PU-Tx), a power allocation policy is derived and utilized for CRN performance analysis under Rayleigh fading. In the third part, the impact of multiple PU-Txs and multiple PU receivers (PU-Rxs) on the outage probability of an underlay CCRN is studied. The outage constraint at the PU-Rx and the peak transmit power constraint of the SUs are taken into account to derive the power allocation policies for the SUs. In the fourth part, analytical expressions for the outage probability and symbol error probability for CCRNs are derived where signal combining schemes at the SU receiver (SU-Rx) are compared. Finally, the fifth part applies a sleep/wake-up strategy and the $\min(N;T)$ policy to an underlay CRN. The SUs of the network operate as wireless sensor nodes under Nakagami- m fading. A power consumption function of the CRN is derived. Further, the impact of M/G/1 queue and fading channel parameters on the power consumption is assessed.

